

How investment risk management is changing

Combining factor models and simulation

Room 5N.4.6

11:00 – 12:50 Friday 27 January 2017

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27 January 2017

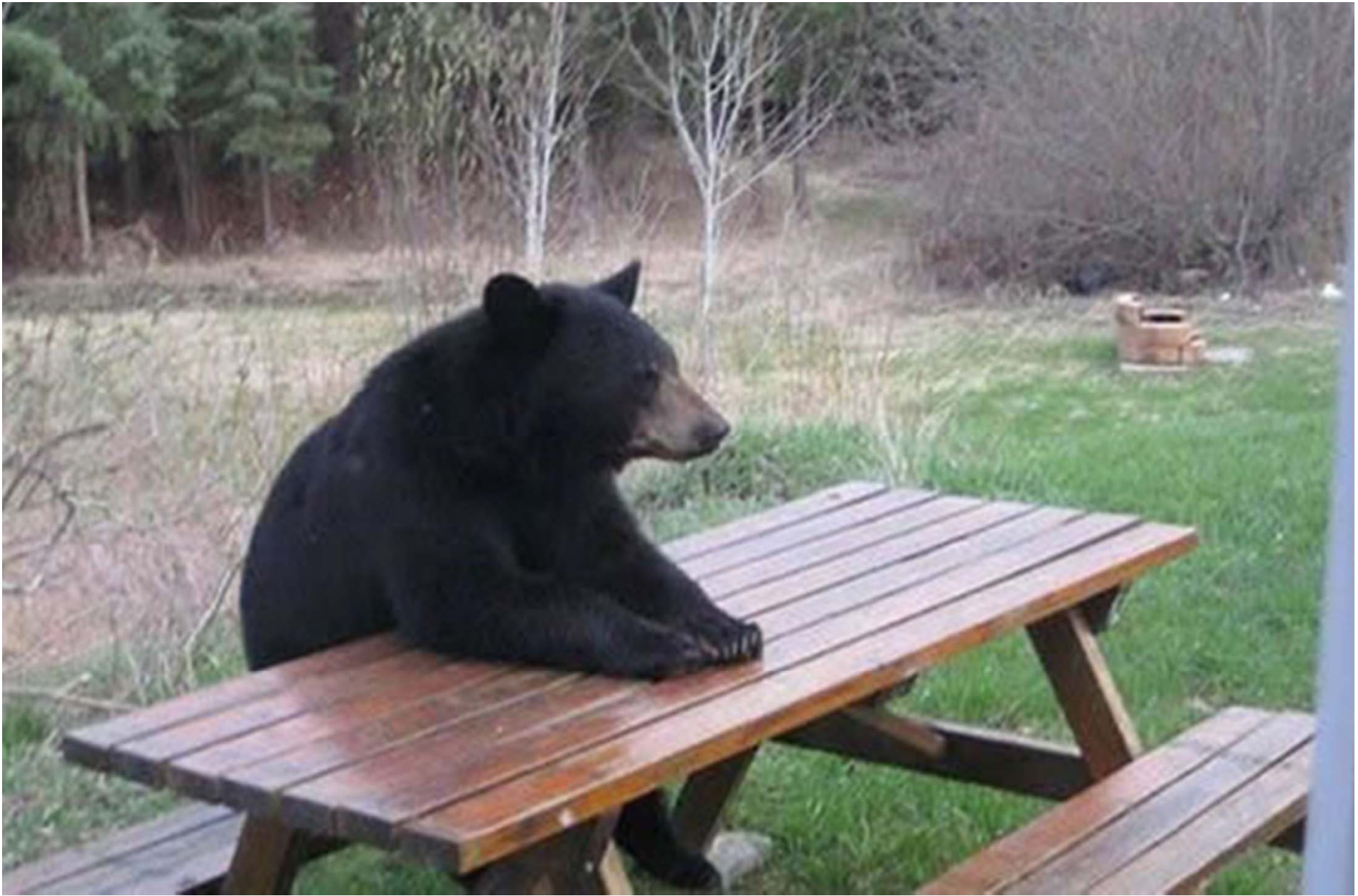
Empowering
the Financial World





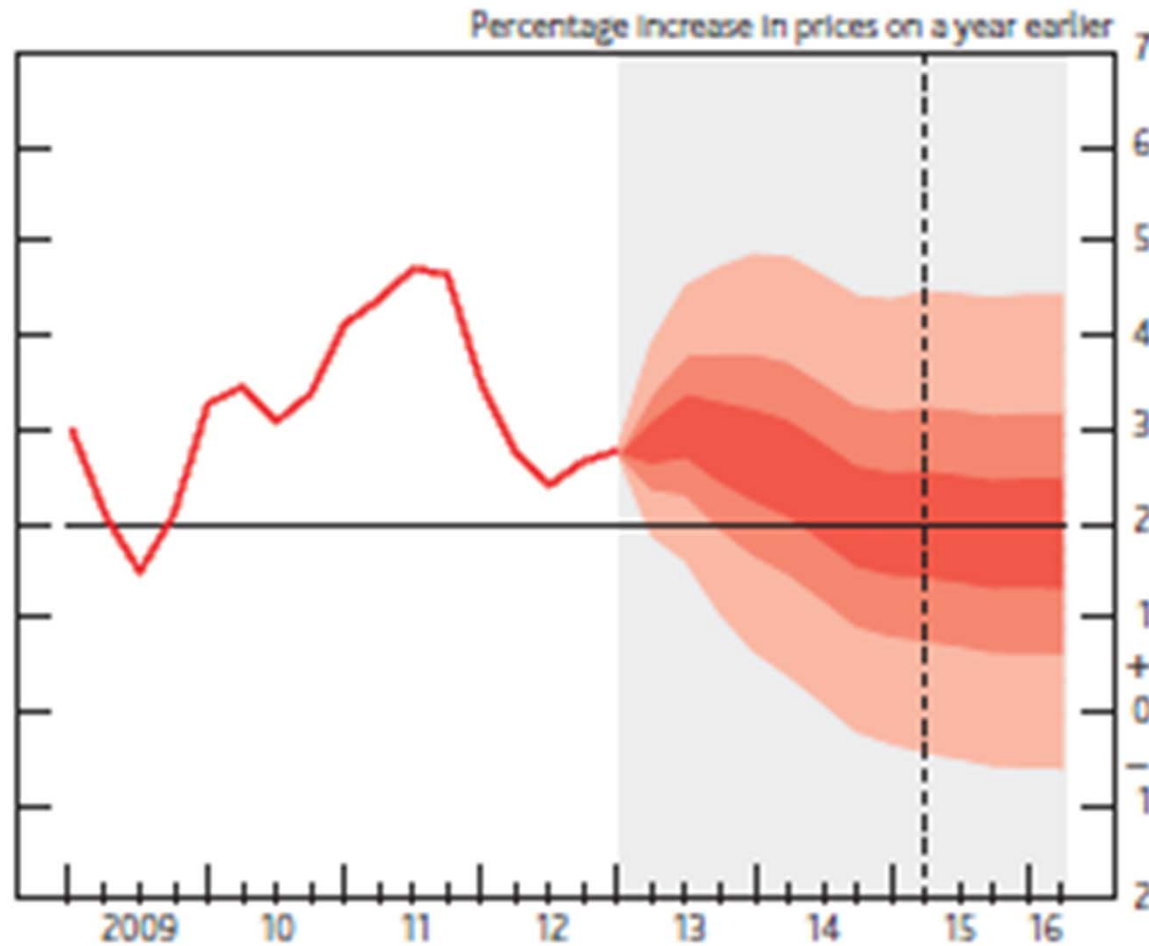
Lecture Plan

- **Overview – What I will talk about (5 min)**
 - Some generic observations, plus specifics about how risk management is actually practised today
 - Risk management is both a science and an art
 - I will also ask the question: how do we think about the future behaviour of markets?
- **Fundamentals of Market Risk**
 - Uncertainties and distributions (10 min)
 - Macro and Market Shocks (5 min)
- **How to estimate market risk?**
 - Factor Model approaches (10 min)
 - PCA and fundamental models (15 min)
- **Monte Carlo simulation approaches**
 - Basic ideas of Monte Carlo approaches (10 min)
 - Need to combine LFM and simulations (10 min)
- **Stress testing and scenario analysis**
 - Combining factor model and simulation approaches (15 min)
- **Where are we headed? (5 min)**
- **Questions and Conclusions (5 min)**

