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# Dependence of – and Long Memory in – Exchange Rate Returns: Statistics, Robustness, Time Aggregation*

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**Abstract**

Empirical research on financial market data reported a number of properties of return time series which are considered as 'stylized facts'. In this contribution, measures of dependence of foreign exchange rate returns are discussed. Furthermore, statistics of long memory are considered. The robustness of the statistics is analyzed using the bootstrap and considering different subsamples. It is also analyzed how time aggregation affects these properties.

Based on the results, robust stylized facts are selected which – together with similar results for unconditional distributions provided in a companion paper – will result in an objective function for an indirect estimation method of agent based models.

**Keywords:** Long memory;BDS-test; rescaled range statistic; Hurst exponent; GPH estimator; unit root; ADF-test; GARCH.
**JEL-classification:** C14, C15, F31

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# 1 Introduction

In the short run, relevant linear dependence of financial market returns would clearly contradict the assumption of market efficiency as introduced by Fama (1970, 1991). However, dependence in higher moments of the return distribution and long memory are often reported as 'stylized facts' of returns on financial assets (Cont, 2001; Granger, 2005) along with the observations of fat tails in unconditional return distributions (Winker and Jeleskovic, 2006). If these features are characteristic for financial market returns, they provide a benchmark for the further development of theoretical models of these markets and for the empirical validation of such models, e.g., agent based models (Gilli and Winker, 2003; Lux and Schornstein, 2005). This benchmark can equally be used for other kinds of simulation models of exchange rate returns.[1]

While Winker and Jeleskovic (2006) concentrate on properties of the unconditional return distribution of foreign exchange series, the focus of this paper is on dependence in foreign exchange return series. In particular, dependence of and long memory in higher order moments of the return series is analyzed. We identify those measures which can be considered as 'stylized facts' in the sense of providing sense of non-trivial statistics, which are robust over time and across assets (Cont, 2001). Keeping the purpose of using these statistics as benchmarks for model evaluation in mind, it is also necessary to provide information on their distributional properties. To this end, a bootstrap approach is used.

Although measures for the unconditional return distributions are already covered in Winker and Jeleskovic (2006), the literature provides still a plentitude of properties with regard to dependence in higher order moments and long range dependence. In this paper, we try to cover the most popular of these statistics, but cannot claim to provide an exhaustive analysis. Nevertheless, our analysis should be broad enough to allow for selecting appropriate statistics which might serve eventually as a benchmark for validating agent based models along the lines proposed by Gilli and Winker (2003).

This paper is organized as follows. Section 2 introduces the exchange rate time series used for the empirical analysis and their empirical moments. Furthermore, we provide an overview on deviations from the independence assumption which will be covered in this contribution and present the results of the BDS-test, which can be considered as a portmanteau test of such deviations. Section 3 is devoted

[1]E.g., Ding *et al.* (1993, pp. 94ff) analyze different time series models with regard to their ability to match sample autocorrelation functions.

to models and statistics of dependence in moments of the return distribution. The issue of long memory is covered by Section 4, while stationarity is the subject of Section 5. The robustness of the all statistics is assessed in Section 6 by analyzing subsamples of the data and application of the bootstrap method. Finally, the impact of time aggregation is discussed in Section 7. Section 8 summarizes our main findings and the implications for indirect estimation approaches.

# 2 Data, Empirical Moments and BDS-Test

## 2.1 The Data

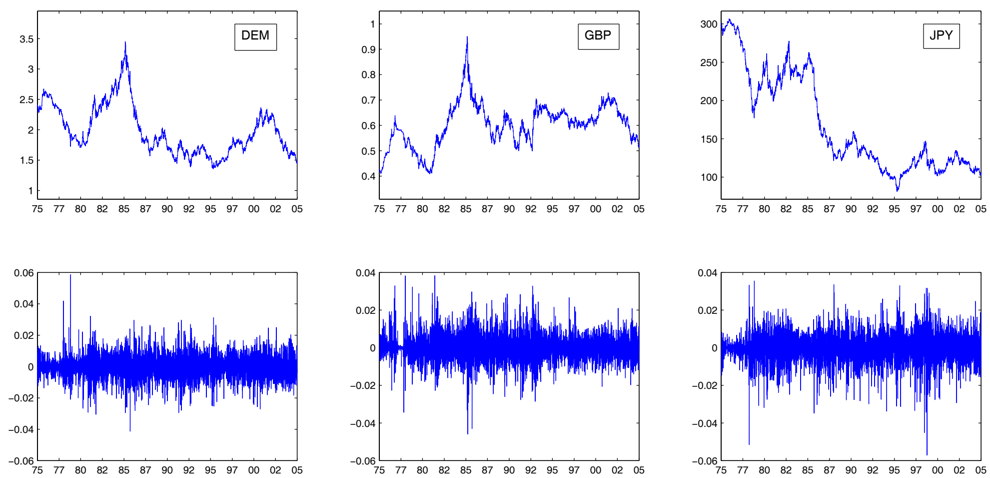
For the empirical analysis, we use foreign exchange time series at daily frequency (5-day-week). The sample includes 8088 observations of the German D-Mark (DEM), British Pound (GBP), French Franc (FRF), Swiss Franc (CHF), and the Japanese Yen (JPY) against the US Dollar in volume notation for the period 1st of January 1974 to 4th of January 2005.[2] The values for the German D-Mark and the French Franc are the implicit rates after the introduction of the Euro in 1999. Each time series contains 284 missing values due to bank holidays. We use the sample 1.1.1975 to 31.12.2004 for the following analysis if not stated otherwise. For the observations following a missing value, the return is calculated on the base of the last preceding valid observation. The return time series are calculated as logarithmic differences of the price series and denoted by $r_i$, $i \in \{DEM, GBP, FRF, CHF, JPY\}$.

Figure 1 shows the exchange rate time series for the German D-Mark, the British Pound and the Japanese Yen against the US Dollar in the upper panel, and the corresponding return time series in the lower panel.

While it might appear difficult to spot autocorrelation of the return series in figure 1, the volatility clustering is quite obvious, i.e. periods of high volatility in returns tend to be persistent as are low volatility periods. In contrast to these short run dependencies in the second moment it is difficult if not impossible to judge the relevance of long term dependencies in some (higher) moments of the return series. Therefore, after considering short run dependencies in Section 3, testing procedures for long range dependence will be presented in Section 4. Finally, looking at the exchange rate series themselves, the issue of stationarity comes into play. Section 5 is devoted to tests for stationarity and fractional integration.

[2] The data were retrieved from `http://fx.sauder.ubc.ca/data.html`, a site of the Sauder School of Business, University of British Columbia.

Figure 1: Exchange Rate Time Series and Returns for *DEM*, *GPB*, and *JPY*




Before turning to all these potential deviations from the independence assumption in detail, we first present the unconditional moments of our return series in the following subsection, and the BDS-test at the end of this section.

## 2.2 Empirical Moments and Tests for Normality

In Table 1, we provide the empirical skewness, kurtosis and the Komogorov-Smirnov statistic. While the empirical skewness is close to zero for most series, the observed kurtosis is clearly above the value of three for a normal distribution. This finding is supported by the values of the Kolmogorov-Smirnov statistic (KS) reported in the last column. For the test of the null hypothesis of a normal distribution, the moments of the normal distribution have to be estimated. Therefore, the modified statistic and critical values from Lilliefors (1967) are used for the test. The null hypothesis has to be rejected at any conventional level of significance.

## 2.3 Deviations from Independence and BDS-test

Brock *et al.* (1996) introduce the BDS-test to distinguish between a deterministic white noise chaos and a stochastic white noise process.[3] The case of white noise

[3]The approach is based on earlier work by Brock *et al.* (1987).

Table 1: Test Statistics for Normality Assumption

| Series | Obs. | Mean | Std.dev. | Skew. | Kurt. | KS |
|---|---|---|---|---|---|---|
| $r_{DEM}$ | 7554 | -6.8E-05 | 0.0065 | 0.0314 | 5.987 | 0.0490 |
| $r_{FRF}$ | 7554 | 1.1E-05 | 0.0064 | 0.1000 | 7.169 | 0.0544 |
| $r_{GBP}$ | 7554 | 2.6E-05 | 0.0061 | 0.1300 | 6.677 | 0.0633 |
| $r_{CHF}$ | 7554 | -0.00011 | 0.0076 | 0.0438 | 13.699 | 0.0527 |
| $r_{JPY}$ | 7554 | -0.00014 | 0.0066 | -0.4858 | 7.377 | 0.0704 |

chaos can also be interpreted as linear dependence in higher conditional moments. Therefore, the BDS-test can also be called a test for non-linear dependence of the original process (Liu *et al.*, 1992; Kanzler, 1998). We use the BDS-test in this sense, i.e. as a portmanteau test for any kind of time based dependence in the return series. In fact, according to Kanzler (1998), theoretically, the BDS-test is sensitive against a huge variety of linear and non-linear, deterministic and stochastic alternatives and does not depend on the existence of higher moments.[4] Therefore, we apply the BDS-test to the residuals of a linear model, e.g., an ARMA model. Although we do not expect to find significant short run linear dependence in the return series, we also apply the BDS-test to such residuals.

The BDS-test is based on the concept of the correlation integral and its generalization for subsequences of time series (Liu *et al.*, 1992). For a given return series $r_t$, the correlation integral is defined as

$$C_1(\varepsilon) = \lim_{T\to\infty} \frac{1}{T^2} \cdot NB\,, \tag{1}$$

where $NB$ denotes the number of pairs $(i, j)$ such that the corresponding returns $r_i$ and $r_j$ have similar values, i.e. such that $|r_i - r_j| < \varepsilon$. For a local approximation of $C_1(\varepsilon)$ by $C_1(\varepsilon) \approx \varepsilon^\nu$, the exponent $\nu$ is called the correlation exponent. In principle, it might be used to distinguish stochastic white noise from white chaos, but also low-dimensional chaos from high-dimensional chaos.

[4]Davidson and Sibbertsen (2005, p. 269) propose to use the McLeod and Li (1983) portmanteau test as a test for nonlinearity. However, the asymptotic distribution of this test hinges on the assumption of the existence of finite high order moments. Nevertheless, we also run the McLeod and Li test and find clear indication for deviations from the independence assumption. The results are available on request from the authors.

The generalization of (1) to subsequences of $r_t$ of length $m$ is given by

$$C_m(\varepsilon) = \lim_{T\to\infty} \frac{1}{T^2} \cdot NB_m\,, \tag{2}$$

where $NB_m$ denotes the number of pairs $(i, j)$ such that all corresponding elements of $[r_{i-m+1}, \ldots, r_i]$ and $[r_{j-m+1}, \ldots, r_j]$ are close to each other, i.e. satisfy the condition $|r_{i-k} - r_{j-k}| < \varepsilon$ for $k = 0, \ldots, m-1$. Thereby, $m$ is called the embedding dimension. Again, a local approximation $C_m(\varepsilon) \approx \varepsilon^{\nu_m}$, for small $\varepsilon$ is considered. If $r_t$ is a stochastic white noise process, the correlation exponent $\nu_m$ is equal to $m$ for all $m$.

If the observations $r_t$ are independent, the joint probability of satisfying the $\varepsilon$-condition in (2) should be equal to the product of probabilities of satisfying the individual conditions, i.e., $C_m(\varepsilon) = [C_1(\varepsilon)]^m$. This motivates the BDS-statistic $S_m(\varepsilon) = C_m(\varepsilon) - [C_1(\varepsilon)]^m$, which is asymptotically normal distributed under the null hypothesis of i.i.d. distributed $r_t$.

For the application of the BDS-test to sample data, the unobserved probabilities $C_1(\varepsilon)$ and $C_m(\varepsilon)$ have to be replaced by sample estimates, i.e.

$$C_{m,T}(\varepsilon) = \frac{2}{(T-m+1)(T-m)} \sum_{i=1}^{T-m+1} \sum_{j=1}^{T-m+1} \prod_{k=0}^{m-1} I_\varepsilon(r_{i+k}, r_{j+k})\,, \tag{3}$$

where $I_\varepsilon$ indicates the indicator function, i.e. $I_\varepsilon(x, y) = 1$, if $|x-y| < \varepsilon$ and zero otherwise.

Normalizing the estimate $C_{m,T}(\varepsilon) - [C_{1,T-m+1}(\varepsilon)]^m$ by an estimate of its asymptotic standard deviation and multiplication with $\sqrt{T-m+1}$ leads to the BDS-statistic, which is asymptotically standard normal distributed under the null hypothesis of independence (Brock *et al.*, 1996).

It should be noted that one has to set a priori the relevant dimensional distance $\varepsilon$ and the embedding dimension $m$ for the test. As proposed by Kanzler (1998), we choose the dimensional distance $\varepsilon$ to be equal to 0.7 times the standard deviation of the return series. The values of the embedding dimension are chosen to be 2, 3, and 4 as appears to be common practice in the literature.[5] The results for the original time series and for the residuals after fitting ARMA(1,1) models to these time series are summarized in Table 2.

[5]Increasing the embedding dimension $m$ would result in very small values of $C_{m,T}$ and a high variance of its estimator. However, high-dimensional chaos and long range dependence might not be uncovered using small values of $m$.

Table 2: BDS statistics

| | Return Time Series | | | Residuals of ARMA(1,1) Models | | |
|---|---|---|---|---|---|---|
| m = | 2 | 3 | 4 | 2 | 3 | 4 |
| $r_{DEM}$ | 9.611 | 14.708 | 19.541 | 9.793 | 14.878 | 19.734 |
| $r_{FRF}$ | 11.699 | 18.082 | 24.447 | 11.709 | 18.084 | 24.402 |
| $r_{GBP}$ | 14.939 | 20.080 | 25.167 | 15.331 | 20.753 | 25.975 |
| $r_{CHF}$ | 8.996 | 12.049 | 15.663 | 8.997 | 12.162 | 15.896 |
| $r_{JPY}$ | 13.144 | 17.322 | 22.751 | 13.231 | 17.501 | 22.907 |

As expected, the results for the original series and the residuals of a simple ARMA(1,1) specification do not differ substantially. Furthermore, the null hypothesis of independence has to be rejected for all exchange rates and all embedding dimensions at any conventional level of significance. Thus, a more detailed analysis of dependence structures seems indicated. It is conducted in the following sections.

# 3 Conditional Moments of Return Distribution

The returns themselves should not contain a relevant amount of serial correlation. Otherwise the assumption of market efficiency would be clearly contradicted Fama (1970, 1991). Nevertheless, it is a well known observation that absolut or squared returns show persistence, i.e. significant positive autocorrelation over some lags (Ding *et al.*, 1993; Cont, 2001). Consequently, this section is devoted to measures of sample autocorrelation for different functionals of the return series and to the classical approach for modelling volatility clustering, the (G)ARCH model.

