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Abstract

Ait-Sahalia and Lo (2000) and Panigirtzoglou and Skiadopoulos (2004) have argued that Economic VaR (E-VaR), calculated under the option market implied risk neutral density is a more relevant measure of risk than historically based VaR. As industry practice requires VaR at high confidence level of 99%, we propose Extreme Economic Value at Risk (EE-VaR) as a new risk measure, based on the Generalized Extreme Value (GEV) distribution. Markose and Alentorn (2005) have developed a GEV option pricing model and shown that the GEV implied RND can accurately capture negative skewness and fat tails, with the latter explicitly determined by the market implied tail index. Here, we estimate the term structure of the GEV based RNDs, which allows us to calibrate an empirical scaling law for EE-VaR, and thus, obtain daily EE-VaR for any time horizon. Backtesting results for the FTSE 100 index from 1997 to 2003, show that EE-VaR has fewer violations than historical VaR. Further, there are substantial savings in risk capital with EE-VaR at 99% as compared to historical VaR corrected by a factor of 3 to satisfy the violation bound. The efficiency of EE-VaR arises because an implied VaR estimate responds quickly to market events and in some cases even anticipates them. In contrast, historical VaR reflects extreme losses in the past for longer.

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1 Introduction

Value-at-Risk (VaR) has become the most popular measure for risk management. Value-at- Risk, denoted by VaR (q, k), is an estimate, for a given confidence level q, of the maximum that can be lost from a portfolio over a given time horizon k. An alternative measure of risk is the Economic VaR (E-VaR) proposed by Ait-Sahalia and Lo (2000) and calculated under the option-implied risk neutral density. It has been argued that E-VaR is a more general measure of risk, since it incorporates investor risk preferences, demand–supply effects, and market implied probabilities of losses or gains, Panigirtzoglou and Skiadopoulos (2004). E-VaR can be seen as a forward looking measure to quantify market sentiment about the future course of financial asset prices, whereas historical or statistically based VaR (S-VaR) is backward looking, based on the historical data. With the development in 1993 of the traded option implied VIX index for the SP-500 returns volatility over a 30 calendar day horizon, the so called "investor fear gauge", a significant move toward the use of a market implied rather than a historical measure of risk in practical aspects of risk management has occurred. Policy makers such as the Bank of England use traded option implied risk neutral density, volatility and quantile measures to gauge market sentiment regarding future asset prices. [1]

Given the industry standard for 10 day VaR at high confidence levels of 99%, it is important to correctly model the distribution of the extreme values of asset returns, as it is well known that the probability distributions of asset returns are not Gaussian especially at short time horizons (see, Cont, 2001). In the management of risk, the modelling of asymmetries and the asymptotic behaviour of the tails of the distribution of losses is important. Extreme value theory is a robust framework to analyse the tail behaviour of distributions. Extreme value theory has been applied extensively in hydrology, climatology and also in the

[1]For the VIX index see www.cboe.com/micro/vix/ and for the Bank of England option traded implied probability density functions, volatility and quantile measures see, www.bankofengland.co.uk/statistics/impliedpdfs/ . In particular, market risk premia for a given holding period is estimated as payoffs from volatility swaps which effectively take the difference between realized volatility and the option implied volatility.

insurance industry (see, Embrechts et. al. 1997). Despite early work by Mandelbrot (1963) on the possibility of fat tails in financial data and evidence on the inapplicability of the assumption of log normality in option pricing, a systematic study of extreme value theory for financial modelling and risk management has only begun recently. Embrechts et. al. (1997) is a comprehensive source on extreme value theory and applications.[2] Dacorogna et. al. (2001) develop a VaR estimate based on the extreme value Pareto distribution for the tails of the distribution which is then empirically estimated from high frequency data using a bootstrap method for the Hill estimator.

In this paper we propose Extreme Economic Value at Risk (EE-VaR) as a new risk measure, which is calculated from an implied risk neutral density that is based on the Generalized Extreme Value (GEV) distribution. It has been shown in Markose and Alentorn (2005) that the GEV option pricing model not only accurately captures the negative skewness and higher kurtosis of the implied risk neutral density (RND), but it also delivers the market implied tail index that governs the tail shape. It is important to note that the GEV does not pose a priori restrictions on the tail shape as the GEV distribution encompasses the thin and short tailed class of the Gumbel and Weibull , respectively, along with the fat tailed Fréchet.[3] Indeed, one of the main findings from Alentorn and Markose (2006) and Alentorn (2007) is that the daily implied tail shape parameter estimated without maturity effects from the GEV RND model indicates that market perception of fat tailed behaviour of extreme events is interspersed with thin and short tailed Gumbel and Weibull values.[4] Hence, the assumption of the GEV parametric model for the RND overcomes problems, associated with the estimation of the risk neutral density function to flexibly include extreme values and fat tails, which are often encountered with many non-parametric methods and with the use of parametric models such as the Gaussian.[5] In this paper we

[2]Embrechts (1999, 2000) considers the potential and limitations of extreme value theory for risk management. Without being exhaustive here, De Haan et. al. (1994) and Danielsson and de Vries (1997) study quantile estimation. Bali (2003) uses the GEV distribution to model the empirical distribution of returns. Mc Neil (1999) gives an extensive overview of extreme value theory for risk management. See also Dowd (2002, pp.272-284).