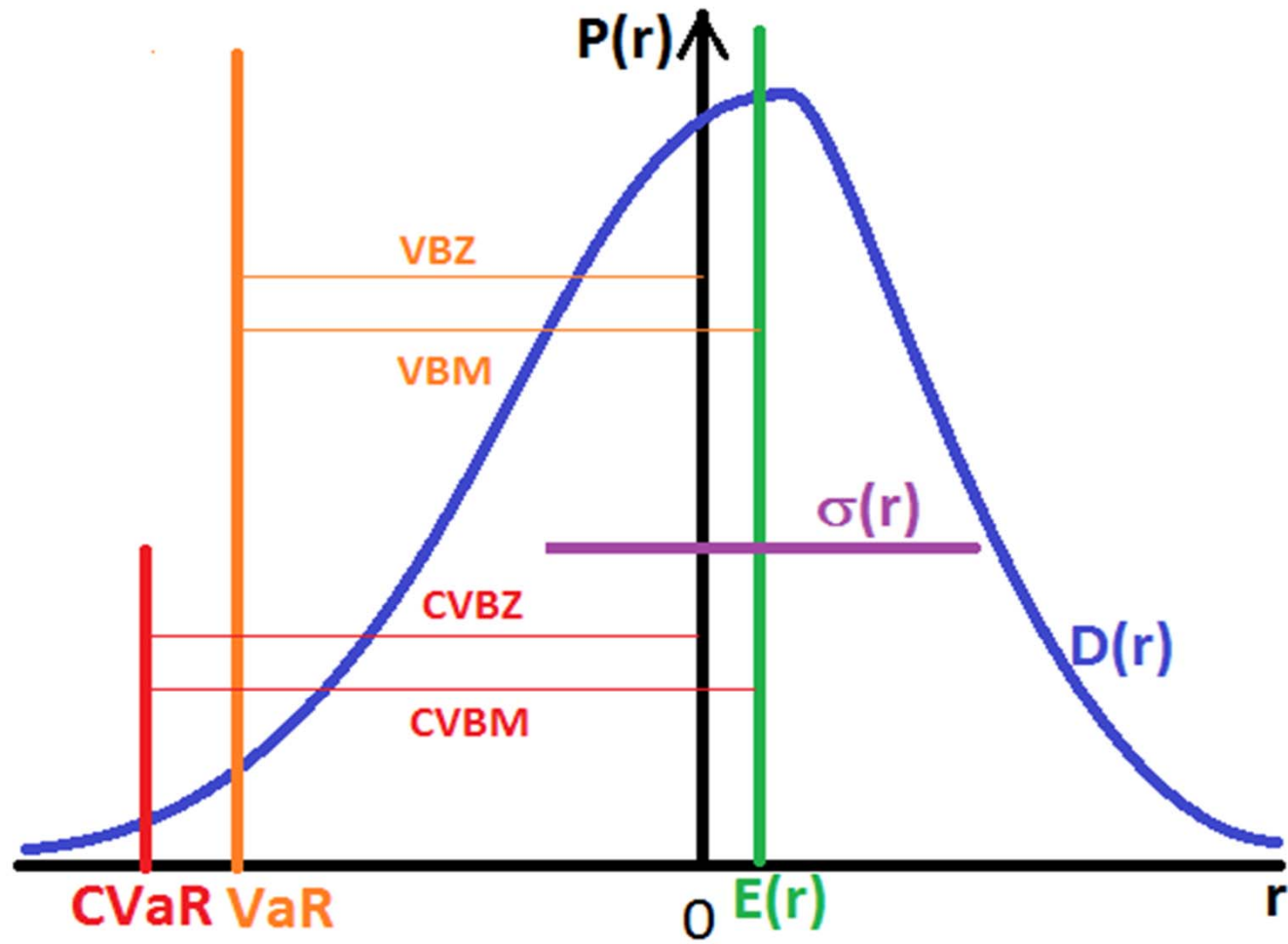


Definition of Investment (or Market) Risk

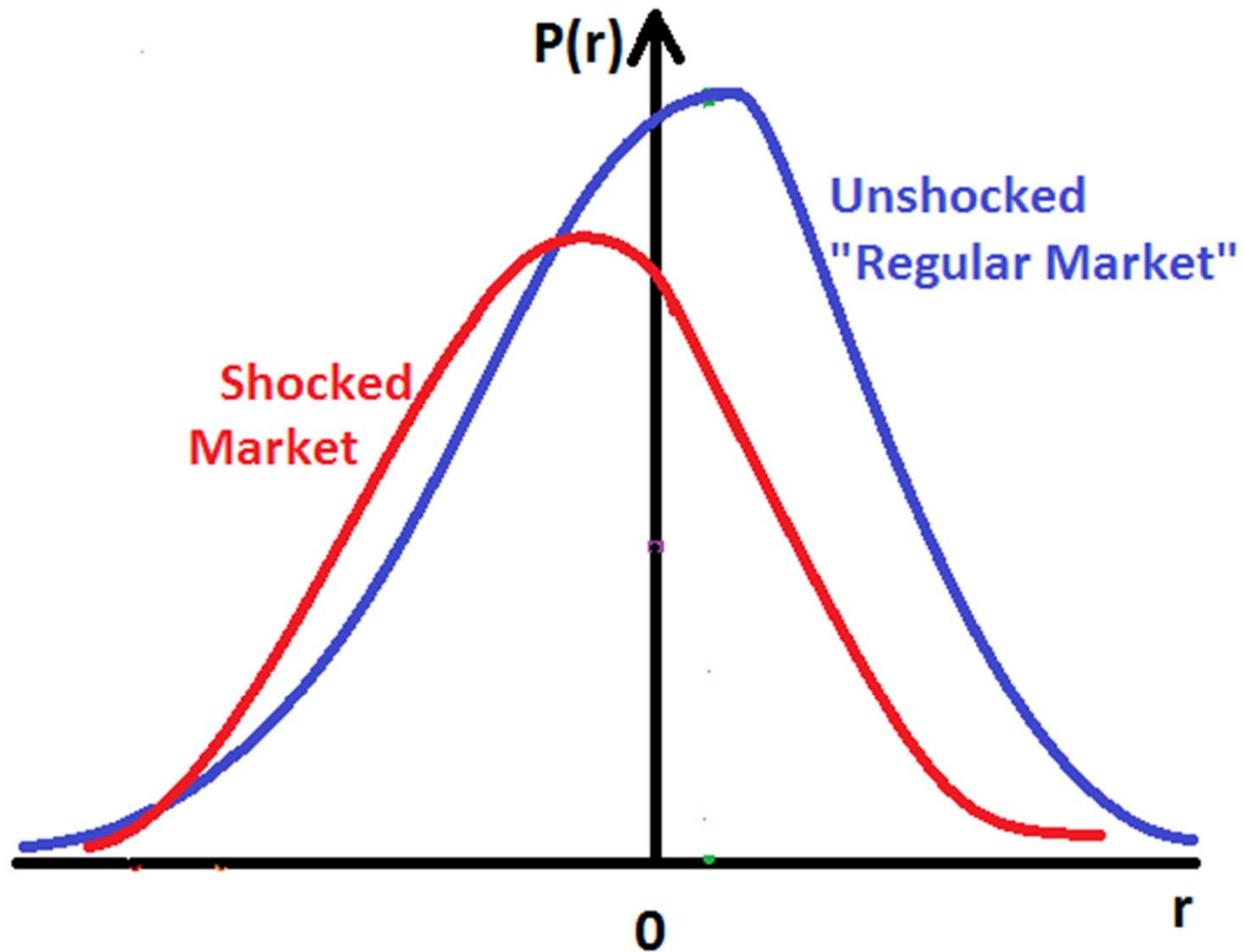
- **Market Risk** is the uncertainty in the future path of my wealth derived from the uncertainty in the future path of market prices