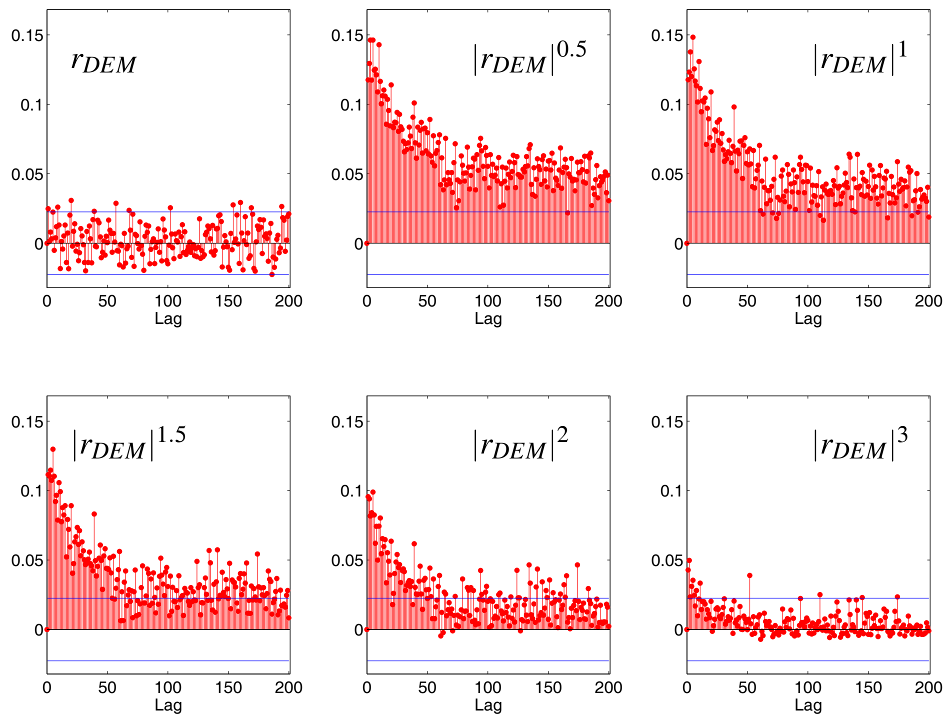
## 3.1 Sample Autocorrelation

The sample autocorrelation functions for $r_{DEM}$ and $|r_{DEM}|^d$ for different positive values of $d$ are shown in Figure 2.[6] The dotted horizontal lines indicate the 95%

[6]The corresponding figures for the other exchange rates are provided in the appendix.

asymptotic confidence interval under the null hypothesis that $r_{DEM}$ is independently and identically distributed.[7]

Figure 2: Sample autocorrelation for $r_{DEM}$ and $|r_{DEM}|^d$ for different values of $d$.




The sample autocorrelation function for $r_{DEM}$ shows a few marginally significant positive values. All other autocorrelation coefficients are not significantly different from zero, while positive and negative values are of similar frequency. The picture changes drastically when moving to the second and third plots in the upper panel corresponding to $|r_{DEM}|^{0.5}$ and $|r_{DEM}|$. Here, we find a similar result as reported by Ding *et al.* (1993, pp. 86ff) for the S&P 500 stock returns. All autocorrelation coefficients up to lag 200 are positive. For $d = 0.5$, all but one coefficient are significantly positive at the 5% level, while for $d = 1$ still 189 out of 200 (94.5%) are significantly positive. These results are strong evidence for a

[7]For our sample of 7554 observations, the confidence interval is $\pm 1.96/\sqrt{7554} = \pm 0.0225$. Note that if $r_{DEM}$ is i.i.d., the same applies to any transformation of $r_{DEM}$, in particular to $|r_{DEM}|^d$. Thus, the same asymptotic confidence intervals apply under the i.i.d. assumption.

high persistence in low order moments of absolute returns.

The picture changes gradually as we move to higher order moments in the lower panel of Figure 2. For $d = 1.5$ (left most plot), the first 50 autocorrelations are still significantly positive, while the overall number of significant coefficients shrinks to 143. While all coefficients are positive for $d = 1.5$, we observe 4 negative, though insignificant, values for $d = 2$ and 58 (all insignificant) for $d = 3$. While increasing $d$, the number of significant coefficients decreases to 85 for $d = 2$ and 12 for $d = 3$. Thus, positive autocorrelation and long memory is most pronounced for small powers of absolute returns.

A more detailed picture of sample autocorrelations of $|r_{DEM}|^d$ for different values of $d$ is given in Table 3. At first sight, it becomes obvious that negative autocorrelations appear to be the exception for small powers of $|r_{DEM}|$. Furthermore, the maximum coefficient values for different lags are usually found for $d = 0.5$ or $d = 0.75$, i.e. at slightly lower exponents than reported by Ding *et al.* (1993, p. 87) in their application to S&P 500 stock returns.

Table 3: Autocorrelations of $|r_{DEM}|^d$

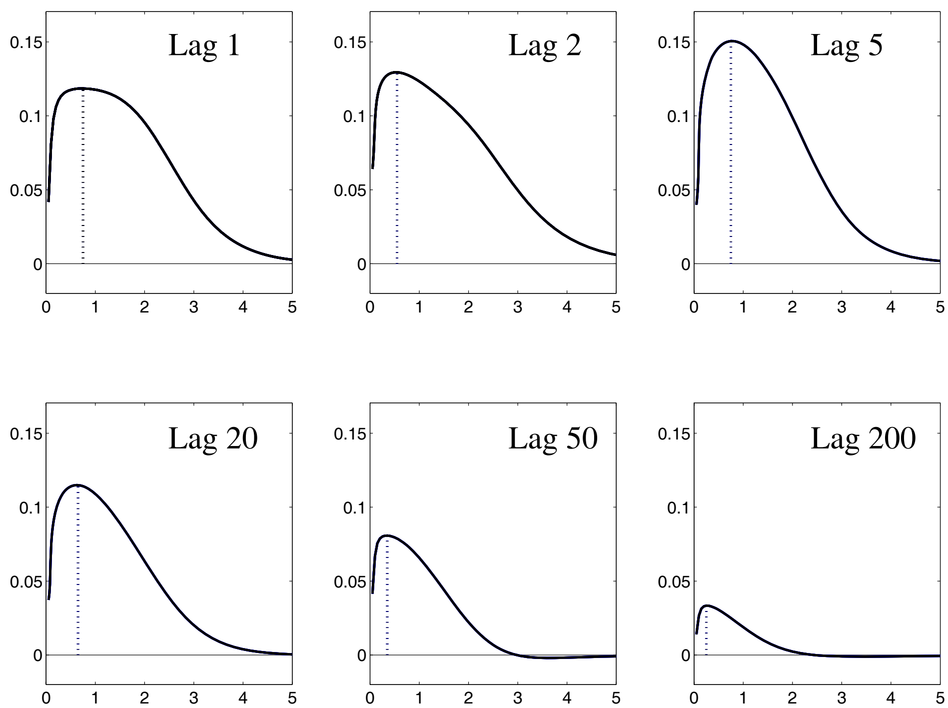
| | Lag | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d$ | 1 | 2 | 3 | 4 | 5 | 10 | 20 | 30 | 40 | 50 | 100 | 200 |
| 0.125 | 0.091 | 0.107 | 0.124 | 0.083 | 0.099 | 0.118 | 0.083 | 0.058 | 0.050 | 0.072 | 0.050 | 0.030 |
| 0.25 | 0.110 | 0.123 | 0.140 | 0.104 | 0.129 | 0.138 | 0.103 | 0.070 | 0.065 | 0.080 | 0.056 | 0.033 |
| 0.5 | 0.118 | 0.129 | 0.146 | 0.117 | 0.146 | 0.143 | 0.114 | 0.073 | 0.072 | 0.080 | 0.055 | 0.031 |
| 0.75 | 0.118 | 0.128 | 0.144 | 0.121 | 0.151 | 0.139 | 0.114 | 0.071 | 0.071 | 0.074 | 0.051 | 0.025 |
| 1 | 0.118 | 0.123 | 0.138 | 0.120 | 0.148 | 0.131 | 0.109 | 0.067 | 0.066 | 0.066 | 0.044 | 0.019 |
| 1.25 | 0.116 | 0.117 | 0.128 | 0.115 | 0.141 | 0.119 | 0.100 | 0.062 | 0.059 | 0.055 | 0.038 | 0.013 |
| 1.5 | 0.112 | 0.110 | 0.115 | 0.107 | 0.130 | 0.106 | 0.089 | 0.057 | 0.050 | 0.044 | 0.031 | 0.008 |
| 1.75 | 0.105 | 0.103 | 0.099 | 0.097 | 0.115 | 0.090 | 0.077 | 0.050 | 0.040 | 0.033 | 0.025 | 0.005 |
| 2 | 0.096 | 0.094 | 0.082 | 0.084 | 0.099 | 0.074 | 0.064 | 0.044 | 0.030 | 0.022 | 0.020 | 0.002 |
| 3 | 0.043 | 0.050 | 0.023 | 0.031 | 0.035 | 0.022 | 0.020 | 0.016 | 0.005 | 0.000 | 0.006 | -0.001 |

A different view on the dependence of the sample autocorrelation at a given lag for varying exponents $d$ is provided in Figure 3.[8] The left most plot in the upper panel shows the autocorrelation at the first lag for $d = 0.05, 0.10, 0.15, \ldots, 5$. Obviously, this first order autocorrelation is a smooth function of $d$, which reaches as single maximum. A similar shape results for lags 2 and 5 shown in the other

[8]The corresponding plots for the other exchange rates are provided in the appendix.

two plots in the upper panel and for lags 20, 50, and 200 provided in the lower panel. However, the maximum autocorrelation is obtained for different values of $d$. It can be observed that with increasing lag length, the exponent $d$ resulting in the maximum autocorrelation coefficient (dashed lines) tends to decrease.[9]

Figure 3: Dependence of Autocorrelation of $|r_{DEM}|^d$ on $d$ for Different Lags.




## 3.2 GARCH-Models

The results of the previous subsection indicate that autocorrelation of $r^2$ might not be the most important form of dependence in exchange rate returns. Nevertheless, keeping in mind that our aim is not the construction of an optimal time series model for exchange rate returns, but the selection of robust 'stylized facts', it seems adequate to proceed with an analysis of the traditional approach for mod-

[9]Granger (2005, p. 38) reports as a stylized fact (Taylor effect) that the slowest decline is usually found for $d = 1$.

eling volatility clustering, i.e. the class of autoregressive conditional heteroskedasticity (ARCH) models.[10]

ARCH models have been introduced by Engle (1982).[11] The standard specification of an ARCH($p$) for a return time series $r_t$ is given by

$$r_t = h_t^{1/2}\varepsilon_t\,,\; h_t = \omega + \sum_{i=1}^{p}\alpha_i r_{t-i}^2\,,\; \varepsilon_t \sim N(0,1)\,. \tag{4}$$

Thus, the current volatility of $r_t$ depends on past realizations of squared returns. Bollerslev (1986) presents the generalized ARCH (GARCH) model by adding lagged values of the unobserved volatility $h_t$ to the variance equation:

$$r_t = h_t^{1/2}\varepsilon_t\,,\; h_t = \omega + \sum_{i=1}^{p}\alpha_i r_{t-i}^2 + \sum_{i=1}^{q}\beta_i h_{t-i}\,,\; \varepsilon_t \sim N(0,1)\,. \tag{5}$$

A further modification is the exponential GARCH (EGARCH) model developed by Nelson (1991), where the conditional variance depends both on the absolute size and the sign of the lagged innovations:

$$\ln h_t = \omega + \sum_{i=1}^{p}\alpha_i(\phi\varepsilon_{t-i} + \gamma[|\varepsilon_{t-i}| - E|\varepsilon_{t-i}|]) + \sum_{i=1}^{q}\beta_i \ln h_{t-i} \tag{6}$$

Therefore, this model is able to capture asymmetries in returns, namely that negative returns have a stronger impact on the dynamic of prices than positive returns (leverage effect). However, given the functional symmetry in exchange rate returns, it is doubtful whether an EGARCH specification might be adequate for this market. Nevertheless, we provide estimates of the EGARCH(1,1) specification as a further potential benchmark.

The estimation results for the ARCH(1), the GARCH(1,1) and also for the EGARCH(1,1) model are presented in Tables 4, 5, and 6, respectively. We stick to the GARCH(1,1) framework as it appears to be the standard in applications to financial market time series. Furthermore, when comparing GARCH-models with up to three lagged values of $r_t^2$ and $h_t$, respectively, the Schwarz information criterion is minimized for the GARCH(1,1) model in most cases.[12]

[10] In fact, Ding *et al.* (1993, p. 94ff) indicate that simple GARCH models cannot reproduce the type of autocorrelation pattern described in the previous subsection and, consequently, should be considered as misspecified.

[11] For an introduction to ARCH models and modifications thereof see, e.g., Gourieroux (1997).

[12] The exceptions are $r_{FRF}$ with a minimum for the GARCH(3,3) and $r_{JPY}$ with a minimum for the GARCH(2,2) specification.

In all specifications, a constant has been estimated for the mean equation. However, this constant has never turned out to be significant. Therefore, it is not reported in the tables. The estimates remain almost unaffected if the constant is removed from the estimated specification. The numbers in paranthesis provide the estimated standard errors.

Table 4: Estimation Results for ARCH(1) Model

| ARCH(1) | $r_{DEM}$ | $r_{FRF}$ | $r_{GBP}$ | $r_{CHF}$ | $r_{JPY}$ |
|---|---|---|---|---|---|
| $\omega$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| $\alpha_1$ | 0.1392 | 0.1331 | 0.2065 | 0.1266 | 0.2112 |
| | (0.009) | (0.010) | (0.012) | (0.008) | (0.011) |

The estimates for $\alpha_1$ in the ARCH(1) model are highly significant. Nevertheless, the scaled residuals $\varepsilon_t = r_t/\sqrt{h_t}$ are still heavy tailed and exhibit some short term dependence. Therefore, the ARCH(1) model does not appear to be a relevant benchmark.

Table 5: Estimation Results for GARCH(1,1) Model

| GARCH(1,1) | $r_{DEM}$ | $r_{FRF}$ | $r_{GBP}$ | $r_{CHF}$ | $r_{JPY}$ |
|---|---|---|---|---|---|
| $\omega$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| $\alpha_1$ | 0.0708 | 0.0895 | 0.0646 | 0.0658 | 0.0578 |
| | (0.003) | (0.003) | (0.003) | (0.004) | (0.002) |
| $\beta_1$ | 0.9210 | 0.9103 | 0.9227 | 0.9270 | 0.9398 |
| | (0.004) | (0.003) | (0.004) | (0.004) | (0.002) |

The GARCH(1,1) model appears superior to the ARCH(1) specification: The relevant parameters $\alpha_1$ and $\beta_1$ are highly significant. However, the (normalized) residuals of the GARCH(1,1) are still too fat-tailed – though less pronounced than for the simple ARCH(1) – to maintain the assumption of normal distributed $\varepsilon_t$.

Furthermore, we find that the coefficients $\alpha_1$ and $\beta_1$ sum up to values close to one corresponding to an almost integrated process. This finding might result from long memory components in the returns which have to be fitted in the

GARCH(1,1) model through its exponentially decreasing autocorrelation function. Consequently, it seems appropriate to reconsider GARCH-type effects in a long memory setting, e.g., the FIGARCH model discussed in Section 5.

The parameter estimates for the GARCH(1,1) specification are highly significant. The model provides a reasonable approximation to our data. Thus, the estimates for the GARCH(1,1) specification will be used as one possible benchmark in indirect estimation approaches despite of the fact that the residuals are still fat-tailed.[13]

Table 6: Estimation Results for EGARCH(1,1) Model

| EGARCH(1,1) | $r_{DEM}$ | $r_{FRF}$ | $r_{GBP}$ | $r_{CHF}$ | $r_{JPY}$ |
|---|---|---|---|---|---|
| $\omega$ | -0.3483 | -0.4203 | -0.5041 | -0.2532 | -0.4139 |
| | (0.024) | (0.018) | (0.018) | (0.015) | (0.014) |
| $\alpha_1\phi$ | -0.0073 | 0.0125 | 0.0057 | -0.0230 | -0.0148 |
| | (0.004) | (0.004) | (0.003) | (0.003) | (0.004) |
| $\alpha_1\gamma$ | 0.1581 | 0.2022 | 0.1723 | 0.1312 | 0.1725 |
| | (0.007) | (0.006) | (0.006) | (0.004) | (0.005) |
| $\beta_1$ | 0.9775 | 0.9736 | 0.9634 | 0.9843 | 0.9716 |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) |

Finally, we consider the results of the EGARCH(1,1) specification. As pointed out before, it is difficult to provide a rational for the leverage effect when considering exchange rate returns. Thus, on the one hand, it is surprising to find significant estimates,[14] but, on the other hand, it is less surprising that the sign of these effects differs across currencies. For these reasons, the EGARCH(1,1) model does not represent a useful benchmark for indirect estimation or simulation approaches when exchange rate return series are considered.