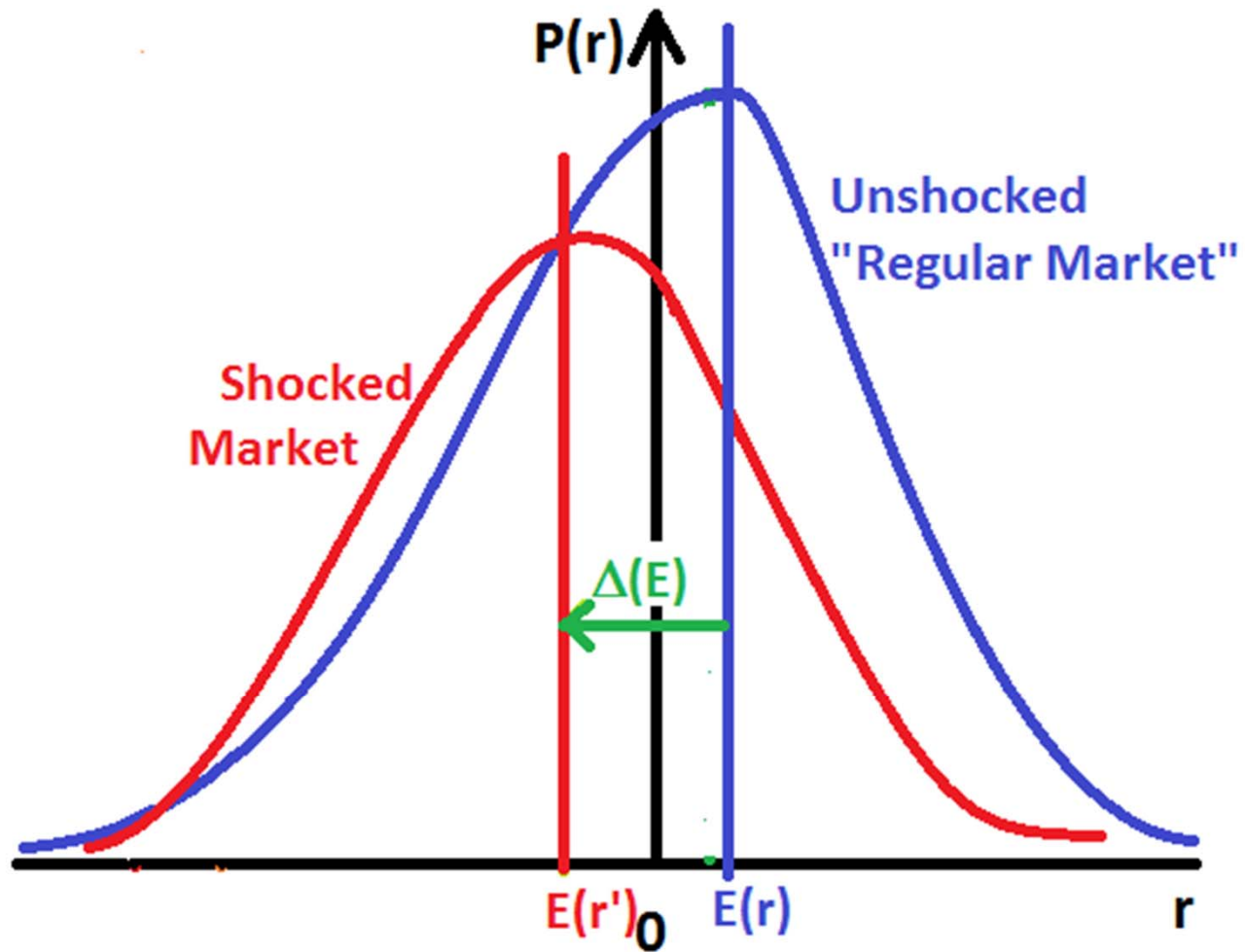
Statistics of a Return Distribution



Regular and Shocked Market Distributions



Expected Loss under Market Shock Scenario



Assumptions of Factor Modelling and Simulation

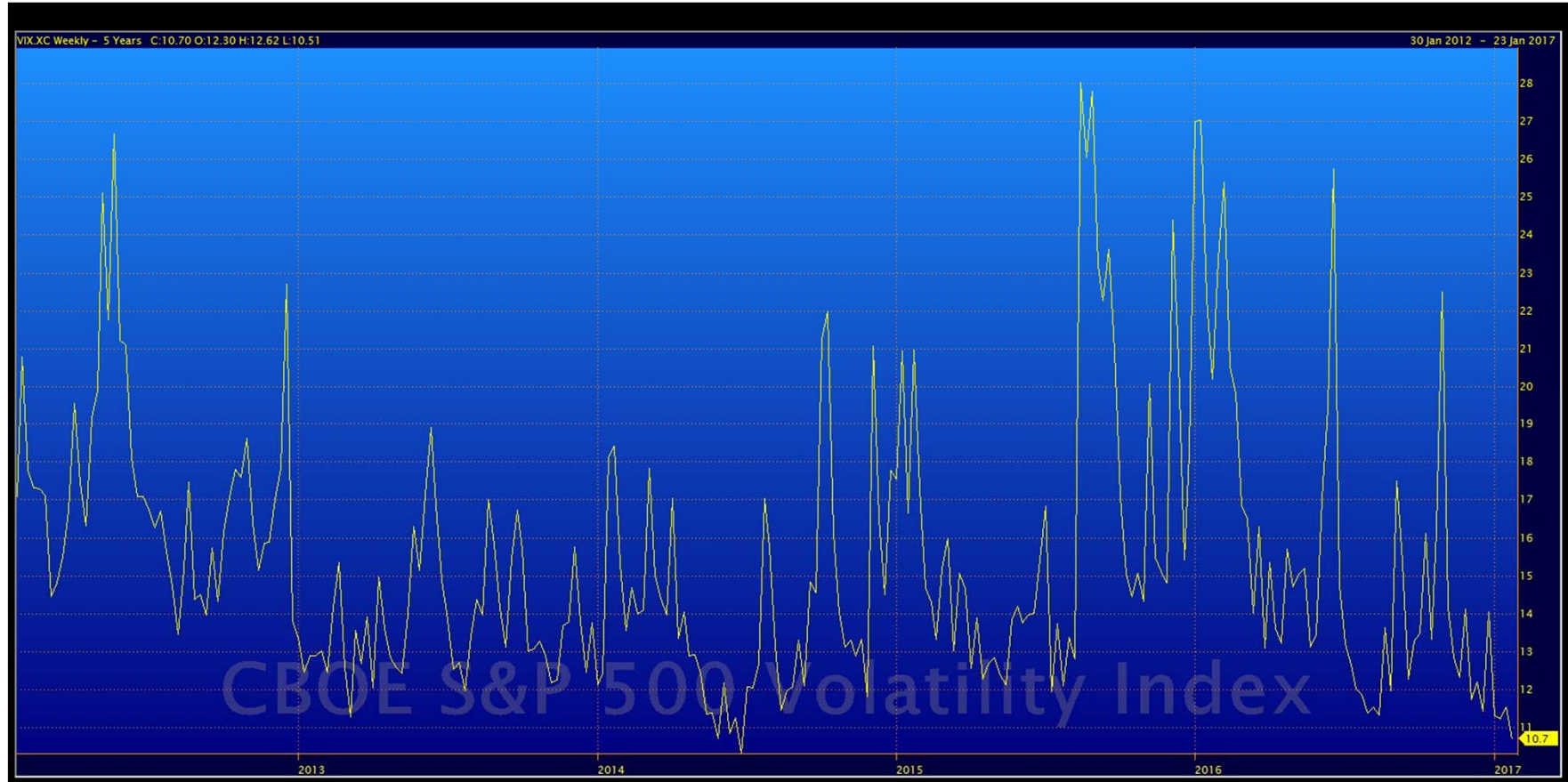
- **We do not know the shape of either the blue or the red distributions**
- **In practise many return distributions are skewed and “fat-tailed”**
- **Even if we had “complete” histories for every asset, these future distributions would still be unknowable**
 - Even if we assume stationary distributions, there are sampling errors associated with any historically-based risk modelling approach
 - Hence we make some modelling assumptions, both about the factor nature of market risk and about the distributions of returns to those factors
- **Multi-factor models are a well-established approach to estimating asset-level exposures to systematic risk factors, and hence providing intuition into risk forecasts**

Market Risk in practise

- In 2008 I was working at Deutsche Bank proprietary trading business, and the volatility of our fund went from about 15% to over 60% in just a few weeks
- We also lost EUR 120 Million during that time

- In 2012 a trader called “The London Whale” incurred \$6.2 billion in trading losses and his firm, JP Morgan, was later fined \$920 million.
- Jamie Dimon is the CEO of JP Morgan, one of the most respected people in finance:
- “In December 2011, as part of a firm-wide effort we instructed the team to reduce risk-weighted assets and associated risk. To achieve it, they embarked on a complex strategy that entailed adding positions that it believed would offset the existing ones. This strategy, however, ended up creating a portfolio that was larger and ultimately resulted in even more complex and hard-to-manage risks.”
- Later he said this was "the stupidest and most embarrassing situation I have ever been a part of."

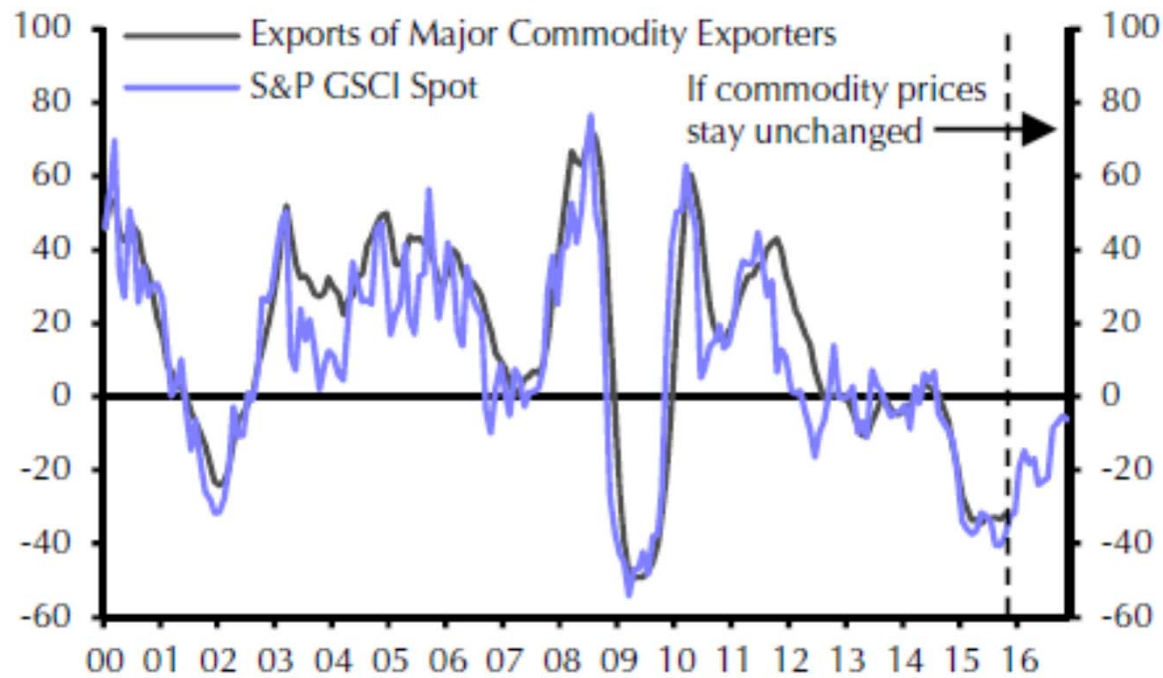
Volatility Shocks since 2012 – The VIX





Correlation of Macro and Financial Shocks

CHART 6: EXPORTS FROM MAJOR COMMODITY EXPORTERS
& GLOBAL COMMODITY PRICES (% Y/Y)



Sources – Thomson Datastream, CEIC, CE

How risk is driven by systematic factors

- **Investment risk is the uncertainty in the future path of my wealth, driven by the uncertainty in the future path of asset prices**
- **That uncertainty in the future path of prices is driven by shocks to expectations (“news”)**
 - Macro shocks – systematic
 - Policy shocks – systematic
 - Political shocks – partly systematic
 - Market-specific effects – partly systematic
 - Sector-specific news – partly systematic
 - Company announcements – mostly specific

Portfolio volatility depends on correlations

- The return of a portfolio in any period is a linear “weighted” combination of the returns to all the assets held

- $R_p = \langle w' \rangle \cdot \langle r \rangle = \sum (w_i * r_i)$

- Thinking of distributions, this means that the expected value of the portfolio return is simply the weighted combination of the expected returns of all the assets held, but the variance (square of the volatility) of the portfolio return depends upon a new set of parameters – the correlations (or covariances) of all assets to each other.
- We can represent all these parameters in a single matrix (the covariance or “risk” or “Markovitz” matrix) V defined by
- $V_{ij} = \text{covar}(r_i, r_j) = \text{corr}(r_i, r_j) * \sigma(r_i) * \sigma(r_j)$

Portfolio volatility depends on correlations

- Then we can write the portfolio variance (as an algebraic identity)
- $\sigma^2(R_p) = \langle w \rangle' V \langle w \rangle$
- This is the famous Markovitz risk term
- So we have turned a very difficult problem of statistics (estimating the distribution of a portfolio made up of large numbers of random variables), into a seemingly simple problem of linear algebra
- In 1955 this was a huge advance in thinking
- What's the catch?
 - There are a lot of correlations to estimate
 - Risk is more than just volatility
- It gets even better! If we assume a factor model of asset returns...

Multi Factor Models

$$r_i = \sum_{k=1}^m b_{ik} f_k + \epsilon_i$$

- » This multi-factor model assumes that asset systematic returns are driven by a common (shared) set of factors. This model can be applied across all asset classes.
- » The specific (“idiosyncratic” or “residual”) term is supposed to be uncorrelated to (“independent of”) the factor term
- » Thus we represent any asset (or portfolio) as “a bundle of exposures to risk factors” – we also call this the “risk profile” of the asset/portfolio
- » This is a very big assumption! But there is a very big payoff in terms of the mathematics of volatility

Mathematics of vol in Multi Factor Models

$$r_i = \sum_{k=1}^m b_{ik} f_k + \epsilon_i$$

- » Write this as $\langle r \rangle = B \cdot \langle f \rangle + \langle \epsilon \rangle$
- » where B is the matrix formed from the betas (asset exposures to factors)
- » Instead of asset covariances, now focus on factor covariances
- » If X is the matrix of factor covariances, we find
- » $\sigma^2(R_p) = \langle w \rangle \cdot V \cdot \langle w \rangle = \langle w \rangle \cdot [B^T X B + \Delta] \cdot \langle w \rangle$
- » $\sigma^2(R_p) = \langle w \rangle \cdot V \cdot \langle w \rangle = \langle w \rangle \cdot B^T X B \cdot \langle w \rangle + \langle w \rangle \cdot \Delta \cdot \langle w \rangle$

The Factor Model payoff

- What is the benefit of all of this maths?
- We can “explain” portfolio risk (variance) purely in terms of factors – no need to worry about asset-level covariances!
- $\sigma^2(R_p) = \langle w \rangle \cdot B^T X B \cdot \langle w \rangle + \langle w \rangle \cdot \Delta \cdot \langle w \rangle$
- $\sigma^2(R_p) =$ systematic (factor) risk term + specific (non-factor) risk term
- We can “attribute our risk” to the factor exposures we know about
- In 1975 this was a huge advance in thinking
 - There are no longer a lot of correlations to estimate
 - Still, risk is more than just volatility – that is why we will also discuss Simulation Methods

The method of Principal Components

- The method of principal components is a data-analytic technique that obtains linear transformations of a group of correlated variables such that certain optimal conditions are achieved.
- The most important of these conditions is that the transformed variables are uncorrelated
- I think of it as a way of separating “signal” from “noise”
- Reference: “*A User’s Guide To Principal Components*”, J Edward Jackson, Wiley (1991)

Eigenvalues and Principal Components

- The method of principal components is based on a key result from matrix algebra: Any $p \times p$ symmetric, nonsingular matrix, such as the covariance matrix \mathbf{S} , may be reduced to a diagonal matrix \mathbf{L} by pre-multiplying and post-multiplying it by a particular orthonormal matrix \mathbf{U} such that $\mathbf{U}'\mathbf{S}\mathbf{U} = \mathbf{L}$
 - The diagonal elements of \mathbf{L} , (l_1, l_2, \dots, l_p) are called the eigenvalues or characteristic roots of \mathbf{S} .
 - The columns of \mathbf{U} , $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_p$ are called the eigenvectors or characteristic vectors of \mathbf{S} .
 - Expressed as timeseries, the portfolios formed from the eigenvectors may be considered as transformed model factors

- Then the matrix of factor covariances $\mathbf{X} = \mathbf{I}$

- The algebraic expression for portfolio volatility is further simplified:

$$\sigma^2(R_p) = \langle \mathbf{w}' \rangle \cdot \mathbf{B}^T \mathbf{B} \cdot \langle \mathbf{w} \rangle + \langle \mathbf{w}' \rangle \cdot \Delta \cdot \langle \mathbf{w} \rangle$$

- This is the simplest possible representation of portfolio volatility, in terms of the betas to the “orthogonal basis” of PCA factors

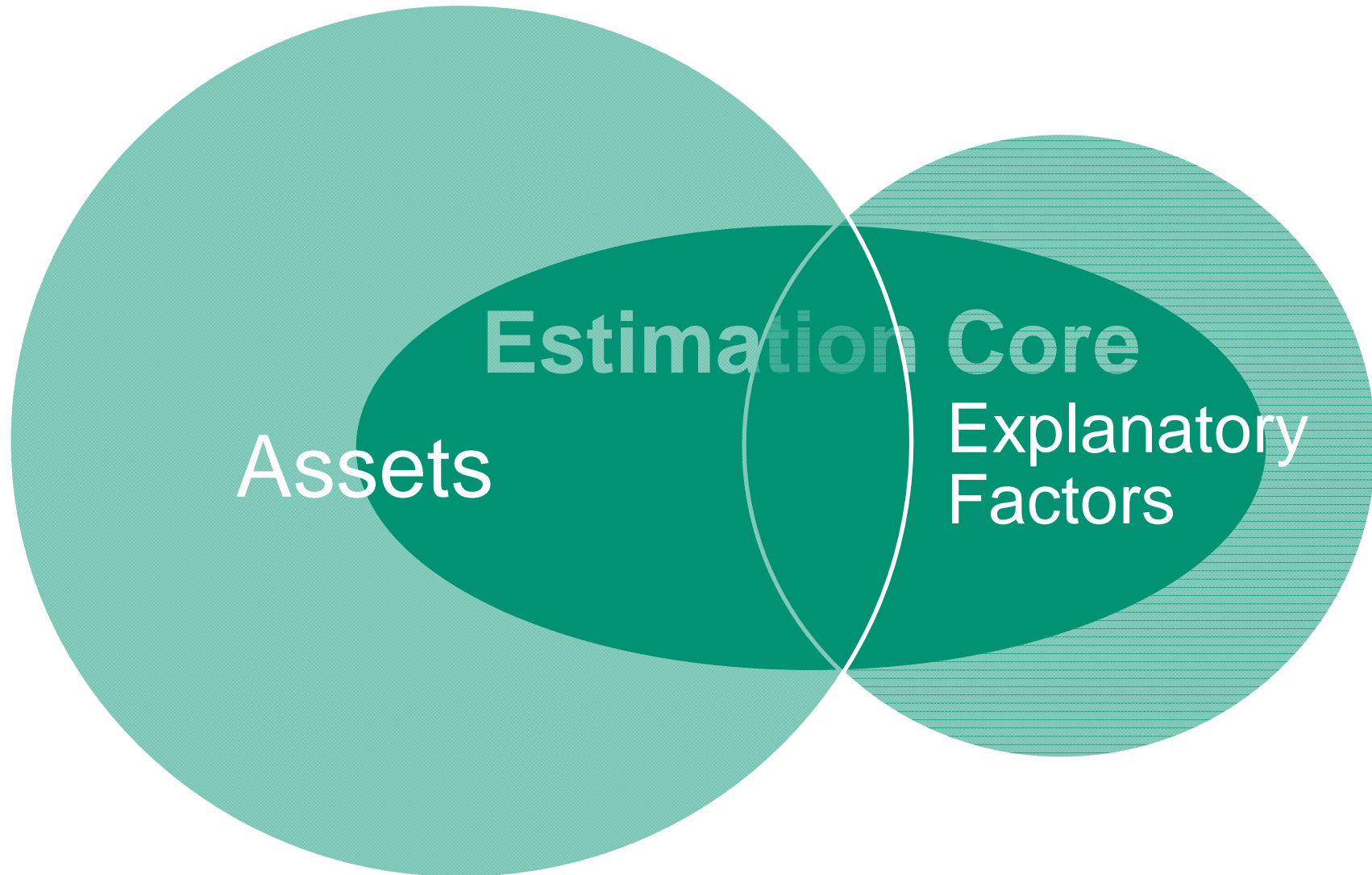
The APT PCA Factor Model

- **First released as a commercial product in 1986**
- **Motivated by well-established pricing theory and macro-economic approaches**
 - Price changes in all liquid assets are the result of buying and selling under conditions of arbitrage
 - *APT aims to capture the co-movement of asset prices associated with the systematic driving terms of their markets*
 - There are many observable “Explanatory Factors” which market participants watch
 - shocks to these factors are always associated with shocks to asset prices
 - Examples: FX rates, Interest Rates, Credit spreads, Commodities, Equity Market Indices, Inflation, Vol Indices...
 - The riskiness of an investment strategy is intuitively understood in terms of exposures to the true “systematic” driving terms of the markets plus an element of “specific” or “diversifiable” risk (eg for a pure stock-picking strategy)
 - Portfolios can be constructed to take advantage of correlation via diversification

The APT PCA Factor Model Estimation Core

- **The Estimation Core includes a rich set of more than 300 market and macro Explanatory Factors plus the historical asset returns**
 - Include FX, Equity, Rates, Credit, Commodity, Inflation, Volatility
 - For equity include Styles, Industry, Country, Region
 - For bonds include Shift, Twist & Butterfly curve factors plus Credit Spreads
 - For hedge funds include Strategy factors
 - May include GDP, aggregate property, private equity, other unlisted assets plus other genuine macro factors (labour markets, fund flows etc)
 - These explanatory factors allow for flexible risk attribution and scenario analysis across multiple asset classes

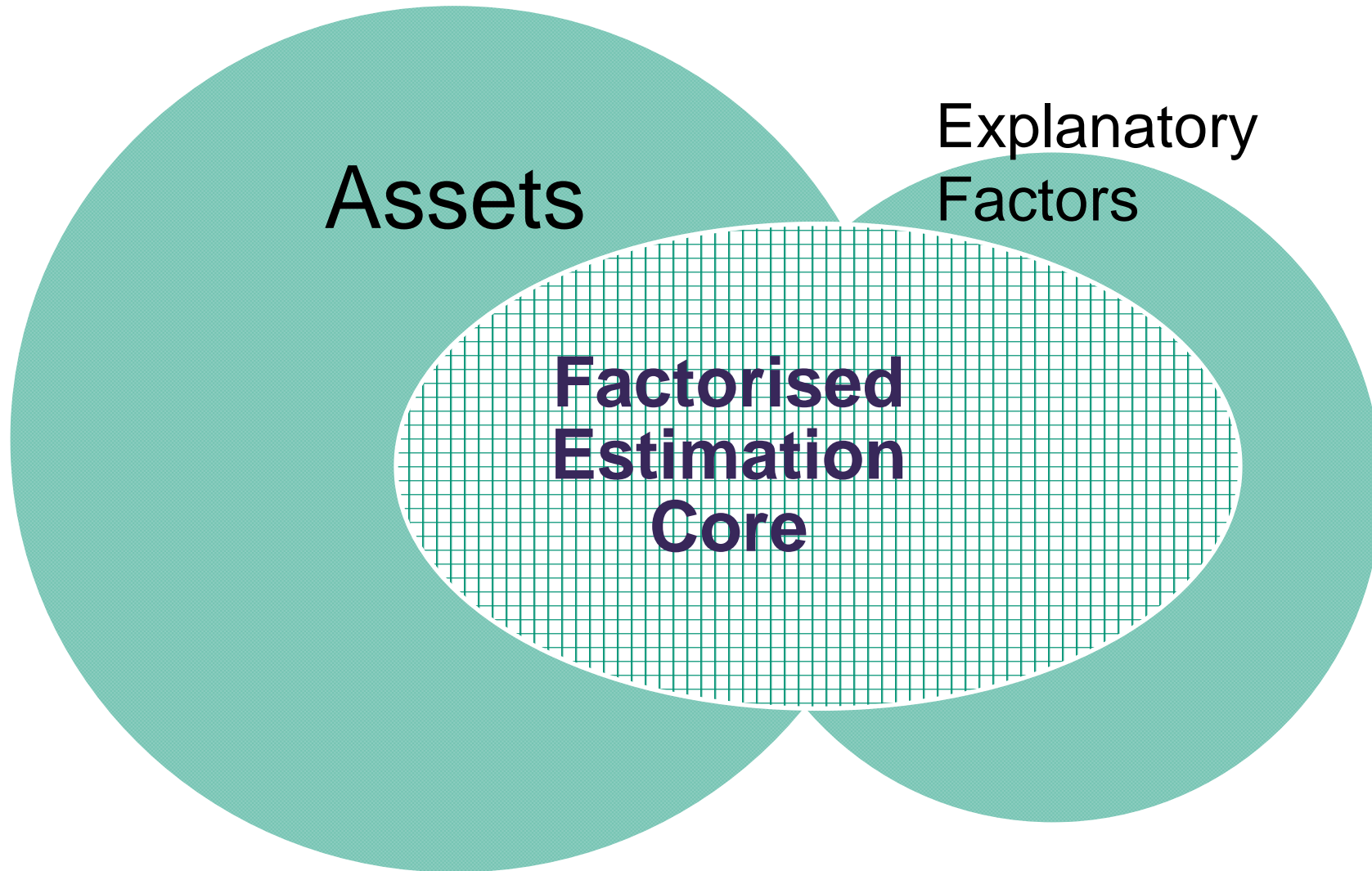
Factor Model Step: Definition of Estimation Core