As an alternative to the GARCH class of models, *stochastic volatility models (SV)* have been proposed. While in a GARCH model, volatility depends on past squared returns deterministically, the SV models specify the variance as a latent stochastic process. An additional stochastic term in the variance equation allows

[13]The fat-taildness of GARCH(1,1) residuals might be used as a further 'stylized fact' of return time series.

[14]Pagan (1996) also reports weak evidence for leverage effects using a different modelling approach and monthly data.

for a more flexible approximation of volatility. For example, Kim *et al.* (1998) and Carnero *et al.* (2004) point out that a Gaussian AR(1)-SV performs well for modelling real data with high kurtosis and low first-order autocorrelation of squares, while the GARCH(1,1) model in these cases requires to assume a fat-tailed conditional distribution (e.g., Student-t). On the other side, in the applications considered by Kim *et al.* (1998), the Gaussian AR(1)-SV is dominated by the t-GARCH model. Of course, more sophisticated specifications of the SV model might again lead to an improvement. Consequently, it has to be considered an open question which class of models has to be preferred for modelling conditional volatility of financial market time series. In fact, Carnero *et al.* (2004) point out that both the GARCH and the SV models allow for an ARMA-presentation and, thus, the autocorrelation patterns of squares from both processes might be similar. Furthermore, the estimation of the SV model is complicated by the presence of the random shocks in the latent variance equation. In fact, estimation requires non standard procedures like, e.g., the Kalman filter resulting in an even higher computational load than the GARCH model.[15] For these reasons, we consider only the GARCH model as a benchmark statistic for conditional heteroscedasticity in the following.

# 4 Long Memory

The autocorrelation patterns for different powers of absolute returns provide a typical indication of long memory. This finding is supported by the estimated parameters of the different GARCH specifications. Consequently, in this section, we consider explicit statistics for long memory taking into account that autocorrelation structures do not provide a complete characterization of long memory properties (Davidson and Sibbertsen, 2005, p. 267).

A standard characterization of long memory is by the hyperbolic decline of the autocorrelation coefficients $\rho_{(k)}$ for higher order lags $k$. Thereby, $\rho_{(k)}$ refers to the autocorrelation coefficients of $r_t$, $|r_t|$, and $r_t^2$, respectively, or any other power of $|r_t|$. For an approximation

$$\rho_{(k)} \sim c \cdot (k)^{2H-2}, \qquad \text{as} \qquad k \to \infty, \tag{7}$$

the rate of decay $H$ is called *Hurst exponent*. This parameter can be used to describe a self-similar property of a stochastic process (Taqqu and Teverovsky,

[15]To our knowledge, none of the standard econometric software packages like EViews, GAUSS, Matlab, or SAS includes a standard routine for the estimation of SV models.

1998, p. 177-217). Furthermore, the Hurst exponent $H$ describes the overall shape of the autocorrelation function of the process. For $1/2 < H < 1$, the process exhibits positive autocorrelation and long-range dependence, i.e. long memory. Estimates of $H$ can be obtained by different methods. In the sequel, we consider the two most widely used approaches, namely the *R/S-analysis* and the *GPH-estimator*.

### 4.1 R/S-Analysis

The range over standard deviation statistic (*R/S-statistic*), or simply rescaled range statistic, was originally developed by the English hydrologist Hurst (1951) for the analysis of river discharges. Mandelbrot (1971) was the first to apply this concept to financial markets in order to detect long-range dependence. The rescaled range statistic is defined by the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Let $r_t$ be the return for an asset in period $t$ (the same analysis is repeated for $|r_t|$ and $r_t^2$). Then, the average return over a period of length $n$, i.e. the observations $1, \ldots, n$ is given by

$$\bar{r}_n = \frac{1}{n} \sum_{t=1}^{n} r_t \,. \tag{8}$$

The difference between the maximum and the minimum accumulated deviation from the mean over a period of length $n$ is called the range ($R_n$), i.e.:

$$R_n = \max_{1 \leq k \leq n} \sum_{t=1}^{k} (r_t - \bar{r}_n) - \min_{1 \leq k \leq n} \sum_{t=1}^{k} (r_t - \bar{r}_n) \,. \tag{9}$$

To rescale this range, it is divided by the usual standard deviation

$$S_n = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (r_t - \bar{r}_n)^2} \,. \tag{10}$$

Finally, the ratio $R_n/S_n$, or $R/S$ for short, is called the rescaled range statistic.

Mandelbrot and Wallis (1969) suggest estimating the Hurst exponent $H$ by regressing the logarithm of $Q_n$ against the logarithm of the sample size $n$. Thereby, $Q_n$ denotes average of $R_n/S_n$ over all contiguous intervals of length $n$.[16] In Table 7, the values of $1/\sqrt{(n)}Q_n$ are given in the column labeled $Q_{classical}$. Critical

[16]For the calculation of the Hurst exponent and Q-statistics we use the algorithm provided by Peters (1994, p. 62).

values for $1/\sqrt{n}Q_n$ under the null hypothesis of no long memory are provided by Lo (1991, Table II).

This estimate of $H$ appears superior to several other methods, in particular for time series with large skewness and/or kurtosis (Lo, 1991). The column labeled '$H_{R/S}$' in Table 7 provides the estimates of the Hurst exponent $H$ using this standard approach. The estimates of $H$ for the return series $r_t$ are all close to 0.5. Thus, they do not provide a clear indication for the existence of long-range dependence in the return series. In contrast, the estimated Hurst exponents for absolute and squared returns are clearly above 0.5 indicating some long-range dependence in absolute and squared returns.

Unfortunately, to our knowledge, no standard asymptotic distribution theory is available for this estimates of $H$. A further possible drawback of the classical R/S analysis is that the estimated Hurst exponent appears to be biased towards a value close to 0.72 (Mandelbrot and Wallis, 1969). Finally, a potentially more important shortcoming of the R/S analysis is a bias of the estimate of $H$ resulting from short term dependence, e.g., in the sense of ARMA-processes (Lo, 1991; Jacobsen, 1996).

In order to take into account the latter issue, Lo (1991) proposes the *modified rescaled range statistic* by replacing $S_n$ in $Q_n = R_n/S_n$ through

$$S(n,q) = \sqrt{S_n^2 + 2\sum_{k=1}^{q} w_k(q)\hat{\gamma}_k}\,, \tag{11}$$

for some window length $q < n$, where $\hat{\gamma}_k$ denotes the $k$–th order autocovariance of $r_t$, $|r_t|$, and $r_t^2$, respectively, and the weights $w_k(q)$ are given by

$$w_k(q) = 1 - k/(q+1)\,. \tag{12}$$

The modified R/S-statistic has the main advantage that it is insensitive to many types of short term dependence (Jacobsen, 1996). However, a correct choice of the window length $q$ is crucial. Lo (1991) proposes to select $q$ following a data-dependent formula similar to the one proposed by Andrews (1991):

$$q = \left(\frac{3n}{2}\right)^{\frac{1}{3}} \cdot \left(\frac{2\hat{\rho}_{(1)}}{1-\hat{\rho}_{(1)}^2}\right)^{\frac{2}{3}}\,, \tag{13}$$

where $\hat{\rho}_{(1)}$ is the estimated first order autocorrelation coefficient. The results for this modified version of the test are provided in the column headed $Q_{mod.}(q = opt)$

in Table 7. The asymptotic critical values for the resulting statistic $1/\sqrt{n}\tilde{Q}_n = 1/\sqrt{n}R_n/S(n,q)$ are the same as for the standard version (see Lo (1991, Table II)).[17]

Table 7: Estimates of the Hurst Exponent and Q-statistics from the R/S Analysis

| Estimates | $H_{R/S}$ | $Q_{classical}$ | $Q_{modified}(q = opt)$ |
|---|---|---|---|
| $r_{DEM}$ | 0.6200 | 1.8290 | 1.7855 |
| $\lvert r_{DEM}\rvert$ | 0.8730 | 5.2393* | 1.4754 |
| $r^2_{DEM}$ | 0.7645 | 3.6448* | 1.9671 |
| $r_{FRF}$ | 0.6188 | 1.8971 | 1.8622 |
| $\lvert r_{FRF}\rvert$ | 0.8940 | 5.7482* | 1.6177 |
| $r^2_{FRF}$ | 0.7860 | 3.6836* | 2.1387* |
| $r_{GBP}$ | 0.5934 | 1.6196 | 1.5391 |
| $\lvert r_{GBP}\rvert$ | 0.9374 | 7.0080* | 1.7975 |
| $r^2_{GBP}$ | 0.8506 | 5.1924* | 2.6162* |
| $r_{CHF}$ | 0.6069 | 1.6202 | 1.6202 |
| $\lvert r_{CHF}\rvert$ | 0.8268 | 4.4768* | 1.1942 |
| $r^2_{CHF}$ | 0.6937 | 2.9968* | 1.2809 |
| $r_{JPY}$ | 0.6275 | 1.7053 | 1.6545 |
| $\lvert r_{JPY}\rvert$ | 0.9192 | 5.7620* | 1.5261 |
| $r^2_{JPY}$ | 0.8381 | 4.0958* | 1.9527 |

Rejection of the null hypothesis of no long memory at the 1% level is indicated by $^*$.

The results of the test based on the classical estimate of $Q_n$ clearly support the findings from the estimates of $H$ based on the R/S statistic. For the return series themselves, no long memory is found, while the absolute and squared returns exhibit significant long memory properties. The results become weaker when taking into account short run dependence considering the modified $Q_n$ statistic.[18]

[17]For a more detailed description of the R/S statistic, its modification and its properties, the reader is referred to Lo (1991) and Taqqu *et al.* (1999).

[18]However, Taqqu *et al.* (1999) indicate that under the null hypothesis of long memory, the choice of $q$ according to equation (13) might be too large resulting in too small values of the modified Q-statistic.

## 4.2 GPH-Estimator

The notion of long-range dependence is closely linked to the concept of fractional integration (see also section 9). In fact, the following relations hold between the Hurst exponent $H$ and the degree of fractional integration $d$:[19]

$$d = H - 1/2 \tag{14}$$

for finite variance processes, and

$$d = H - 1/\alpha \tag{15}$$

for infinite variance processes, where $\alpha$ is the tail index (Taqqu and Teverovsky, 1998). Given the results of the estimation of the tail index for our data set reported in Winker and Jeleskovic (2006), we might assume that the return processes exhibit a finite variance.

The most widely used approach for estimating the fractional integration parameter $d$ has been introduced by Geweke and Porter-Hudak (1983) and is called the GPH estimator (Andersson, 2002). It is based on the periodogram of the time series

$$I(\lambda_j) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} r_t \cdot e^{i\lambda_j}\right|^2 , \tag{16}$$

where $\lambda_j = 2\pi j/n$ for $j = 1,\ldots,m$ denotes the frequency and $n$ the number of observations. Close to the origin, the spectral density of a long-memory process is proportional to $|\lambda|^{1-2H}$. The regression of the periodogram against the frequency $\lambda$ in logarithms provides an estimator of the fractal coefficient. The GPH estimator uses a slightly different version, by regression on $ln|2\sin(\lambda/2)|$ instead of $ln(|\lambda|)$. For very small frequencies, both approaches are equivalent (Taqqu and Teverovsky, 1996). Thus, the GPH estimate is obtained from the following regression (Diebold and Rudebusch, 1991):

$$ln[I(\lambda_j)] = \hat{\beta_0} + \hat{\beta_1} ln[4\sin^2(\lambda_j/2)] + \varepsilon_j \tag{17}$$

This estimator is consistently and asymptotically normal (Diebold and Rudebusch, 1991; Jeng, 1999).[20] The estimator should include only the frequencies

[19] Nevertheless, as pointed out by Granger (2005), fraction integration is just one possible source of long memory besides, e.g., stochastic breaks.

[20] The asymptotic properties of the GPH-estimator hold for processes with $d \in (-1/2, 1/2)$. (Hurvich and Ray, 1995) proposed the data tapering method in order to overcome this stationarity restriction. We also calculated the estimates based on their modified approach. The qualitative results do not differ from the ones provided in Table 8.

near the origin. To this end, Geweke and Porter-Hudak (1983) proposed to choose the number of frequencies according to $m = n^{0.5}$. In our application, we consider also the exponents 0.55, 0.575 and 0.6 as proposed by Jeng (1999). The results are provided in Table 8.

Table 8: GPH Estimator for different $m$

| m= | 0.5 | 0.55 | 0.575 | 0.6 |
|---|---|---|---|---|
| $r_{DEM}$ | 0.0642 | 0.0910 | 0.1071 | 0.0715 |
| $\lvert r_{DEM}\rvert$ | 0.4506 | 0.3862 | 0.3144 | 0.3435 |
| $r^2_{DEM}$ | 0.3570 | 0.3227 | 0.2814 | 0.2788 |
| $r_{FRF}$ | 0.0485 | 0.0223 | 0.0535 | 0.0453 |
| $\lvert r_{FRF}\rvert$ | 0.4127 | 0.4065 | 0.3521 | 0.3411 |
| $r^2_{FRF}$ | 0.3192 | 0.2839 | 0.2597 | 0.2500 |
| $r_{GBP}$ | 0.0469 | 0.0338 | 0.0935 | 0.0784 |
| $\lvert r_{GBP}\rvert$ | 0.4626 | 0.4900 | 0.4354 | 0.4338 |
| $r^2_{GBP}$ | 0.3671 | 0.4399 | 0.4183 | 0.4200 |
| $r_{CHF}$ | 0.0786 | 0.0541 | 0.0731 | 0.0511 |
| $\lvert r_{CHF}\rvert$ | 0.3721 | 0.4379 | 0.3824 | 0.3714 |
| $r^2_{CHF}$ | 0.1239 | 0.1702 | 0.1349 | 0.1487 |
| $r_{JPY}$ | 0.0758 | 0.0581 | 0.0541 | 0.0737 |
| $\lvert r_{JPY}\rvert$ | 0.5022 | 0.4216 | 0.4249 | 0.3970 |
| $r^2_{JPY}$ | 0.3690 | 0.2918 | 0.3150 | 0.3211 |

The estimates of the degree of fractional integration $d$ are close to zero for all return series and all choices of $m$. Consequently, using the naive estimates of the standard error of the least squares regression, the null hypothesis of $d = 0$, i.e. no long memory, cannot be rejected for the return series. However, for the absolute and squared returns, the estimates of $d$ are much larger and often between 0.3 and 0.5. Furthermore, the null hypothesis of $d = 0$ has to be rejected at the 1% level for the absolute and squared returns for all currencies under consideration and for all choices of $m$.[21] Therefore, the estimates of $d$ for the return series, the absolute and squared returns might be used as a further benchmark of long memory behaviour.

[21]These qualitative findings remain unchanged when replacing the least squares standard deviation by the asymptotic standard deviation of the GPH-estimator (Geweke and Porter-Hudak, 1983).

## 4.3 Parametric Models of Long-Memory

A further alternative for modelling long-memory consists in parametric models such as the ARFIMA-model for powers of (absolute) returns (Granger, 1980) and the FIGARCH-model for volatility (Baillie *et al.*, 1996).

For the ARFIMA-model, we only report results of the application to absolute return motivated by the findings from section 3.1. In fact, the application to returns does not result in any significant parameter estimate,[22] while the results for squared returns are similar to the ones for absolute returns though with smaller and for some exchange rates insignificant estimates of the long-memory parameter.