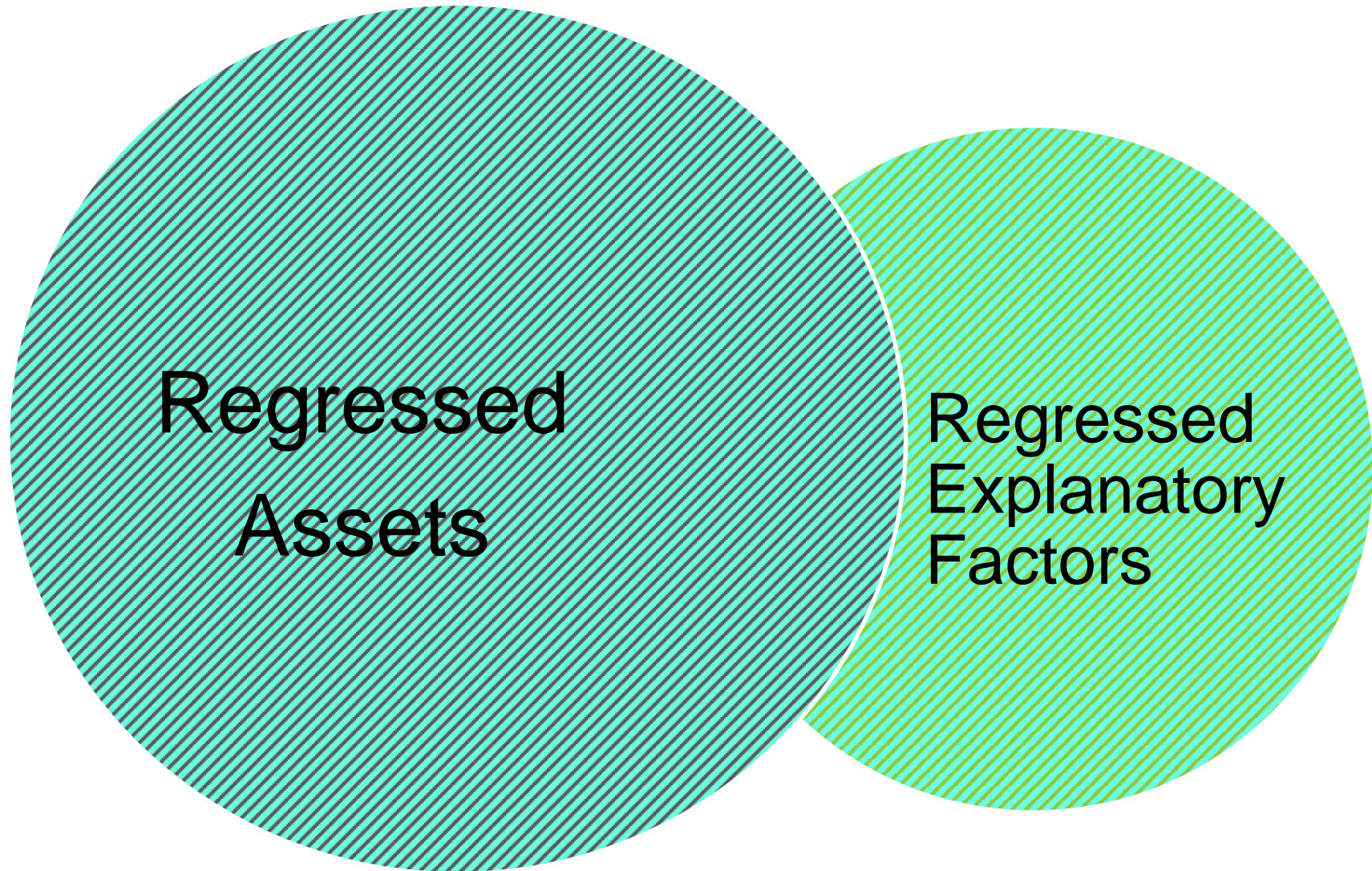
Covariances estimated from historical returns

- The covariances for the Estimation Core are calculated from the historical data
- If $r_i(t)$ is the timeseries of returns on asset i
- $S_{ij} = \text{covar}(r_i, r_j)$ is called the Covariance Matrix S
- S is obviously symmetric and its diagonal elements are the variances of the p assets within the Estimation Core

Factor Model Step 2: Factorisation



Factor Model Step 3: Robust Regression



The APT Multi-Asset-Class Factor Model

- **Attribution and Scenario Analysis are always based entirely on explanatory factors**
- **We use principal components analysis to identify a set of PCA factors that span the maximum amount of shared market volatility**
 - PCA factors are uncorrelated – we can demonstrate persistence
 - PCA factors robustly specify the genuinely systematic risk
 - The PCA factors do NOT represent asset-class specific risk factors, but (as FMPs) can be understood as combinations of explanatory factors across asset classes
- **Every underlying asset and explanatory factor is represented in the factor model by a vector of betas to the PCA factors, plus a residual term**
 - The model is grounded in historical data for every asset class, not based only on finance theory
 - The empirical distributions of PCA factors are available within the estimation of each risk model

More than 500 Explanatory Factors in the APT models

Type	Used For	Based On
Industry, Country, Region, Style	Equity Attribution via Risk Chart	Conventional market indices (MSCI, FTSE, Barclays Agg etc) plus 8 APT-defined styles
Yield Curve: Shift-Twist-Butterfly plus Credit Spreads.	Fixed Income Attribution via Credit FIRA. Also for Bond Scenario Analysis	Calculated for 26 currencies and the full set of borrower classification and rating. Use both bond and CDS data.
Macro	MAC Attribution at total portfolio level via RiskScan. Also for Monte-Carlo based Stress Testing	Includes FX, Property, Bond performance indices, Hedge Fund strategies, Commodities, Inflation, Yield Curves, CDS indices, Volatility

Numbers of MAC APT Components

Asset Class	Number of APT Components
Equities/Credit	20
FX	27 (purely systematic)
Bonds/Rates	30
Commodities	20
Total (Global MAC)	97

- We use different numbers of APT components depending on the asset class coverage which the factor model is designed for



Monte Carlo simulation approaches

- **Simulation allows us to go beyond simple parametric measures of risk**
- **Three good reasons for this**
 - Financial return distributions are NOT Gaussian
 - Financial returns are NOT independently distributed
 - Many financial assets have non-linear payoffs (optionality)
- Because we believe there is a degree of randomness about asset returns (both the systematic AND the specific returns) simulation is based on “random draws” – hence “Monte Carlo”
- Monte Carlo simulation is a powerful tool to generate “future paths” directly, by random draws from any underlying distribution
- **Monte Carlo Simulation**
 - Take an assumed distribution for shocks to pricing factors or asset returns
 - Draw repeatedly from that distribution and re-price your portfolio
 - Look at the outcome distribution and provide risk measures

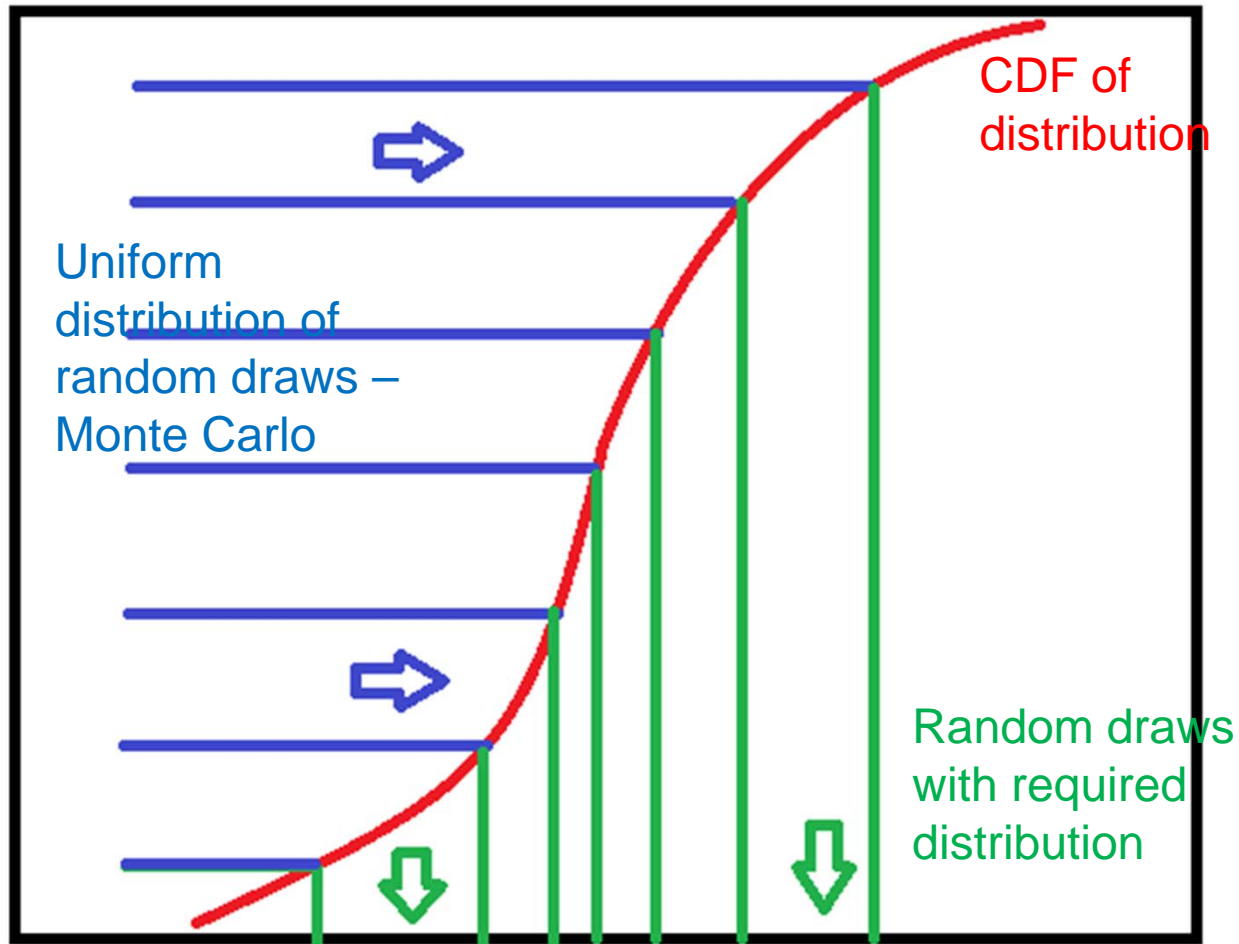
Monte Carlo simulation approaches

- **Definition (from standard Econometrics textbook)**
 - *Monte Carlo simulation is a method of analysis based on artificially recreating a chance process (usually with a computer), running it many times, and directly observing the results.*
- **The markets share something in common with a lottery or a casino**
 - Each period's outcome has an essentially unpredictable element
 - That is expressed as a probability distribution
 - We will need to make some assumption about this distribution (eg CDF)