Formally, an ARFIMA$(p,d,q)$ process is given by

$$\Phi(B)(1-B)^d|r_t| = \Psi(B)\varepsilon_t\,, \tag{18}$$

$$\text{where } (1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)B^k}{\Gamma(-d)\Gamma(k+1)}\,, \tag{19}$$

and $B$ is the backshift operator ($B(r_t) = r_{t-1}$), $\Phi(B)$ and $\Psi(B)$ are lag polynomials of order $p$ and $q$, respectively, with all roots of the characteristic polynomial outside the unit circle, $d$ stands for the fractional integration parameter and $\Gamma$ denotes the usual gamma function. $\varepsilon_t$ is assumed to be a white noise process. An ARFIMA$(p,d,q)$ process is stationary and invertible for $d \in (-0.5, 0.5)$. Table 9 summarized the estimation results for ARFIMA$(1,d,1)$ models fitted to the absolute exchange rate returns. All parameters are significant at the 5% level of significance. It turns out, that the estimated fractional degree of integration $d$ is quite similar for the different exchange rates. The results of ARFIMA$(1,d,1)$ seem to be very stabil over different absolute exchange rate returns. Furthermore, the order of magnitude of the estimated values of $d$ are similar to those obtained by the GPH estimator at least for $m = 0.6$.

We also considered a long memory modification of the GARCH model, the FIGARCH$(1,d,1)$ model proposed by Baillie *et al.* (1996). However, the estimation results turned out to be not robust enough to be considered as a stylized fact for exchange rates. In fact, the estimation results differ substantially among different exchange rates and different subsamples of the data. For some subsamples, the estimates of $d$ are not significantly different from zero. Consequently, we decided not to include this model for further consideration in the present context.[23]

[22] The estimation of the ARFIMA-models is performed using the Arfima package 1.04 for Ox with default settings.

[23] A further model which might be of interest in this context is the long-memory stochastic

Table 9: Estimation results for ARFIMA$(1,d,1)$-model for absolute exchange rate returns

| Return | AR(1) | d | MA(1) | Intercept |
|---|---|---|---|---|
| $\lvert r_{DEM}\rvert$ | 0.2625 | 0.3565 | -0.5782 | 0.0046 |
| $\lvert r_{FRF}\rvert$ | 0.2493 | 0.3461 | -0.5491 | 0.0044 |
| $\lvert r_{GBP}\rvert$ | 0.3305 | 0.3739 | -0.6255 | 0.0041 |
| $\lvert r_{CHF}\rvert$ | 0.4228 | 0.3767 | -0.7022 | 0.0053 |
| $\lvert r_{JPY}\rvert$ | 0.3782 | 0.3469 | -0.6302 | 0.0044 |

# 5 Stationarity

## 5.1 Unit root tests

According to Fama's definition of market efficiency (Fama, 1970), "a market in which prices always fully reflect available information is called efficient". This definition leads to the *martingale model* of asset prices, which includes the random walk as a special case. A martingale is a stochastic process $P_t$ which satisfies the following condition:

$$E[P_{t+1} - P_t | P_t, P_{t-1}, ...] = 0\,. \tag{20}$$

Thus, the martingale hypothesis implies that the forecast of tomorrow's price minimizing the mean-squared forecast error is simply today's price. Although the martingale has the disadvantage, that it does not take into account the trade-off between the risk and the expected return and although it has been shown that the martingale property is neither a necessary nor a sufficient condition for rationally determined asset prices, it is used as a benchmark model in modern theories of asset prices.

The simplest version of a martingale process, the random walk with independently and identically distributed increments, is given by

$$P_t = \mu + \alpha P_{t-1} + \varepsilon_t \tag{21}$$

where the shocks $\varepsilon_t$ are *iid*-distributed, $\alpha = 1$ and $\mu$ is a deterministic drift term. This model produces a non-stationary time series and exhibits the *unit root prop-*

volatility model. However, Casas and Gao (2005) point out that the estimation of this model is very involved. Therefore, it might not be useful for an application in the context of indirect simulated inference and is not considered in the present contribution.

*erty*. The model can be easily extended to relax the assumption of identically and/or independently distributed increments.

Given the linkage to the theory of efficient markets, it appears reasonable to consider the unit root property as a relevant property of financial market time series and to test its presence for actual samples.[24] A standard test of the hypotheses $\alpha = 1$ versus the alternative $\alpha < 1$ is the Dickey-Fuller (DF) test (Dickey and Fuller, 1979), which has been generalized to the augmented Dickey-Fuller (ADF) test in order to take into account possible serial correlation of the $\varepsilon_t$ and to allow for deterministic trend terms. Although many alternative unit root tests have been proposed and discussed in the literature,[25] we stick to the standard ADF test for our purposes. Given that the unit root property of asset prices is well documented in the literature and does not appear to depend on the specific testing methodology (Lux and Schornstein, 2005), this choice might be justified.

As suggested by Krämer (2002), we apply the ADF unit root test on the logarithms of the asset prices. The results are summarized in Table 10, which also describes the specification of the ADF test with regard to deterministic terms and lag length. The selection of the test specification is based on the Schwarz information criterion with at least a constant included as deterministic term.[26] For all considered asset prices, the null hypothesis of a unit root could not be rejected at the 5% level of significance.[27] Thus, our findings are in line with the theoretical expectation. However, the values of the test statistic – and also the specification of the deterministic terms – differ markedly across the exchange rates. Thus, it will be of interest to analyze the distribution of these values which is considered in section 6.

[24]It should be taken into account that tests of the unit root property are not equivalent with tests of the random walk hypothesis. The null hypothesis of unit root tests includes also non random walk processes. Thus, these tests are clearly not designed for detecting predictability, but are in fact insensitive to it by construction (Campbell *et al.*, 1997).

[25]In particular, modifications of the DF test taking into account possible conditional heteroskedasticity of the error terms would be of interest in the present application (Seo, 1999).

[26]This constraint does not affect the qualitative findings with the exception of the JPY/US–rate where removing the drift term leads to a borderline rejection of the unit root hypothesis. However, the robustness of the statistic, e.g., in the bootstrap analysis is increased to a relevant extent.

[27]The 5% critical values in Table 10 are taken from MacKinnon (1996).

Table 10: Results of ADF-Tests of Exchange Rates

| Exchange Rate | Specification | Test Statistic | 5% Critical Value |
|---|---|---|---|
| log(DEM/US) | no drift, no lags | -1.2256 | -2.565 |
| log(FRF/US) | no drift, no lags | -0.0255 | -2.565 |
| log(GBP/US) | with drift, lag 1 | -2.5417 | -2.862 |
| log(CHR/US) | no drift, no lags | -1.7850 | -2.862 |
| log(JPY/US) | with drift, no lags | -1.3795 | -2.862 |

# 6 Robustness

The empirical findings for the exchange rates presented in the previous sections provide clear evidence for dependence in some moments, in particular, conditional heteroskedasticity, long range dependence and non stationarity. Although these findings are observed for all exchange rates considered, some additional robustness results are required before identifying them as 'stylized facts' to be used as benchmark in simulated method of moments approaches, e.g., for estimating the parameters of agent based models or multifractal processes.

As proposed in Winker and Jeleskovic (2006), we use different approaches in order to assess the robustness of the statistics. First, the analysis is repeated for rolling window subsamples of the data. Second, a simple sample split into three subperiods is used to detect possible regime changes. Finally, the distribution of the statistics is assessed by means of bootstrap techniques taking into account a possible time dependence of the data.

## 6.1 Rolling Windows

When considering a rolling windows analysis, the choice of the window length becomes crucial when assessing (long range) dependence structures in the data. In order to obtain reliable estimates of the statistics, a large sample size is required. However, in order to spot possible structural changes, the window length should not be too long. Consequently, we consider window lengths of 1 000 and 2 500 days, respectively.

The following discussion is restricted to the window lengths of 2 500 days. The qualitative findings are similar for a window length of 1 000. These results are available on request. We do not present results for the unconditional moments such as mean or standard deviation. These results do not differ from the ones presented in Winker and Jeleskovic (2006) obtained from a simple bootstrap.

We start with an analysis of the BDS-statistic. Figure 4 exhibits the values of this test for a window length of 2 500 days and an embedding dimension of 4 for $r_{DEM}$. The plot shows the values of the statistic against the last point included in the rolling window. Results for the other exchange rates are provided in the appendix. The results for embedding dimensions of 2 and 3 and for a window length of 1 000 are qualitatively similar.[28]

Figure 4: BDS-Statistic for Rolling Windows (2 500 Days)

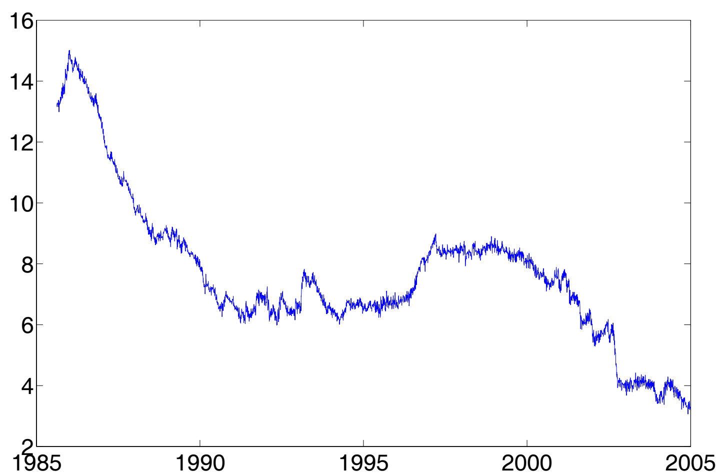


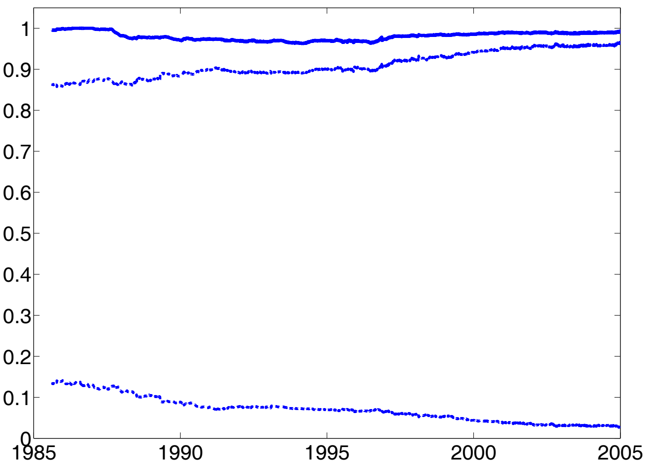

Figure 4 exhibits a clear downward trend of the BDS-statistic as the rolling window moves to the right indicating that the deviation from then null hypothesis become less pronounced for more recent time periods. Nevertheless, all statistics remain above the asymptotic critical values at the 5% level. Although the trend is unwelcome when the test statistic is used as a benchmark, the low volatility of the test statistic is a positive feature.

Figure 5 shows results of the estimated GARCH(1,1) model for $r_{DEM}$. The estimates are quite stable, but exhibit a trend over time resulting in an increased estimate of the GARCH-effect at the expense of the ARCH-effect, while the sum of both coefficients (upper line) remains almost unaltered and close to one.[29]

[28]These results are available on request.

[29]This corresponds to the stylized fact "(v)" described by Granger (2005, p. 38).

Figure 5: Estimates for GARCH(1,1) Model for Rolling Windows (2 500 Days)




In comparison to the GARCH estimates, the statistics from the $R/S$-analysis are more volatile. The left panels of Figure 6 exhibits the estimated degree of fractional integration according to the $H_{R/S}$ statistic for $r_{DEM}$ and $r^2_{DEM}$, respectively.[30] Although both series show some fluctuation over time, the order of magnitude of the estimate for the whole samples is confirmed. Similar findings are obtained for the classical and the modified $Q$-statistics, respectively.[31]

When considering the GPH estimates of the degree of fractional integration (right panels of Figure 6), we obtain similar results. For the returns themselves, the estimates are close to zero for all windows. In contrast, the estimates for the squared returns appear to be less robust hinting at some increasing trend in the long memory property. Figure 6 summarizes the findings for $m = 0.55$.

Figure 7 shows the estimated $d$ parameter from the rolling windows analysis of the Hurst (left), GPH (middle) and ARFIMA$(1,d,1)$ (right) estimates for the absolute returns $|r_{DEMR}|$.

Again, despite of some fluctuations, the long memory properties of the absolute returns appear to be quite robust. Furthermore, we might spot some similarity of the fluctuations of the estimates provided by the GPH and the ARFIMA$(1,d,1)$ model.

Finally, we consider the behaviour of the ADF-statistic under rolling win-

[30] Corresponding results for the other exchange rates are available upon request from the authors.

[31] Again, the detailed results are available upon request.

Figure 6: Hurst (left panels) and GPH$_{m=0.55}$ (right panels) Estimator of Degree of Fractional Integration for Rolling Windows (2 500 Days) for $r_{DEM}$ (upper panels) and $r^2_{DEM}$ (lower panels)

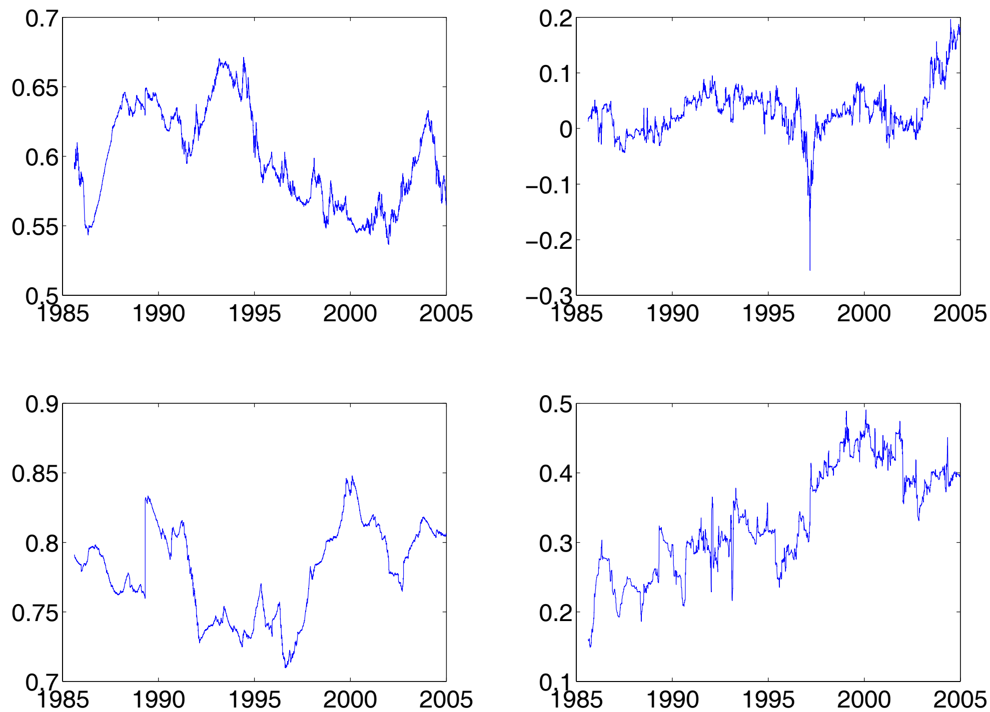
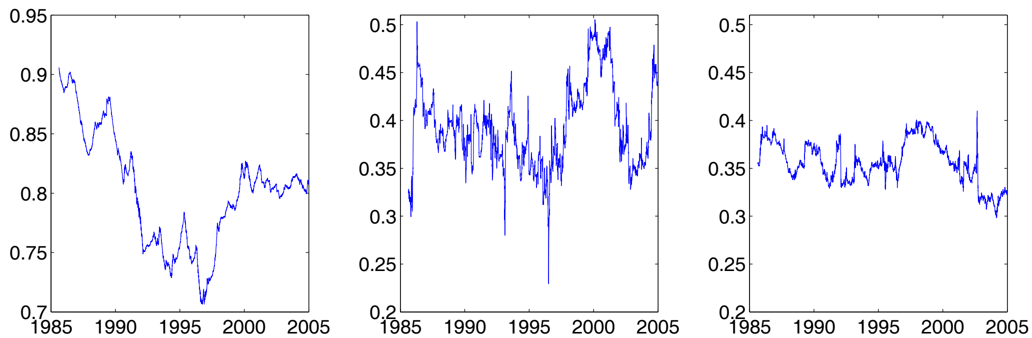


Figure 7: Estimates of $d$ by Hurst (left), $\text{GPH}_{m=0.55}$ (middle) and ARFIMA$(1,d,1)$ (right) Model for Rolling Windows (2 500 Days) for $|r_{DEM}|$




dows. Figure 8 shows the results for the exchange rate $DEM/US-\$$ and a window length of 2 500, which is representative for the other results. Obviously, the stationarity properties of the exchange rate as measured by the ADF-statistic undergo regime shifts over the sample period. This highly unwelcome feature of the ADF-statistic might be a result of the GARCH-component in the time series. Consequently, we also implemented the modified test proposed by Elliott *et al.* (1996) which is based on a local-to-unity detrending procedure and appears to be more robust in settings with GARCH-effects. However, for the exchange rates, the results of the unit root tests are not more robust than those of the ADF-test.