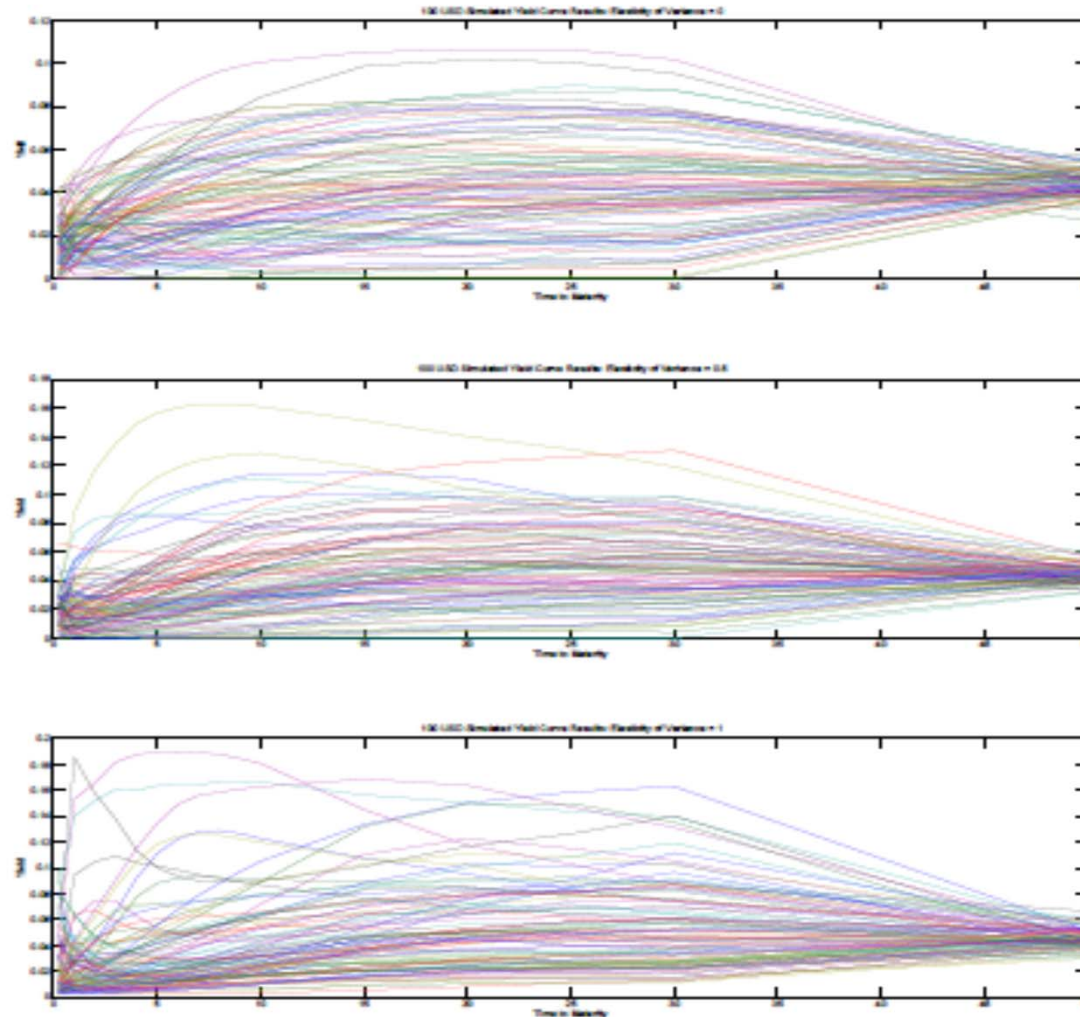
Simulation on the APT Components

- The beauty of the APT model is that every underlying asset is already represented as a linear combination of standardised PCA factors (the risk profile)
- The APT components are independent by construction, so we can use the same set of these components to model ANY underlying asset within a simulation
- This means we can use (up to 97) factors rather than 200k assets when simulating – this is “Monte Carlo on the Components”
- Each of these components can be assumed to have a distribution of returns which may be Gaussian (unlikely) or fat-tailed

Transforming uniform random draws into any specified distribution



Example simulations for a maturing bond, with different interest rate volatilities



How Monte Carlo may be implemented

- **Choose an APT model with appropriate number of factors (20, 50, 76, 96)**
- **Choose a distribution for the underlying assets**
 - Gaussian distribution assumption
 - Fat-tailed = “Generalised Pareto Distribution” assumption for tail behaviour
 - Alpha-stable with fat-tail index
- **Choose a set of pricing functions for derivatives (swaps, futures, options): choose analytic pricing functions for computational speed**
 - Black-Scholes for European options
 - Barone-Adesi-Whaley for American options
 - Garman-Kohlhagen for fx options
 - Black76 for options on rates, swaptions
- **Choose a time-step (daily, weekly, monthly)**
- **Choose a simulation horizon (1-day, 2-week, 52-weeks)**
- **Choose a required number of simulations (50k, 100k, 200k)**

Monte Carlo Outputs

- **Most importantly: we can display the entire distribution of simulated portfolio value outcomes**
- **From the distribution of simulated outcomes we can calculate the required risk statistic**
 - Second-moment measures (volatility, tracking error)
 - Higher moments (skew, excess kurtosis)
 - Fractile measures (median, 95% VaR)
 - Hybrid measures (Conditional VaR)
- **We can also calculate some other interesting measures by observing the behaviour of individual assets, or sub-portfolios, or explanatory factors within the simulation**
 - Standalone VaR
 - Incremental VaR
 - Implied Factor Shocks

Risk over Time within Simulations

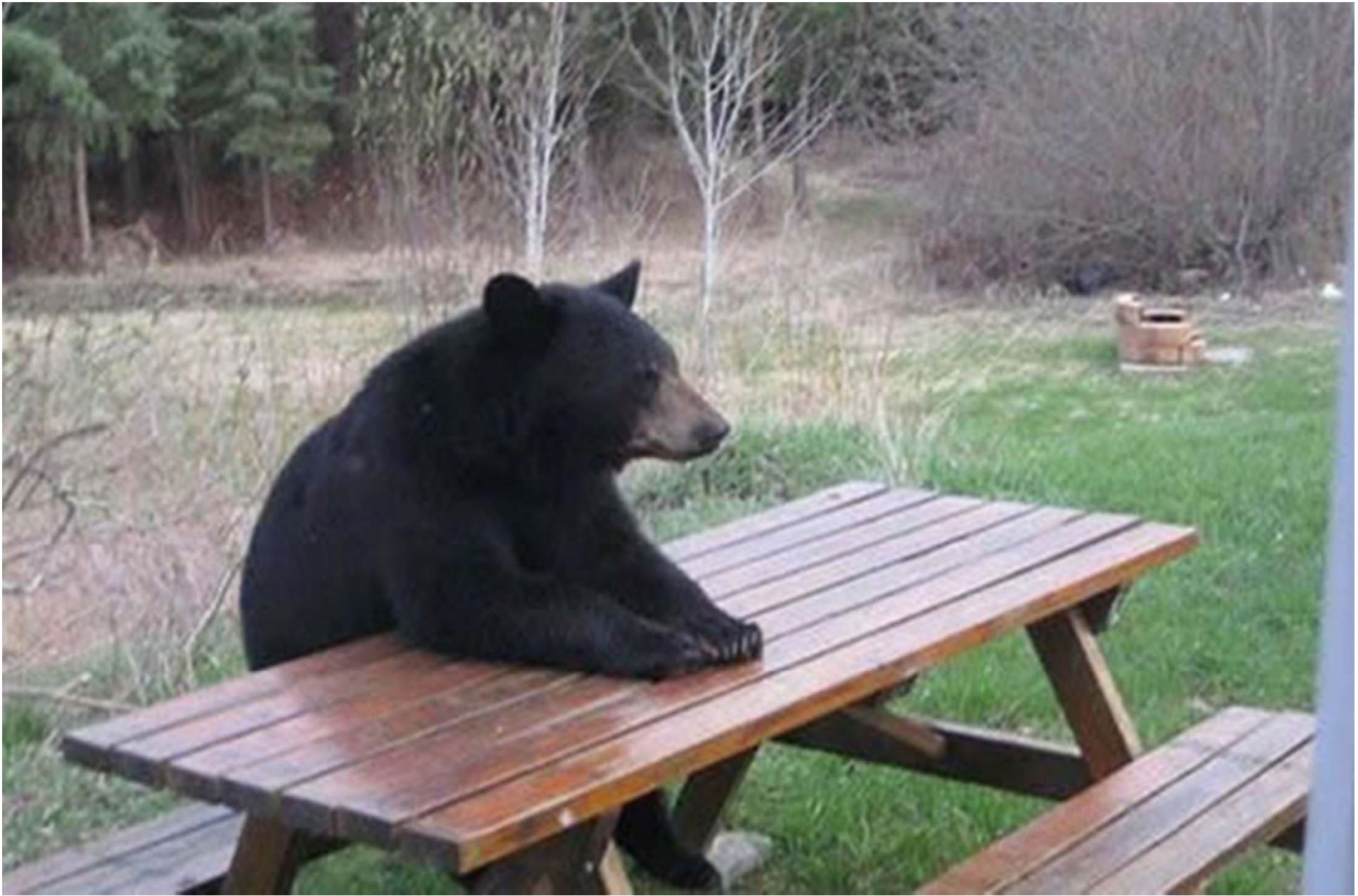
- **Monte Carlo Simulations provide a method for estimating multi-period risk**
- **Typically we still assume the same distribution of returns for underlying assets in every period**
- **However derivate assets are re-priced under new assumptions in successive periods**
- **We could also change the distribution assumptions over successive time periods (eg to model mean-reverting behaviour)**
- **This means that MC simulations will give different estimates for VaR than analytic estimates, which are derived simply from vol estimates and then scaled by $\sqrt{\text{time}}$**

Constrained Monte Carlo

- **We do not always bother estimating the entire distribution of outcomes**
 - A constrained simulation is one in which we only pay attention to outcomes which are constrained – typically those where some variable falls within pre-specified limits
 - We can do this very simply by just “throwing away” the outcomes that do not meet the constraints
- **For example, look only at outcomes with a portfolio loss between 4.9% and 5.1%**
 - Then consider the values of some explanatory factor (say a market index) for this constrained set of outcomes. We will call the median shock to this index δF the “implied factor shock” associated with a portfolio loss of 5%
 - Then look at another set of simulations constrained to have an outcome factor shock = δF +/- a small amount
 - See where the distribution of portfolio returns was centred, and how much dispersion there is
- **This is the beginning of “Reverse Stress Testing”**

Strengths and Weaknesses of Simulation

- **Strengths of the Simulation approach**
 - Can handle optionality
 - Can be applied to historical distributions (non-parametrically)
 - Can handle non-Gaussian parametric distributions
 - Can be extended via constraints
- **Weaknesses of the Simulation approach**
 - Does not provide breakdown of risk measures to systematic/specific parts
 - Does not provide factor attribution of risk measures



Approaches to Scenario Analysis

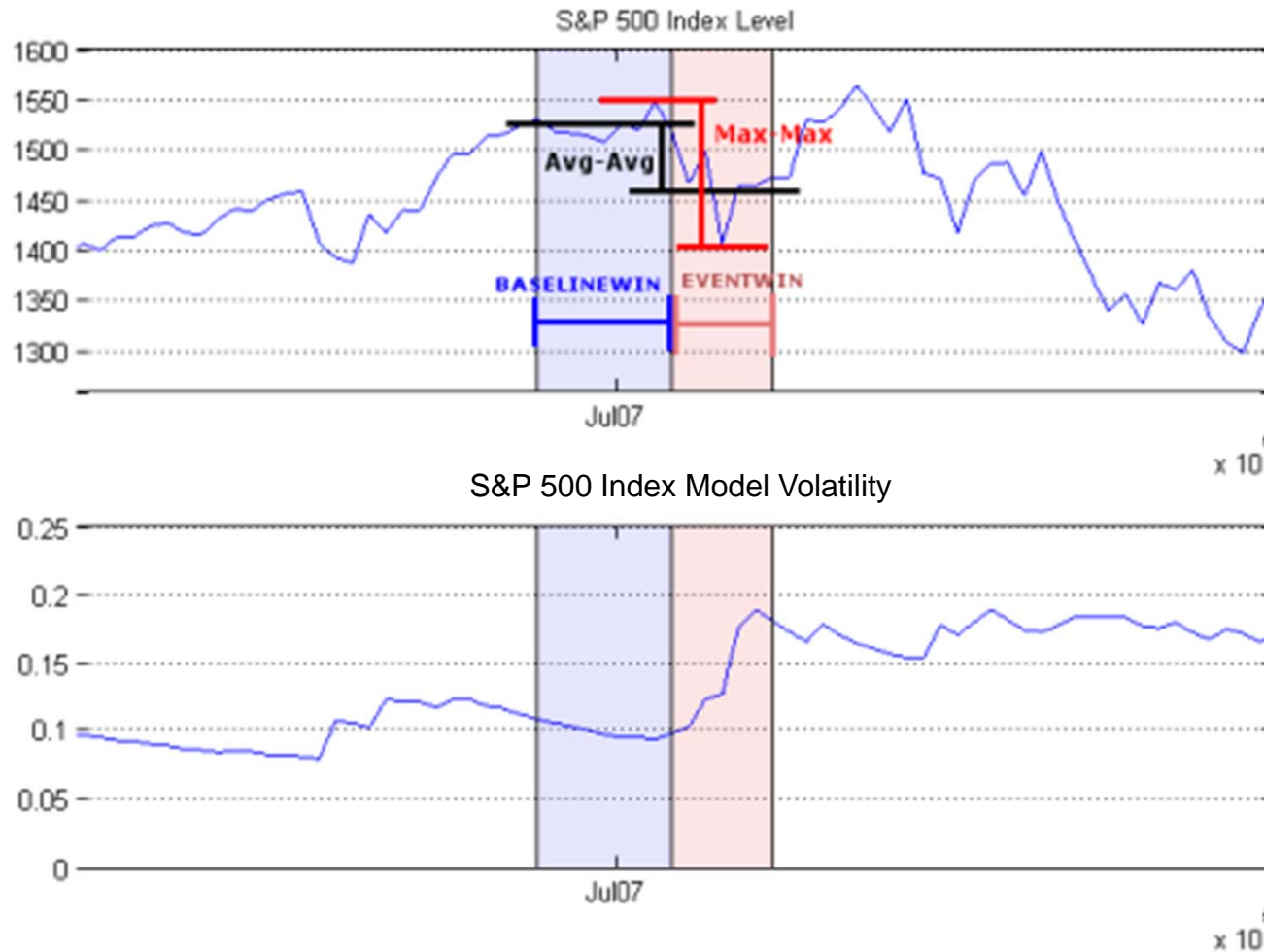
- **Different approaches to Scenario Analysis**
 - Historical Scenario Analysis
 - Forward-looking Stress Testing
- **Here is where we will try to think a bit more about how the future may actually come about**
 - But of course we always have to start with the past

Historical Scenario Analysis

- **Aim of Scenario Analysis: to quantify the likely effects of market shocks on portfolio value and riskiness**
 - Simplest approach – define “market shocks” in terms of shocks to level and volatility of a set of explanatory risk factors
 - Translate those shocks to the portfolio which is represented as a bundle of exposures to those same explanatory risk factors
 - Essentially we are “shifting” and “stretching” the distribution of portfolio returns estimated from the risk model
- **As a concrete example**
 - For historical scenarios, we look at the behaviour of the set of explanatory factors over a specified 14-week period
 - 8 weeks before the event date, 6 weeks after the event date

Historical Scenario Shocks

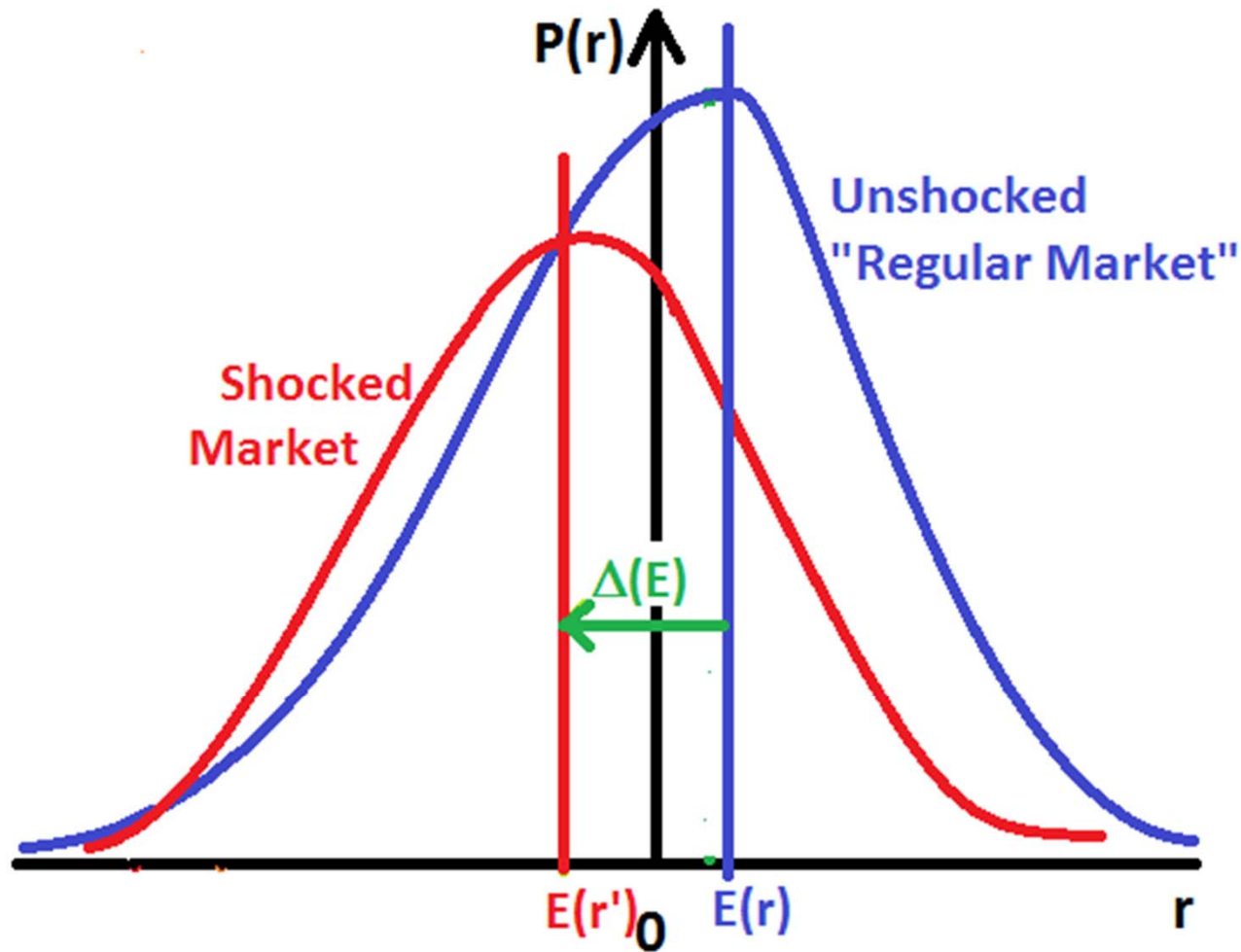
- Example: July 2007 - calculation of historical scenario shock to explanatory factor



Example Historical Scenario – July 2007

Explanatory Factor	Level Shock	Original Volatility	Volatility Shock	Shocked Volatility	Nature of Vol Shock
Regions - North America	-9.4%	24.4%	233.3%	81.4%	Relative Multiplier
Regions - Europe Ex UK	-9.2%	31.8%	127.5%	72.3%	Relative Multiplier
Regions - UK	-9.7%	29.6%	137.1%	70.2%	Relative Multiplier
Regions - Japan	-8.4%	23.2%	27.7%	29.6%	Relative Multiplier
Regions - Pacific Ex Japan	-11.9%	32.3%	143.1%	78.4%	Relative Multiplier
Resources	-12.9%	34.4%	106.8%	71.2%	Relative Multiplier
Basic Industries	-10.8%	31.5%	111.0%	66.4%	Relative Multiplier
General Industrials	-9.7%	29.3%	130.3%	67.4%	Relative Multiplier
Cyclical Consumer Goods	-8.5%	29.2%	126.0%	65.9%	Relative Multiplier
Non-Cyclical Consumer Goods	-5.3%	16.5%	153.8%	41.9%	Relative Multiplier
Cyclical Services	-10.4%	23.7%	174.3%	65.0%	Relative Multiplier
Non-Cyclical Services	-7.1%	19.4%	109.1%	40.6%	Relative Multiplier
Utilities	-7.0%	20.7%	110.7%	43.6%	Relative Multiplier
Financials	-11.4%	33.8%	194.6%	99.7%	Relative Multiplier
Information Technology	-7.7%	24.7%	161.5%	64.6%	Relative Multiplier
US Tbill - Yield	-25.7%	75.7%	109.0%	158.2%	Relative Multiplier
US Long Rate - Yield	-12.2%	30.0%	12.5%	33.7%	Relative Multiplier
Oil	13.6%	42.2%	-28.6%	30.1%	Relative Multiplier
Japan Yen	7.5%	11.0%	78.5%	19.7%	Relative Multiplier
United Kingdom Pound	3.9%	13.6%	62.6%	22.2%	Relative Multiplier
Eurobloc Euro	3.9%	13.3%	129.6%	30.5%	Relative Multiplier