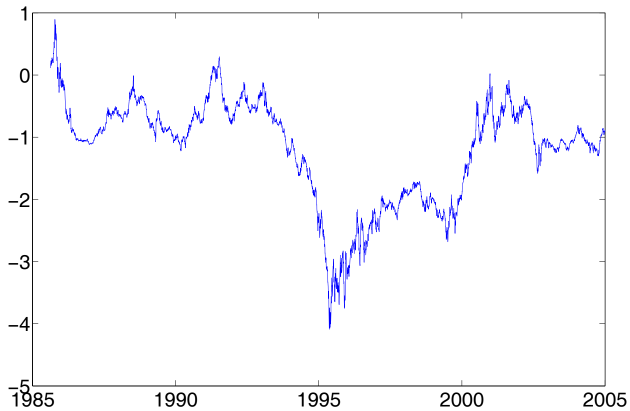
## 6.2 Subsamples

As a complement to the rolling window analysis, we also considered three non overlapping subperiods of the sample, namely the periods 1975-1984, 1985-1994, and 1995-2004. Given that the length of these subsamples is quite similar to our larger window length of 2 500, we do not expect different results. However, presenting these results in Tables 11 and 12[32] might provide an easier access to the main results than considering a large number of time series plots as presented above.

The results for the BDS-test reaffirm the tendency of a decrease over times already observed in Figure 4. However, even for the last subperiod, all test statistics

[32]The results for the other exchange rates are provided in Table 16 in the appendix.

Figure 8: ADF-Statistic for Rolling Windows (2 500 Days)




are significant at the 5% level with the sole exception of $r_{FRF}$ for a low embedding dimension. For the GARCH-estimates, the increase in the $\beta$ coefficient is also manifest, while the sum of $\alpha$ and $\beta$ remains fairly constant. A striking result, however, is obtained for the ADF-test.[33] While the null hypothesis of non stationarity could not be rejected at any conventional level of significance for the first and last subperiod, it has to be rejected for all exchange rate series in the second subperiod. Obviously, a standard unit root model is not adequate for the exchange rate time series. Alternatives like stochastic unit roots or stochastic breaks will be considered in future research.

The results for the different measures of long-memory presented in Table 12 indicate that these properties are rather robust when considering different subsamples, i.e. satisfy a central condition of ‘stylized facts’.

## 6.3 Bootstrap

A more general approach to analyze the robustness of statistics and to obtain estimates of their distribution is the bootstrap method. Given that our focus is on conditional moments of the data including measures of long memory, a block

[33] Note that the ADF-test is applied to the exchange rate series themselves.

Table 11: Test Statistics for Subperiods I

| | | BDS-Test | | | GARCH(1,1) | | | |
|---|---|---|---|---|---|---|---|---|
| | | embedding dim. | | | | | | |
| | Obs. | 2 | 3 | 4 | $\alpha$ | $\beta$ | $\alpha+\beta$ | ADF |
| 1975 - 1984 | | | | | | | | |
| $r_{DEM}$ | 2505 | 6.59 | 10.19 | 13.22 | 0.133 | 0.863 | 0.996 | 0.537 |
| $r_{FRF}$ | 2505 | 8.61 | 12.45 | 15.37 | 0.160 | 0.840 | 1.000 | 2.235 |
| $r_{GBP}$ | 2505 | 8.28 | 10.63 | 12.51 | 0.107 | 0.879 | 0.984 | 1.813 |
| $r_{CHF}$ | 2505 | 9.25 | 12.19 | 14.64 | 0.094 | 0.906 | 1.000 | -1.101 |
| $r_{JPY}$ | 2505 | 10.69 | 13.33 | 15.51 | 0.082 | 0.917 | 0.999 | -1.734 |
| 1985 - 1994 | | | | | | | | |
| $r_{DEM}$ | 2531 | 4.62 | 5.94 | 6.93 | 0.071 | 0.898 | 0.969 | -3.501 |
| $r_{FRF}$ | 2531 | 5.34 | 7.32 | 8.51 | 0.079 | 0.892 | 0.971 | -3.316 |
| $r_{GBP}$ | 2531 | 7.14 | 8.55 | 9.51 | 0.053 | 0.936 | 0.989 | -3.126 |
| $r_{CHF}$ | 2531 | 2.71 | 3.66 | 4.88 | 0.054 | 0.914 | 0.968 | -3.138 |
| $r_{JPY}$ | 2531 | 3.56 | 4.36 | 5.39 | 0.057 | 0.884 | 0.941 | -3.400 |
| 1995 - 2004 | | | | | | | | |
| $r_{DEM}$ | 2516 | 2.08 | 2.40 | 3.28 | 0.027 | 0.964 | 0.991 | -0.903 |
| $r_{FRF}$ | 2516 | 1.11 | 1.57 | 2.77 | 0.026 | 0.967 | 0.993 | -0.847 |
| $r_{GBP}$ | 2516 | 4.49 | 6.18 | 6.73 | 0.039 | 0.942 | 0.981 | -0.982 |
| $r_{CHF}$ | 2516 | 2.90 | 2.68 | 3.10 | 0.028 | 0.951 | 0.979 | -0.972 |
| $r_{JPY}$ | 2516 | 7.18 | 7.37 | 7.61 | 0.036 | 0.953 | 0.989 | -1.977 |

bootstrap procedure has to be used. All bootstrap samples are drawn from the return series. For the ADF-statistic, price series are generated by cumulating returns over time with start values drawn from historical data. Obviously, we face a trade off between long blocks in order to capture long memory properties on the one hand, and short blocks in order to justify asymptotic arguments. We experimented with different block lengths. In the following, we present the results of a block length of 250 days. For those statistics which appear robust under the bootstrapping procedure, the findings do not change much when using a block length of 100, while the results for the less robust statistics are affected to a larger extent.

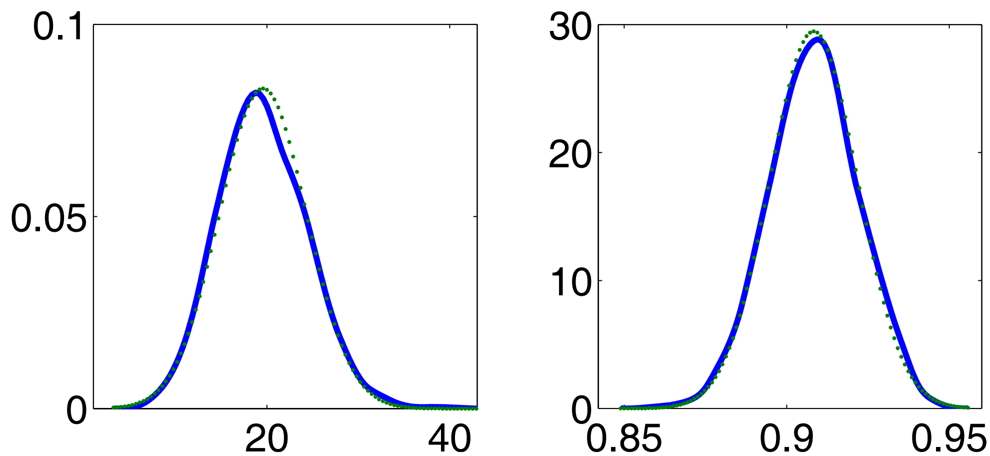
In Figure 9, the results of the block bootstrap are plotted for the BDS statistic and the $\beta$-Parameter of the GARCH(1,1)-estimates. The plots provide density estimates based on a normal kernal for 1 000 bootstrap drawings. In addition to

Table 12: Test Statistics for Subperiods II: $r_{DEM}$ and $r_{FRF}$

| | Obs. | R/S Statistics | | | GPH for $m =$ | | | | ARFIMA |
|---|---|---|---|---|---|---|---|---|---|
| | | $H_{R/S}$ | $Q_{class.}$ | $Q_{mod.}$ | 0.5 | 0.55 | 0.575 | 0.6 | $d$ |
| | | | | | 1975 - 1984 | | | | |
| $r_{DEM}$ | 2505 | 0.590 | 1.687 | 1.683 | -0.015 | 0.031 | 0.066 | 0.077 | – |
| $\|r_{DEM}\|$ | 2505 | 0.906 | 2.136 | 1.634 | 0.351 | 0.308 | 0.264 | 0.282 | 0.369 |
| $r^2_{DEM}$ | 2505 | 0.791 | 2.000 | 1.809 | 0.189 | 0.154 | 0.145 | 0.165 | – |
| $r_{FRF}$ | 2505 | 0.580 | 1.661 | 1.661 | -0.048 | 0.014 | 0.039 | 0.072 | – |
| $\|r_{FRF}\|$ | 2505 | 0.933 | 2.166 | 1.679 | 0.384 | 0.348 | 0.288 | 0.262 | 0.356 |
| $r^2_{FRF}$ | 2505 | 0.757 | 1.950 | 1.828 | 0.230 | 0.178 | 0.151 | 0.141 | – |
| | | | | | 1985 - 1994 | | | | |
| $r_{DEM}$ | 2531 | 0.600 | 1.693 | 1.675 | 0.089 | 0.030 | 0.041 | 0.051 | – |
| $\|r_{DEM}\|$ | 2531 | 0.776 | 2.095 | 1.610 | 0.301 | 0.333 | 0.384 | 0.340 | 0.386 |
| $r^2_{DEM}$ | 2531 | 0.764 | 2.059 | 1.813 | 0.294 | 0.316 | 0.384 | 0.367 | – |
| $r_{FRF}$ | 2531 | 0.594 | 1.681 | 1.659 | 0.116 | 0.043 | 0.050 | 0.051 | – |
| $\|r_{FRF}\|$ | 2531 | 0.794 | 2.126 | 1.635 | 0.339 | 0.359 | 0.422 | 0.377 | 0.355 |
| $r^2_{FRF}$ | 2531 | 0.781 | 2.088 | 1.843 | 0.295 | 0.328 | 0.393 | 0.381 | – |
| | | | | | 1995 - 2004 | | | | |
| $r_{DEM}$ | 2516 | 0.574 | 1.625 | 1.625 | 0.153 | 0.2623 | 0.232 | 0.153 | – |
| $\|r_{DEM}\|$ | 2516 | 0.808 | 1.933 | 1.521 | 0.419 | 0.416 | 0.404 | 0.385 | 0.348 |
| $r^2_{DEM}$ | 2516 | 0.803 | 1.932 | 1.709 | 0.376 | 0.411 | 0.366 | 0.338 | – |
| $r_{FRF}$ | 2516 | 0.572 | 1.618 | 1.618 | 0.154 | 0.219 | 0.193 | 0.124 | – |
| $\|r_{FRF}\|$ | 2516 | 0.800 | 1.939 | 1.569 | 0.461 | 0.372 | 0.348 | 0.306 | 0.332 |
| $r^2_{FRF}$ | 2516 | 0.773 | 1.894 | 1.723 | 0.374 | 0.341 | 0.296 | 0.269 | – |

the kernal estimates, a normal approximation to the bootstrap distribution is shown as dotted lines.

Figure 9: Bootstrap Distributions (Block Bootstrap) for BDS and GARCH(1,1)




These bootstrap distributions look reasonably well, while some of the statistics for measuring long memory appear to be rather unstable as can be observed from the summary information provided in Table 13 for $r_{DM}$.[34] The bootstrap distributions for the BDS statistic are well centered around its empirical values with a 5%–Quantile well above the critical value. Also the parameters of the GARCH(1,1) model are quite well behaved, it becomes obvious that the sum of $\alpha$ and $\beta$ is an even more robust statistic (smaller confidence interval). Regarding the measures of long memory, only the statistics for the absolute returns appear to be rather robust. One possible reason for the bad performance of the measures of long memory might be the relatively low block length. Finally, the missing robustness of the ADF statistic found in the rolling windows and subsample analysis is confirmed by the bootstrap estimates.

Figure 10 exhibits the distribution of the $d$ parameter obtained by the block bootstrap for the Hurst, GPH ($m = 0.55$) and ARFIMA$(1,d,1)$ models for $|r_{DEMR}|$.

Table 13 indicated a smaller variance of the estimates obtained from the ARFIMA model for absolute returns. Thus, it might be considered the best statistics for modelling the stylized fact of long memory in absolute returns for foreign ex-

[34]The corresponding results for the other exchange rates are provided in the appendix.

Table 13: Bootstrap Distribution of $r_{DEM}$

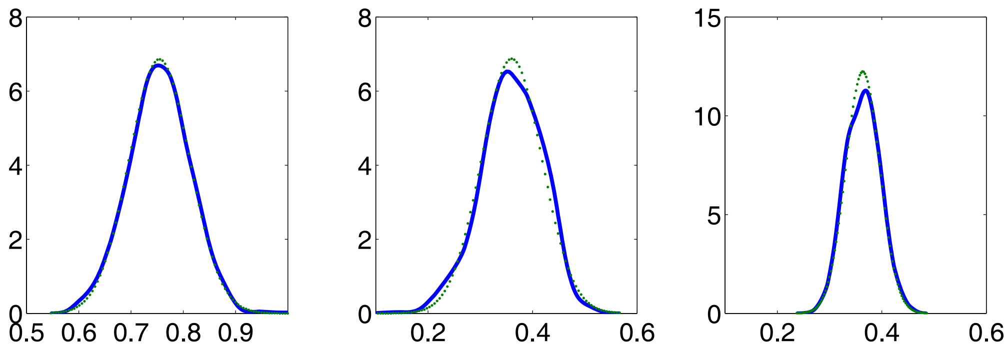
| Statistic | DM/US | Mean | 5% | 50% | 95% | Std.dev. |
|---|---|---|---|---|---|---|
| BDS (dim. 2) | 9.611 | 9.661 | 5.629 | 9.596 | 13.813 | 2.4600 |
| BDS (dim. 3) | 14.708 | 14.727 | 9.137 | 14.604 | 20.782 | 3.558 |
| BDS (dim. 4) | 19.541 | 19.555 | 12.104 | 19.293 | 27.628 | 4.787 |
| GARCH(1,1) $\alpha$ | 0.071 | 0.078 | 0.059 | 0.078 | 0.099 | 0.012 |
| GARCH(1,1) $\beta$ | 0.921 | 0.908 | 0.886 | 0.909 | 0.931 | 0.014 |
| GARCH(1,1) $\alpha+\beta$ | 0.992 | 0.987 | 0.972 | 0.988 | 0.997 | 0.008 |
| $H_{R/S}\ r_{DEM}$ | 0.620 | 0.554 | 0.475 | 0.555 | 0.632 | 0.048 |
| $H_{R/S}\ \|r_{DEM}\|$ | 0.873 | 0.754 | 0.655 | 0.754 | 0.848 | 0.058 |
| $H_{R/S}\ r^2_{DEM}$ | 0.765 | 0.699 | 0.598 | 0.694 | 0.793 | 0.059 |
| $Q_{class.}\ r_{DEM}$ | 1.829 | 1.481 | 1.130 | 1.487 | 1.846 | 0.219 |
| $Q_{class.}\ \|r_{DEM}\|$ | 5.239 | 4.017 | 2.826 | 3.980 | 5.343 | 0.779 |
| $Q_{class.}\ r^2_{DEM}$ | 3.644 | 3.062 | 2.173 | 3.012 | 4.059 | 0.587 |
| $Q_{mod.}\ r_{DEM}$ | 1.785 | 1.451 | 1.110 | 1.455 | 1.812 | 0.214 |
| $Q_{mod.}\ \|r_{DEM}\|$ | 1.475 | 1.120 | 0.825 | 1.109 | 1.442 | 0.196 |
| $Q_{mod.}\ r^2_{DEM}$ | 1.967 | 1.658 | 1.216 | 1.632 | 2.156 | 0.290 |
| GPH ($m = 0.55$) $r_{DEM}$ | 0.091 | 0.044 | -0.041 | 0.044 | 0.128 | 0.051 |
| GPH ($m = 0.55$) $\|r_{DEM}\|$ | 0.386 | 0.360 | 0.255 | 0.361 | 0.447 | 0.058 |
| GPH ($m = 0.55$) $r^2_{DEM}$ | 0.323 | 0.266 | 0.163 | 0.269 | 0.366 | 0.062 |
| ARFIMA $d$ $\|r_{DEM}\|$ | 0.357 | 0.364 | 0.311 | 0.364 | 0.416 | 0.033 |
| ADF (no drift) ($DEM$) | -1.2256 | -0.9532 | -2.4161 | -1.0633 | 0.7706 | 0.9858 |
| ADF (with drift) ($DEM$) | -1.2494 | -1.2209 | -2.6778 | -1.3051 | 0.4996 | 0.9621 |

change rate time series. However, given the high computational cost of estimating the ARFIMA model, it might not be well suited in a framework of simulated indirect inference. In this case, the GPH-estimator might be a more adequate choice.

# 7 Time aggregation

As a final issue, we consider the effect of temporal aggregation on the statistics introduced above. Obviously, short run autocorrelation structures of returns or powers of returns might be affected by temporal aggregation (Ding *et al.*, 1993, p. 92). The same applies to coefficient estimates of the GARCH-model. For mea-

Figure 10: Bootstrap Distributions (Block Bootstrap) of $d$ for Hurst, GPH ($m = 0.55$) and ARFIMA$(1,d,1)$ model




sures of non linearity, long memory, and non stationarity, the effect of temporal aggregation is not obvious a priori.

We reevaluate all statistics presented above after aggregating returns for 5, 20 and 60 observations corresponding roughly to weekly, monthly and quarterly data, respectively. The results are summarized in Tables 14 and 15. According to the BDS statistic, the deviations from the independence assumption decrease under time aggregation. With the exception of the British Pound, the statistic becomes insignificant for the quarterly data, while it is still significant for all exchange rates at the weekly frequency and for most rates at the monthly frequency. A similar tendency can be observed for the parameters of the GARCH(1,1) model. While they still sum up to values close to one at the weekly frequency, their values shrink when looking at the monthly frequency and are close to zero (again, with the exception of the British Pound) for the quarterly frequency.

While the $H_{R/S}$ statistic does not change much under time aggregation, the results for the $Q_{class.}$ statistic and the GPH estimator show the same tendency.[35] The significance of the $Q_{class.}$ statistic for absolute and squared returns found at the daily and weekly frequency disappears for lower frequencies. Also, the GPH estimators decrease and are close to zero for the monthly or lower frequencies.

Finally, ARFIMA estimates exhibit a slightly different behavior comparing to the $H_{R/S}$ and GPH estimators. While the estimates of the long memory parameter do not differ substantially for weekly and monthly data, the estimates are not sig-

[35] This result is also found for the modified $Q$ statistic; not shown.

Table 14: Test Statistics under Time Aggregation I

| Series | Obs. | BDS | | | GARCH | | | ADF | |
|---|---|---|---|---|---|---|---|---|---|
| | | dim. 2 | dim. 3 | dim. 4 | $\alpha$ | $\beta$ | $\alpha+\beta$ | no drift | drift |
| 1-day returns | | | | | | | | | |
| $r_{DEM}$ | 7554 | 8.58 | 11.76 | 14.75 | 0.071 | 0.921 | 0.992 | -1.087 | -1.297 |
| $r_{FRF}$ | 7554 | 9.53 | 13.51 | 17.04 | 0.090 | 0.910 | 0.999 | -0.203 | -1.546 |
| $r_{GBP}$ | 7554 | 13.69 | 17.18 | 19.56 | 0.065 | 0.923 | 0.987 | -0.050 | -2.381 |
| $r_{CHF}$ | 7554 | 9.17 | 11.48 | 14.18 | 0.066 | 0.927 | 0.993 | -1.648 | -1.841 |
| $r_{JPY}$ | 7554 | 12.31 | 14.57 | 16.89 | 0.058 | 0.940 | 0.998 | -2.629 | -1.859 |
| 5-day returns | | | | | | | | | |
| $r_{DEM}$ | 1510 | 6.18 | 7.99 | 8.77 | 0.132 | 0.799 | 0.931 | -1.330 | -1.206 |
| $r_{FRF}$ | 1510 | 3.57 | 5.66 | 6.58 | 0.068 | 0.926 | 0.994 | -1.640 | -0.032 |
| $r_{GBP}$ | 1510 | 5.51 | 6.84 | 7.78 | 0.077 | 0.902 | 0.979 | -2.528 | -1.007 |
| $r_{CHF}$ | 1510 | 2.76 | 4.50 | 5.74 | 0.085 | 0.872 | 0.957 | -1.620 | -1.784 |
| $r_{JPY}$ | 1510 | 6.46 | 8.23 | 9.45 | 0.089 | 0.893 | 0.982 | -1.407 | -1.875 |
| 20-day returns | | | | | | | | | |
| $r_{DEM}$ | 376 | 2.50 | 3.08 | 2.95 | 0.194 | 0.102 | 0.296 | -1.567 | -1.141 |
| $r_{FRF}$ | 376 | 1.66 | 1.76 | 2.15 | 0.046 | 0.883 | 0.929 | -1.804 | 0.005 |
| $r_{GBP}$ | 376 | 3.14 | 3.53 | 4.31 | 0.128 | 0.728 | 0.856 | -2.629 | -1.088 |
| $r_{CHF}$ | 376 | 1.87 | 3.01 | 3.30 | 0.111 | 0.673 | 0.784 | -1.768 | -1.737 |
| $r_{JPY}$ | 376 | 2.70 | 2.61 | 2.70 | 0.118 | 0.735 | 0.853 | -1.481 | -1.691 |
| 60-day returns | | | | | | | | | |
| $r_{DEM}$ | 125 | 0.05 | 0.12 | -0.16 | 0.002 | 0.000 | 0.002 | -1.671 | -1.099 |
| $r_{FRF}$ | 125 | -0.99 | -1.49 | -1.62 | 0.000 | 0.110 | 0.110 | -1.871 | 0.053 |
| $r_{GBP}$ | 125 | 2.61 | 2.48 | 2.42 | 0.501 | 0.017 | 0.518 | -2.790 | -1.072 |
| $r_{CHF}$ | 125 | 0.07 | -0.01 | 0.13 | 0.031 | 0.000 | 0.031 | -1.902 | -1.699 |
| $r_{JPY}$ | 125 | 0.51 | 0.83 | 0.47 | 0.000 | 0.095 | 0.095 | -1.524 | -1.579 |

Table 15: Test Statistics under Time Aggregation II

| | Obs. | $H_{R/S}$ | | | $Q_{class.}$ | | | GPH ($m = 0.55$) | | | ARFIMA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $r$ | $\|r\|$ | $r^2$ | $r$ | $\|r\|$ | $r^2$ | $r$ | $\|r\|$ | $r^2$ | $d$ for $\|r\|$ |
| 1-day returns | | | | | | | | | | | |
| $r_{DEM}$ | 7554 | 0.582 | 0.906 | 0.786 | 1.419 | 6.247 | 4.576 | 0.091 | 0.386 | 0.323 | 0.357 |
| $r_{FRF}$ | 7554 | 0.631 | 0.922 | 0.800 | 1.718 | 6.409 | 4.469 | 0.022 | 0.407 | 0.284 | 0.346 |
| $r_{GBP}$ | 7554 | 0.567 | 0.996 | 0.896 | 1.540 | 9.681 | 7.166 | 0.034 | 0.490 | 0.440 | 0.374 |
| $r_{CHF}$ | 7554 | 0.542 | 0.842 | 0.646 | 1.307 | 5.791 | 3.304 | 0.054 | 0.438 | 0.170 | 0.378 |
| $r_{JPY}$ | 7554 | 0.536 | 0.843 | 0.736 | 1.412 | 5.618 | 3.894 | 0.058 | 0.422 | 0.292 | 0.347 |
| 5-day returns | | | | | | | | | | | |
| $r_{DEM}$ | 1510 | 0.630 | 0.816 | 0.693 | 1.728 | 2.844 | 2.087 | 0.019 | 0.236 | 0.165 | 0.151 |
| $r_{FRF}$ | 1510 | 0.640 | 0.829 | 0.759 | 1.839 | 2.856 | 2.360 | 0.083 | 0.246 | 0.184 | 0.153 |
| $r_{GBP}$ | 1510 | 0.604 | 0.867 | 0.778 | 1.516 | 3.346 | 2.540 | 0.120 | 0.300 | 0.244 | 0.165 |
| $r_{CHF}$ | 1510 | 0.632 | 0.772 | 0.696 | 1.623 | 2.391 | 2.138 | 0.064 | 0.298 | 0.208 | 0.135 |
| $r_{JPY}$ | 1510 | 0.647 | 0.846 | 0.739 | 1.630 | 2.952 | 2.138 | 0.020 | 0.400 | 0.259 | 0.158 |
| 20-day returns | | | | | | | | | | | |
| $r_{DEM}$ | 376 | 0.665 | 0.687 | 0.655 | 1.526 | 1.500 | 1.323 | 0.135 | 0.160 | 0.069 | 0.137 |
| $r_{FRF}$ | 376 | 0.676 | 0.673 | 0.635 | 1.628 | 1.405 | 1.285 | 0.206 | 0.307 | 0.268 | 0.121 |
| $r_{GBP}$ | 376 | 0.638 | 0.782 | 0.701 | 1.417 | 1.970 | 1.720 | -0.010 | 0.256 | 0.123 | 0.147 |
| $r_{CHF}$ | 376 | 0.661 | 0.598 | 0.556 | 1.466 | 1.219 | 1.183 | 0.058 | 0.164 | 0.067 | 0.126 |
| $r_{JPY}$ | 376 | 0.672 | 0.688 | 0.662 | 1.419 | 1.540 | 1.490 | 0.097 | 0.251 | 0.141 | 0.121 |
| 60-day returns | | | | | | | | | | | |
| $r_{DEM}$ | 125 | 0.716 | 0.671 | 0.654 | 1.420 | 1.152 | 1.111 | 0.010 | -0.159 | -0.095 | -0.045 |
| $r_{FRF}$ | 125 | 0.711 | 0.647 | 0.601 | 1.455 | 1.158 | 1.069 | 0.099 | -0.179 | -0.061 | -0.018 |
| $r_{GBP}$ | 125 | 0.593 | 0.732 | 0.587 | 1.220 | 1.357 | 1.101 | -0.161 | 0.296 | 0.141 | 0.212 |
| $r_{CHF}$ | 125 | 0.702 | 0.619 | 0.674 | 1.270 | 1.049 | 1.193 | 0.013 | -0.030 | 0.023 | 0.040 |
| $r_{JPY}$ | 125 | 0.673 | 0.581 | 0.636 | 1.267 | 1.092 | 1.226 | -0.027 | 0.112 | -0.181 | 0.051 |

nificant at the quarterly frequency with the exception of $r_{GBP}$.[36] When interpreting these findings, it should be taken into account that the number of observations on lower frequencies is rather small.

Overall the findings of behaviour of several statistics under time aggregation support the findings in Winker and Jeleskovic (2006) that return distributions for exchange rate become more similar to white noise processes when longer return periods are considered. These results represent another stylized facts of return distributions which might be used for indirect estimation approaches.

# 8 Conclusion

Several 'stylized facts' of the conditional distribution of exchange rate returns are studied using different statistics and methods for the assessment of their robustness for different (sub)samples and under time aggregation. It is found that several features of the data are very robust and suited to serve as benchmark for testing financial market models, e.g. agent based models or multifractal models.

In particular, the BDS statistic, the sum of the two parameters of the GARCH(1,1) model, and some estimates of long memory appear to be useful for this purpose. However, final statements on the statistics related to long memory are not possible given that the bootstrap estimates do not provide reliable information given the available sample length. In particular, it is not possible to use information from the bootstrap procedure to estimate the standard deviation of these statistics. By contrast, the standard unit root test statistic (ADF) appears to be very unstable. Other test statistics or models of alternative unit root hypothesis (stochastic unit root, stochastic breaks) will have to be considered in future work.

Combining the findings of this paper with the results presented in Winker and Jeleskovic (2006), it appears feasible to construct a robust objective function for means of indirect estimation. This is the next issue on our research agenda.

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[36] We point out that the estimates for the lower frequencies are based on the ARFIMA$(0,d,0)$-model as the AR and MA parts have been found to be not significant in most cases.

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# A Autocorrelation Plots

Figure 11: Sample autocorrelation for $r_{FRF}$ and $|r_{FRF}|^d$ for different values of $d$.

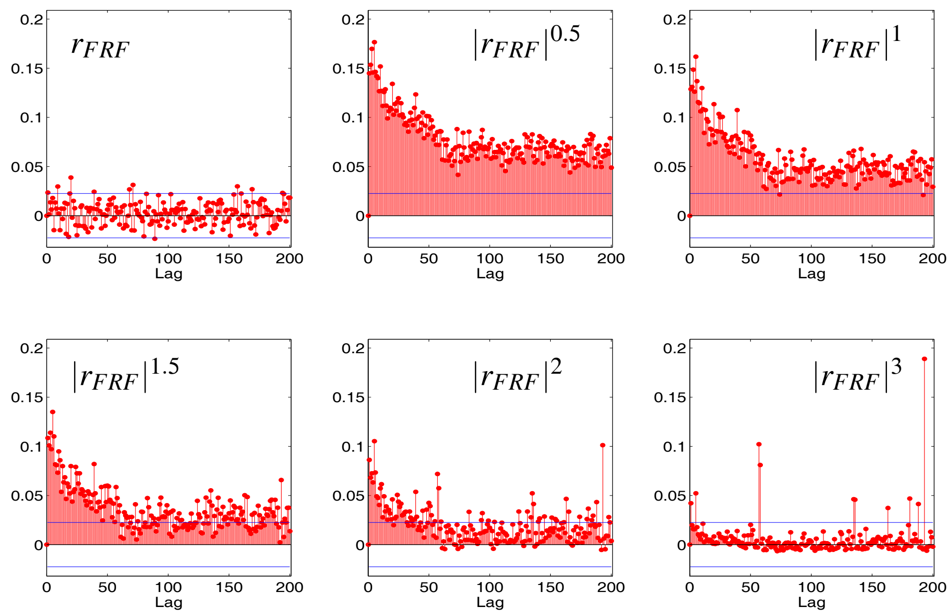


Figure 12: Sample autocorrelation for $r_{GBP}$ and $|r_{GBP}|^d$ for different values of $d$.