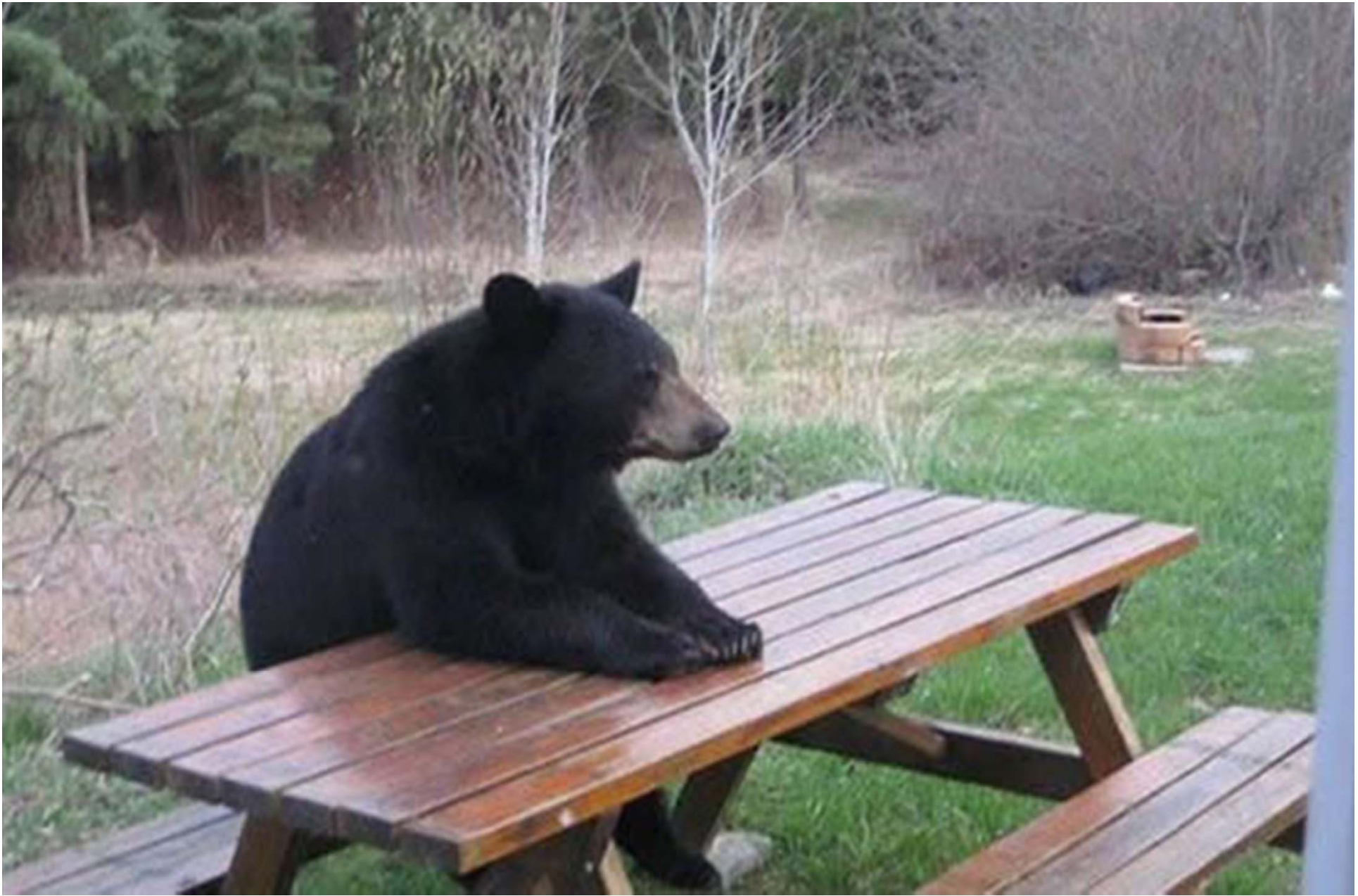
Expected Loss under Market Shock Scenario



Example of Historical Scenario Analysis

8 historical scenarios applied to a single balanced portfolio

Shock Scenario	Portfolio Total VaR (95 %, 2 weeks, Gaussian)	Portfolio Total Volatility	Total Tracking Error	Portfolio Value	Portfolio Value Change	Benchmark Value Change	Portfolio Relative Value Change
Base Scenario	8.9%	27.7%	5.0%	100.0%			
Asian Crisis 1997	17.5%	54.3%	7.1%	98.6%	-1.4%	-2.9%	1.5%
Russian Default 1998	17.9%	55.4%	5.9%	88.4%	-11.6%	-14.2%	2.6%
Tech Bubble 2000	12.9%	40.1%	5.7%	95.3%	-4.7%	-4.5%	-0.2%
9-11 2001	18.6%	57.7%	6.8%	85.3%	-14.7%	-16.4%	1.8%
WorldCom 2002	24.9%	77.1%	6.2%	84.4%	-15.6%	-17.5%	1.9%
Mortgage Crisis 2007	20.3%	62.9%	6.4%	92.9%	-7.1%	-8.2%	1.1%
2008 Financial Crisis	29.3%	90.8%	11.8%	57.4%	-42.6%	-45.1%	2.5%
2010 Euro Sovereign Crisis	14.5%	45.1%	5.7%	80.7%	-19.3%	-20.2%	0.9%



The investment risk report – current best practise

- **Typically generated every day**
 - A summary of key risk measures for a single investment portfolio
 - Sometime aggregated to “firm level”
 - Allows risk managers to see whether certain risk measures are rising or falling
 - Allows risk managers to see whether risk is greater than the investor wants it to be
- **Incorporates factor-based and Monte Carlo measures, plus attribution and historical scenario analyses**

Investment Risk Reporting

Global Income Fund (MSCI World Index)

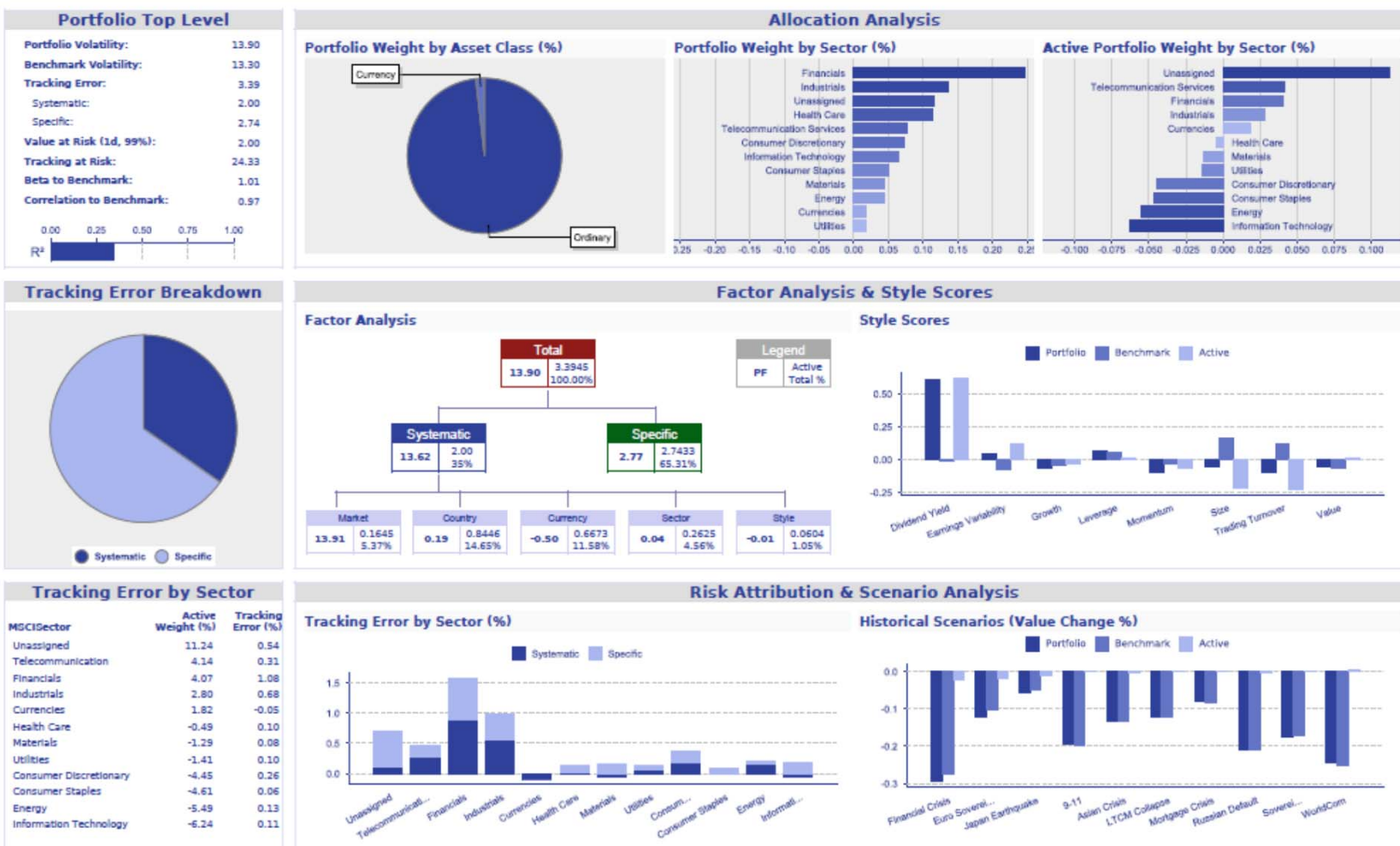
George Boyd-Bowman

Report Date: Jun 26, 2015

Valuation Date: Jul 29, 2015

Market Value (GBP): Not Provided

SUNGARD®



Where are we heading?

Forward-looking Scenario Analysis

- **Effective risk management is about future paths, so the measures we should care about are really those based on forward-looking scenario analysis**
- **We can define forward-looking market shocks in terms of shocks to level and volatility of a set of explanatory risk factors**
- **The assumed shocks may be derived from economist's or strategist's forecasts**
 - Translate those shocks to the portfolio which is represented as a bundle of exposures to those same explanatory risk factors
 - Essentially we are “shifting” and “stretching” the distribution of portfolio returns estimated from the risk model
- **If a forecast is for a zero shock, we must set it to zero**
 - Otherwise the model will assume a consistent shock for each explanatory factor, which is likely to be non-zero

Where are we heading?

Forward-looking Scenario Analysis

- **The real challenge is to produce “plausible” future scenarios**
 - Once we define these, we can use either parametric, or Monte Carlo, methods to forecast the likely effect on my wealth (fund value)
 - But how do we come up with economically sound and plausible scenarios?
 - This is the real “art” of risk management
- **No-one has solved this problem**
 - If we cannot predict the future, perhaps we can at least create enough scenarios to cover most eventualities
 - This approach I have called “wide-field” scenario analysis
 - Some firms are subjecting their portfolios to as many as 1000 forward-looking scenarios every day
 - Another approach, based on Monte Carlo, is to examine the random paths and focus on those which entail the greatest losses
 - This “tail risk” approach is sometimes called “reverse stress testing”, because we reverse the usual question about portfolio losses...

Questions and Conclusions

- **Current best practise is to use a multi-purpose risk reporting model**
 - VaR risk reporting will be based on both factor and Monte Carlo models
 - Reporting includes “granular” asset-level portfolio reporting
 - Attribution and scenario analysis should be based on intuitive explanatory factors
- **Historical scenario analysis is widely used for investment risk management**
 - This provides some context for thinking about future risk
 - But we must never assume that the future will be just like the past
- **Risk management as an art**
 - The art of risk management is to create plausible and coherent scenarios for the future, and then use the powerful models and technologies we have created to turn these into real intelligence for investment decision making

Empowering
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