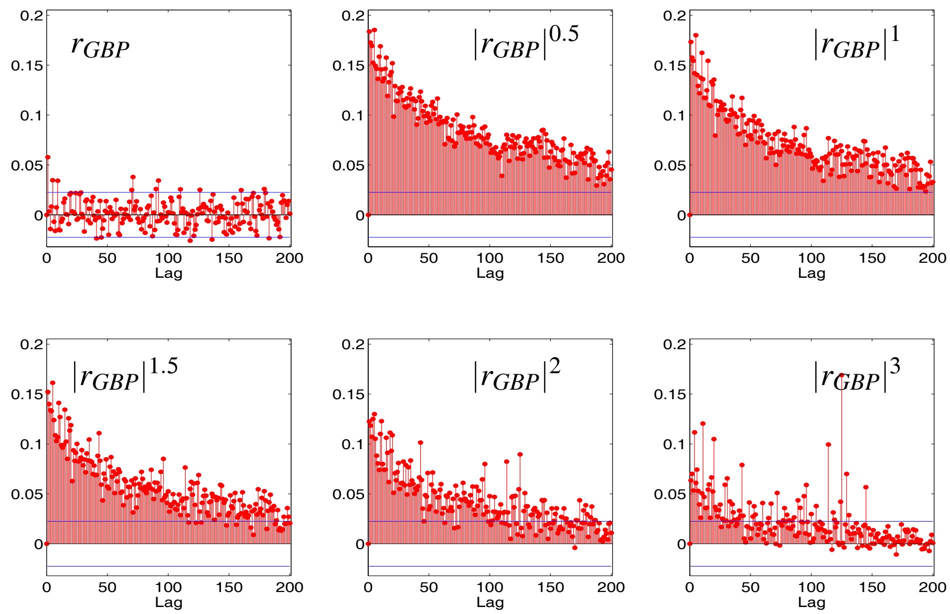



Figure 13: Sample autocorrelation for $r_{CHF}$ and $|r_{CHF}|^d$ for different values of $d$.

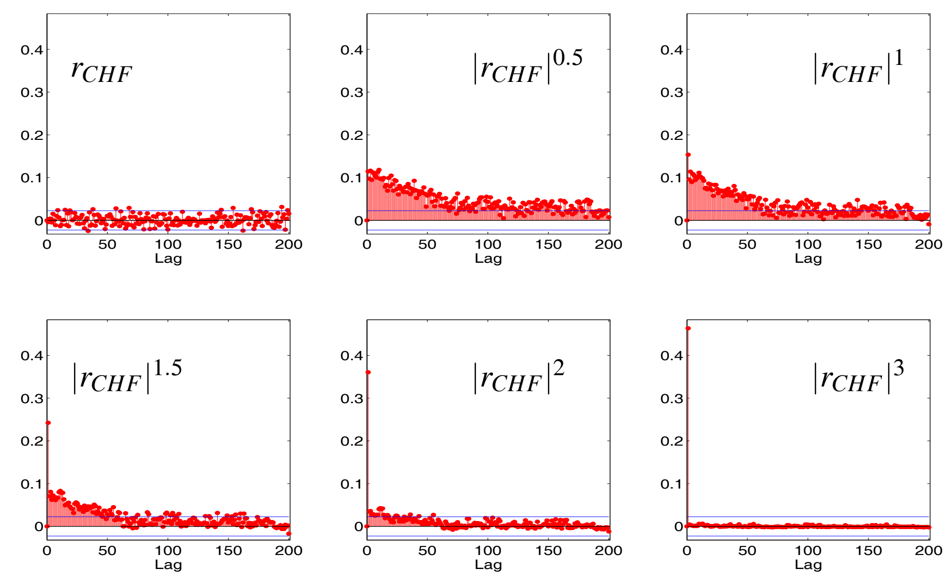


Figure 14: Sample autocorrelation for $r_{JPY}$ and $|r_{JPY}|^d$ for different values of $d$.

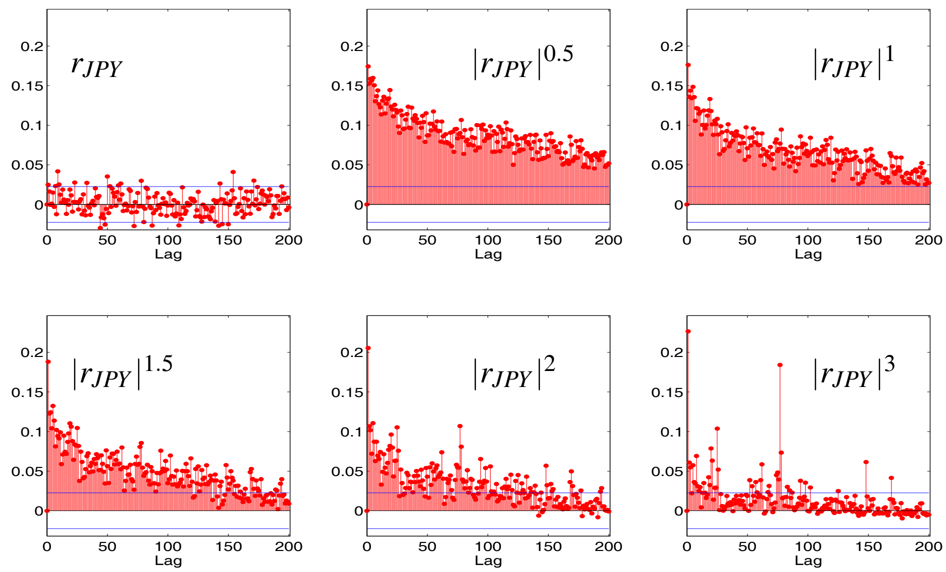



Figure 15: Dependence of Autocorrelation of $|r_{FRF}|^d$ on $d$ for Different Lags.

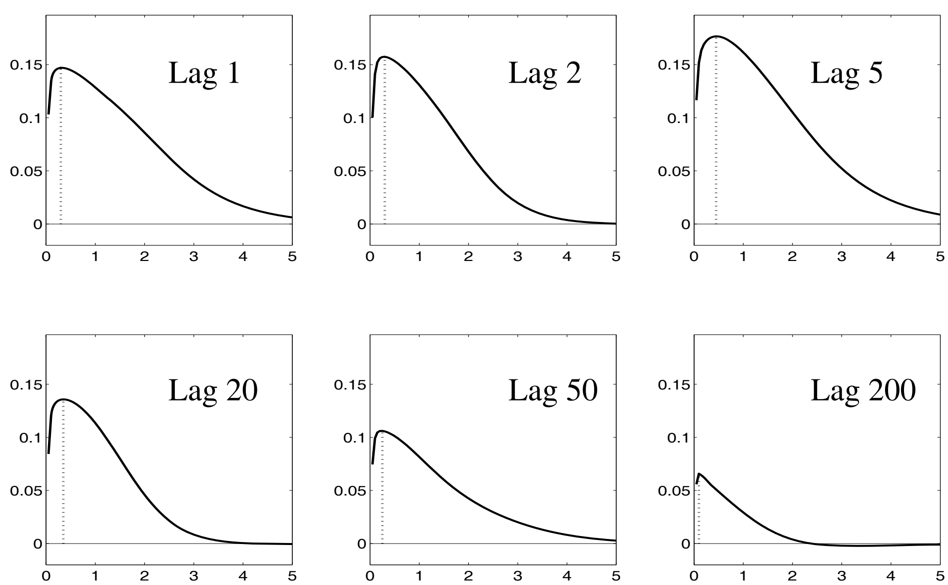


Figure 16: Dependence of Autocorrelation of $|r_{GBP}|^d$ on $d$ for Different Lags.

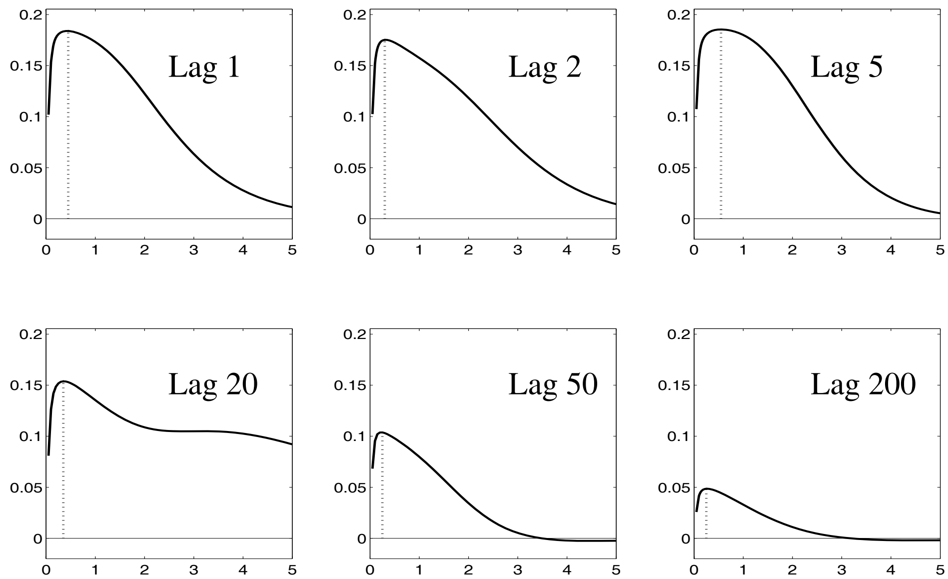



Figure 17: Dependence of Autocorrelation of $|r_{CHF}|^d$ on $d$ for Different Lags.

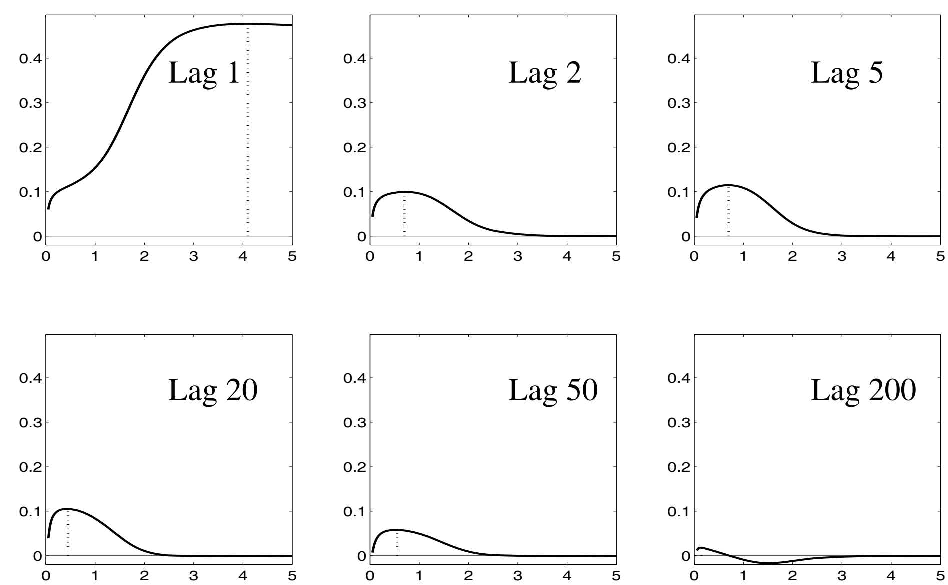
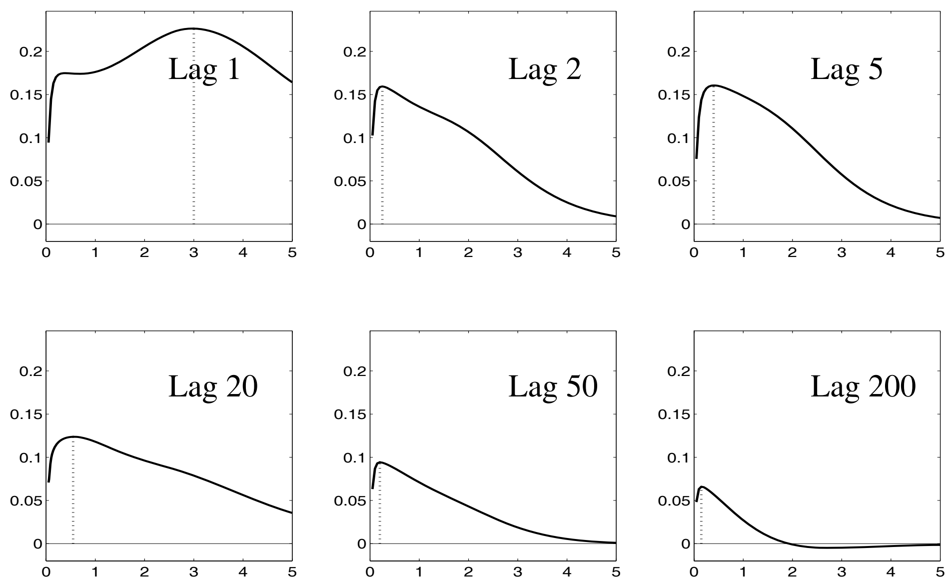


Figure 18: Dependence of Autocorrelation of $|r_{JPY}|^d$ on $d$ for Different Lags.



# B Further Robustness Results

## B.1 Further Results for Subperiods

Table 16: Test Statistics for Subperiods II: $r_{GBP}$, $r_{CHF}$, and $r_{JPY}$

| | Obs. | R/S Statistics | | | GPH for $m =$ | | | | ARFIMA |
|---|---|---|---|---|---|---|---|---|---|
| | | $H_{R/S}$ | $Q_{class.}$ | $Q_{mod.}$ | 0.5 | 0.55 | 0.575 | 0.6 | $d$ |
| 1975 - 1984 | | | | | | | | | |
| $r_{GBP}$ | 2505 | 0.692 | 1.788 | 1.774 | 0.173 | 0.140 | 0.106 | 0.120 | - |
| $\|r_{GBP}\|$ | 2505 | 0.830 | 2.082 | 1.572 | 0.395 | 0.369 | 0.344 | 0.330 | 0.357 |
| $r^2_{GBP}$ | 2505 | 0.734 | 1.944 | 1.726 | 0.292 | 0.280 | 0.291 | 0.258 | - |
| $r_{CHF}$ | 2505 | 0.560 | 1.662 | 1.662 | 0.059 | 0.094 | 0.088 | 0.083 | - |
| $\|r_{CHF}\|$ | 2505 | 0.926 | 2.211 | 1.681 | 0.548 | 0.467 | 0.434 | 0.428 | 0.413 |
| $r^2_{CHF}$ | 2505 | 0.775 | 1.995 | 1.727 | 0.164 | 0.180 | 0.152 | 0.141 | - |
| $r_{JPY}$ | 2505 | 0.661 | 1.818 | 1.811 | 0.109 | 0.181 | 0.131 | 0.207 | - |
| $\|r_{JPY}\|$ | 2505 | 0.913 | 2.326 | 1.797 | 0.398 | 0.404 | 0.350 | 0.356 | 0.3935 |
| $r^2_{JYP}$ | 2505 | 0.773 | 2.147 | 1.881 | 0.243 | 0.265 | 0.241 | 0.239 | - |
| 1985 - 1994 | | | | | | | | | |
| $r_{GBP}$ | 2531 | 0.555 | 1.697 | 1.661 | 0.169 | 0.114 | 0.080 | 0.104 | - |
| $\|r_{GBP}\|$ | 2531 | 0.836 | 2.176 | 1.660 | 0.418 | 0.442 | 0.460 | 0.432 | 0.416 |
| $r^2_{GBP}$ | 2531 | 0.824 | 2.121 | 1.878 | 0.390 | 0.565 | 0.506 | 0.463 | - |
| $r_{CHF}$ | 2531 | 0.621 | 1.708 | 1.692 | 0.097 | 0.087 | 0.063 | 0.076 | - |
| $\|r_{CHF}\|$ | 2531 | 0.926 | 2.054 | 1.629 | 0.346 | 0.319 | 0.325 | 0.311 | 0.371 |
| $r^2_{CHF}$ | 2531 | 0.714 | 2.002 | 1.796 | 0.370 | 0.295 | 0.309 | 0.328 | - |
| $r_{JPY}$ | 2531 | 0.574 | 1.711 | 1.682 | 0.064 | 0.040 | 0.035 | 0.066 | - |
| $\|r_{JPY}\|$ | 2531 | 0.713 | 1.880 | 1.451 | 0.174 | 0.153 | 0.166 | 0.206 | 0.251 |
| $r^2_{JPY}$ | 2531 | 0.685 | 1.774 | 1.614 | 0.058 | 0.124 | 0.126 | 0.154 | - |
| 1995 - 2004 | | | | | | | | | |
| $r_{GBP}$ | 2516 | 0.482 | 1.559 | 1.559 | -0.111 | -0.094 | -0.130 | -0.123 | - |
| $\|r_{GBP}\|$ | 2516 | 0.776 | 1.972 | 1.515 | 0.386 | 0.400 | 0.302 | 0.295 | 0.314 |
| $r^2_{GBP}$ | 2516 | 0.728 | 1.914 | 1.710 | 0.360 | 0.404 | 0.304 | 0.325 | - |
| $r_{CHF}$ | 2516 | 0.540 | 1.557 | 1.557 | 0.050 | 0.090 | 0.090 | 0.052 | - |
| $\|r_{CHF}\|$ | 2516 | 0.748 | 1.918 | 1.468 | 0.390 | 0.315 | 0.274 | 0.261 | 0.312 |
| $r^2_{CHF}$ | 2516 | 0.745 | 1.908 | 1.637 | 0.386 | 0.355 | 0.298 | 0.268 | - |
| $r_{JPY}$ | 2516 | 0.608 | 1.758 | 1.757 | 0.044 | 0.037 | -0.015 | 0.003 | - |
| $\|r_{JPY}\|$ | 2516 | 0.872 | 2.006 | 1.484 | 0.377 | 0.405 | 0.438 | 0.346 | 0.378 |
| $r^2_{JPY}$ | 2516 | 0.831 | 1.935 | 1.617 | 0.347 | 0.397 | 0.380 | 0.252 | - |

# C Further Bootstrap Results

Table 17: Bootstrap Distribution of $r_{FRF}$

| Statistic | FRF/US | Mean | 5% | 50% | 95% | Std.dev. |
|---|---|---|---|---|---|---|
| BDS (dim. 2) | 11.699 | 11.771 | 6.987 | 11.681 | 17.069 | 3.056 |
| BDS (dim. 3) | 18.082 | 18.097 | 11.147 | 17.899 | 25.759 | 4.524 |
| BDS (dim. 4) | 24.447 | 24.438 | 14.666 | 24.103 | 35.317 | 6.295 |
| GARCH(1,1) $\alpha$ | 0.0895 | 0.0957 | 0.0694 | 0.0952 | 0.1232 | 0.0168 |
| GARCH(1,1) $\beta$ | 0.9103 | 0.8942 | 0.8633 | 0.8950 | 0.9228 | 0.0179 |
| GARCH(1,1) $\alpha+\beta$ | 0.9998 | 0.9899 | 0.9672 | 0.9943 | 1.0000 | 0.0114 |
| $H_{R/S}\ r_{FRF}$ | 0.618 | 0.5654 | 0.4868 | 0.5644 | 0.6406 | 0.0477 |
| $H_{R/S}\ \|r_{FRF}\|$ | 0.894 | 0.7600 | 0.6623 | 0.7595 | 0.8571 | 0.0600 |
| $H_{R/S}\ r^2_{FRF}$ | 0.786 | 0.6794 | 0.5787 | 0.6802 | 0.7753 | 0.0602 |
| $Q_{class.}\ r_{FRF}$ | 1.897 | 1.5353 | 1.1655 | 1.5415 | 1.9012 | 0.2293 |
| $Q_{class.}\ \|r_{FRF}\|$ | 5.748 | 4.1717 | 2.8976 | 4.1040 | 5.5700 | 0.8414 |
| $Q_{class.}\ r^2_{FRF}$ | 3.683 | 2.8518 | 2.0010 | 2.7988 | 3.7678 | 0.5570 |
| $Q_{mod.}\ r_{FRF}$ | 1.862 | 1.5046 | 1.1362 | 1.5121 | 1.8617 | 0.2252 |
| $Q_{mod.}\ \|r_{FRF}\|$ | 1.617 | 1.1613 | 0.8481 | 1.1561 | 1.5149 | 0.2063 |
| $Q_{mod.}\ r^2_{FRF}$ | 2.1387 | 1.6731 | 1.2013 | 1.6562 | 2.1698 | 0.2920 |
| GPH ($m=0.55$) $r_{FRF}$ | 0.0223 | 0.0423 | -0.0468 | 0.0457 | 0.1225 | 0.0532 |
| GPH ($m=0.55$) $\|r_{FRF}\|$ | 0.4065 | 0.3840 | 0.2748 | 0.3893 | 0.4811 | 0.0633 |
| GPH ($m=0.55$) $r^2_{FRF}$ | 0.2839 | 0.2403 | 0.1340 | 0.2452 | 0.3384 | 0.0634 |
| ARFIMA $d$ $\|r_{FRF}\|$ | 0.3461 | 0.3564 | 0.3042 | 0.3565 | 0.4091 | 0.0326 |
| ADF (no drift) ($FRF$) | -0.0255 | -0.0679 | -2.1886 | -0.0610 | 2.0042 | 1.2617 |
| ADF (with drift) ($FRF$) | -1.5816 | -1.2862 | -2.6863 | -1.3326 | 0.2896 | 0.8975 |

Table 18: Bootstrap Distribution of $r_{GBP}$

| Statistic | GBP/US | Mean | 5% | 50% | 95% | Std.dev. |
|---|---|---|---|---|---|---|
| BDS (dim. 2) | 14.939 | 14.910 | 10.347 | 14.879 | 19.4025 | 2.630 |
| BDS (dim. 3) | 20.080 | 19.978 | 13.982 | 19.914 | 26.236 | 3.740 |
| BDS (dim. 4) | 25.167 | 24.988 | 16.746 | 24.710 | 33.715 | 5.174 |
| GARCH(1,1) $\alpha$ | 0.0646 | 0.0739 | 0.0547 | 0.0734 | 0.0949 | 0.0124 |
| GARCH(1,1) $\beta$ | 0.9227 | 0.9098 | 0.8818 | 0.9112 | 0.9327 | 0.0155 |
| GARCH(1,1) $\alpha+\beta$ | 0.9873 | 0.9837 | 0.9686 | 0.9843 | 0.9981 | 0.0087 |
| $H_{R/S}\ r_{GBP}$ | 0.593 | 0.5568 | 0.4843 | 0.5569 | 0.6307 | 0.0450 |
| $H_{R/S}\ \lvert r_{GBP}\rvert$ | 0.937 | 0.7891 | 0.7000 | 0.7882 | 0.873 | 6 0.0531 |
| $H_{R/S}\ r^2_{GBP}$ | 0.850 | 0.7515 | 0.6608 | 0.7514 | 0.8382 | 0.0543 |
| $Q_{class.}\ r_{GBP}$ | 1.619 | 1.4839 | 1.1305 | 1.4753 | 1.8572 | 0.2216 |
| $Q_{class.}\ \lvert r_{GBP}\rvert$ | 7.008 | 4.5883 | 3.3714 | 4.5506 | 5.9848 | 0.8076 |
| $Q_{class.}\ r^2_{GBP}$ | 5.192 | 3.6435 | 2.6515 | 3.6039 | 4.7449 | 0.6328 |
| $Q_{mod.}\ r_{GBP}$ | 1.539 | 1.4133 | 1.0796 | 1.4062 | 1.7641 | 0.2097 |
| $Q_{mod.}\ \lvert r_{GBP}\rvert$ | 1.797 | 1.1899 | 0.8830 | 1.1806 | 1.5294 | 0.1949 |
| $Q_{mod.}\ r^2_{GBP}$ | 2.616 | 1.8912 | 1.3960 | 1.8733 | 2.4446 | 0.3206 |
| GPH ($m=0.55$) $r_{GBP}$ | 0.0338 | 0.0470 | -0.0435 | 0.0487 | 0.1369 | 0.0529 |
| GPH ($m=0.55$) $\lvert r_{GBP}\rvert$ | 0.4900 | 0.4119 | 0.3037 | 0.4149 | 0.5143 | 0.0649 |
| GPH ($m=0.55$) $r^2_{GBP}$ | 0.4399 | 0.3473 | 0.2372 | 0.3519 | 0.4484 | 0.0655 |
| ARFIMA $d$ $\lvert r_{GBP}\rvert$ | 0.3739 | 0.3803 | 0.3111 | 0.3813 | 0.4449 | 0.0407 |
| ADF (no drift) ($GBP$) | -1.0103 | -0.5352 | -2.2755 | -0.6193 | 1.2780 | 1.1296 |
| ADF (with drift) ($GBP$) | -2.4484 | -1.2893 | -2.6985 | -1.3219 | 0.2137 | 0.8753 |

Table 19: Bootstrap Distribution of $r_{CHF}$

| Statistic | CHF/US | Mean | 5% | 50% | 95% | Std.dev. |
|---|---|---|---|---|---|---|
| BDS (dim. 2) | 8.996 | 9.059 | 5.426 | 8.944 | 12.833 | 2.2823 |
| BDS (dim. 3) | 12.049 | 12.103 | 7.092 | 11.908 | 17.570 | 3.207 |
| BDS (dim. 4) | 15.663 | 15.789 | 9.323 | 15.469 | 23.221 | 4.277 |
| GARCH(1,1) $\alpha$ | 0.0658 | 0.0711 | 0.0540 | 0.0710 | 0.0885 | 0.0109 |
| GARCH(1,1) $\beta$ | 0.9270 | 0.9158 | 0.8961 | 0.9161 | 0.9344 | 0.0118 |
| GARCH(1,1) $\alpha+\beta$ | 0.9928 | 0.9869 | 0.9705 | 0.9879 | 0.9996 | 0.0090 |
| $H_{R/S}\ r_{CHF}$ | 0.5424 | 0.5418 | 0.4671 | 0.5409 | 0.6212 | 0.0463 |
| $H_{R/S}\ \|r_{CHF}\|$ | 0.8423 | 0.7354 | 0.6381 | 0.7342 | 0.8358 | 0.0596 |
| $H_{R/S}\ r^2_{CHF}$ | 0.6456 | 0.6583 | 0.5346 | 0.6594 | 0.7763 | 0.0726 |
| $Q_{class.}\ r_{CHF}$ | 1.3073 | 1.3835 | 1.0824 | 1.3683 | 1.7241 | 0.1991 |
| $Q_{class.}\ \|r_{CHF}\|$ | 5.7912 | 3.6631 | 2.5895 | 3.6172 | 4.8900 | 0.7241 |
| $Q_{class.}\ r^2_{CHF}$ | 3.3043 | 2.6257 | 1.7124 | 2.5812 | 3.6942 | 0.6024 |
| $Q_{mod.}\ r_{CHF}$ | 1.3098 | 1.3734 | 1.0714 | 1.3584 | 1.7095 | 0.1970 |
| $Q_{mod.}\ \|r_{CHF}\|$ | 1.5399 | 0.9835 | 0.6989 | 0.9711 | 1.2943 | 0.1873 |
| $Q_{mod.}\ r^2_{CHF}$ | 1.6802 | 1.2853 | 0.8680 | 1.2155 | 1.9039 | 0.3234 |
| GPH ($m=0.55$) $r_{CHF}$ | 0.0541 | 0.0345 | -0.0522 | 0.0354 | 0.1189 | 0.0537 |
| GPH ($m=0.55$) $\|r_{CHF}\|$ | 0.4379 | 0.3679 | 0.2678 | 0.3684 | 0.4685 | 0.0612 |
| GPH ($m=0.55$) $r^2_{CHF}$ | 0.1761 | 0.0306 | 0.1300 | 0.3718 | 0.1163 | |
| ARFIMA $d$ $\|r_{CHF}\|$ | 0.3767 | 0.3711 | 0.3183 | 0.3694 | 0.4299 | 0.0334 |
| ADF (no drift) ($CHF$) | -1.7850 | -1.1077 | -2.3879 | -1.1966 | 0.4276 | 0.8620 |
| ADF (with drift) ($CHF$) | -1.6113 | -1.1805 | -2.7464 | -1.2272 | 0.5524 | 0.9907 |

Table 20: Bootstrap Distribution of $r_{JPY}$

| Statistic | JPY/US | Mean | 5% | 50% | 95% | Std.dev. |
|---|---|---|---|---|---|---|
| BDS (dim. 2) | 13.144 | 13.222 | 8.326 | 13.086 | 18.357 | 3.088 |
| BDS (dim. 3) | 17.322 | 17.407 | 10.555 | 17.297 | 24.703 | 4.287 |
| BDS (dim. 4) | 22.751 | 22.757 | 13.271 | 22.435 | 33.092 | 6.050 |
| GARCH(1,1) $\alpha$ | 0.0578 | 0.0718 | 0.0531 | 0.0714 | 0.0926 | 0.0121 |
| GARCH(1,1) $\beta$ | 0.9398 | 0.9192 | 0.8836 | 0.9229 | 0.9414 | 0.0181 |
| GARCH(1,1) $\alpha+\beta$ | 0.9976 | 0.9910 | 0.9624 | 0.9976 | 1.0000 | 0.0130 |
| $H_{R/S}\ r_{JPY}$ | 0.5358 | 0.5547 | 0.4760 | 0.5541 | 0.6384 | 0.0482 |
| $H_{R/S}\ \|r_{JPY}\|$ | 0.8429 | 0.7810 | 0.6841 | 0.7816 | 0.8815 | 0.0605 |
| $H_{R/S}\ r^2_{JPY}$ | 0.7360 | 0.7270 | 0.6310 | 0.7288 | 0.8148 | 0.0553 |
| $Q_{class.}\ r_{JPY}$ | 1.4115 | 1.5117 | 1.1563 | 1.4926 | 1.9378 | 0.2353 |
| $Q_{class.}\ \|r_{JPY}\|$ | 5.6175 | 4.3563 | 3.0031 | 4.2798 | 5.9126 | 0.9147 |
| $Q_{class.}\ r^2_{JPY}$ | 3.8942 | 3.2358 | 2.2999 | 3.2329 | 4.2145 | 0.5843 |
| $Q_{mod.}\ r_{JPY}$ | 1.3740 | 1.4734 | 1.1220 | 1.4533 | 1.8830 | 0.2289 |
| $Q_{mod.}\ \|r_{JPY}\|$ | 1.4757 | 1.1550 | 0.8321 | 1.1411 | 1.5368 | 0.2152 |
| $Q_{mod.}\ r^2_{JPY}$ | 1.8453 | 1.5679 | 1.1911 | 1.5573 | 1.9887 | 0.2521 |
| GPH ($m=0.55$) $r_{JPY}$ | 0.0581 | 0.0382 | -0.0506 | 0.0416 | 0.1157 | 0.0526 |
| GPH ($m=0.55$) $\|r_{JPY}\|$ | 0.4216 | 0.3792 | 0.2655 | 0.3810 | 0.4766 | 0.0628 |
| GPH ($m=0.55$) $r^2_{JPY}$ | 0.2918 | 0.2612 | 0.1652 | 0.2640 | 0.3493 | 0.0567 |
| ARFIMA $d$ $\|r_{JPY}\|$ | 0.3469 | 0.3514 | 0.2713 | 0.3519 | 0.4259 | 0.0458 |
| ADF (no drift) ($JPY$) | -1.9648 | -1.9167 | -3.9373 | -1.9626 | 0.1041 | 1.2100 |
| ADF (with drift) ($JPY$) | -1.3795 | -0.8963 | -2.6937 | -0.9919 | 1.1057 | 1.1533 |