[3]The Gumbel class includes the normal, exponential, gamma and log normal while the Weibull include distributions such as the uniform and beta. Examples of fat tailed distributions that belong to the Fréchet class are Pareto, Cauchy, Student-t and mixture distributions.

[4]Even during extreme events, though the implied tail index results in fat tails for the GEV RND based returns– at all times the first four moments were bounded.

[5]In order to estimate risks at high confidence levels, such as 99% - many non-parametric methods for RND estimation fail to capture tail behaviour of distributions because of sparse data for options traded at very high or very low strike prices. Hence, parametric models have become unavoidable. This, however, replaces sampling error by model error. Markose and

will focus on estimating the term structure of the GEV based implied RNDs, which allows us to calibrate an empirical scaling law for EE-VaR at different confidence levels, and thus, to obtain the daily EE-VaR for any time horizon, without having to employ the widely used but incorrect square root of time scaling rule.

There is a vast literature on the analysis of information implied from option markets. One of the areas that has received the most attention is the study of the implied volatility surfaces, such as in Day and Craig (1988), Ncube (1996), Dumas, Fleming and Whalley (1998) and others. The great majority of studies of implied distributions have focused on the analysis of the distributions at a single point in time for event studies, such as Bates (1991) for the study of the 1987 crash, Gemmill and Saflekos (2000) for the study of British elections, and Melick and Thomas (1996) for the analysis of oil prices during the Gulf war crisis. Starting with the study of the day to day dynamics of implied volatility surfaces (see Cont and Da Fonseca (2002)), recently, Clews et. al. (2001) and Panigirtzoglou and Skiadopoulos (2004) have developed a framework for the analysis of dynamics of implied RND functions.

A problem encountered when looking at the daily dynamics of RNDs, or RND implied measures such as volatility[6] or their associated quantile values, the E-VaR, is the time to maturity and the contract switch effects (see, Melick and Thomas, 1998). RNDs are usually constructed using the options with shortest time to maturity. Since options have a fixed expiry date, this means that both the time horizon of the RND and the holding period of the underlying asset change with time to maturity. The degree of uncertainty decreases as the expiry date approaches. Uncertainty jumps up again when the option with the shortest time to maturity expires, and we switch to options with the next expiration date. For instance, given that options on the FTSE 100 index expire on the third Friday of the expiry month, the jump would occur on the third Monday of the expiry month. Note also that option prices with less than 5 working days to maturity are usually excluded. Thus, the problem associated with obtaining constant horizon RNDs and option implied values for VaR or volatility for the underlying assets from traded options is non-trivial. Clearly, the use of E-VaR

Alentorn (2005) have argued that as the GEV distribution encompasses the 3 main classes of tail behaviour, it mitigates model error and further there is parsimony in the number of parameters necessary to define the distribution.

[6] Alentorn and Markose (2006) give an extensive survey of the studies done on removing maturity effects on implied volatility and higher moments of the RND. Here, we focus on the quantile values, E-VaR.

for risk management is feasible only if it can be calculated and reported daily for a constant time horizon or holding period that is required.

With regard to the traded option implied E-VaR, to our knowledge, there are only three previous studies that have carried out an empirical analysis of E-VaR and two of these study daily constant horizon E-VaR. Ait-Sahalia and Lo (2000) estimated the E-VaR for a 126 day horizon. Clews et al. (2001) have suggested a semi-parametric methodology that can remove maturity effects in the construction of constant horizon RNDs. The methodology consists of interpolating the Black-Scholes implied volatility surface in delta space at a given time horizon, and then deriving the implied RND by calculating the second derivative of the call pricing function, using the Breeden and Litzenberger (1978) result. This methodology is used by the Monetary Instruments and Markets Division at the Bank of England to report daily E-VaR values for the FTSE 100 index at confidence levels ranging from 5% to 95% for the FTSE 100, for a 3 month constant horizon RND. However, with this methodology, it is not possible to construct a constant time horizon implied RND for a time horizon shorter than the shortest maturity available, given that the implied volatility surface in delta space is non-linear. Panigirtzoglou and Skiadopoulos (2004) looked at the E-VaR calculated at 95% confidence level for constant horizons of 1, 3 and 6 months for every 14 days during the year 2001. However, the problem of reporting daily E-VaR at short constant horizons such as 10 days remains and typically semi-parametric methods for RND extraction fail to report E-VaR at 99% confidence level.

In this paper, we focus on obtaining a daily estimate of a constant time horizon GEV based E-VaR using a discrete term structure of RNDs. In Section 2, the new methodology we propose proceeds by first constructing a daily discrete term structure of implied RNDs, using option prices of all maturities available and a cross section of strikes for each maturity. Hence, there is a RND for each maturity available for traded options in a given day. Assuming the parametric GEV model for the RND, we calculate the EE-VaR at different confidence levels as the quantile values for the RND for each available maturity. We exploit the linear behaviour of quantile values vis-à-vis the holding period, k, in the log-log scale to derive an empirical scaling law for different confidence levels, q.[7] One of the advantages of this linear relationship is that it allows us

[7] The empirical evidence for the scaling parameter b in the relationship, $VaR(q,k) = VaR(q,1)k^b$, which is linear in logs has been studied by Hauksson (2001), Menkens (2004) and Provizionatou et. al.(2005) in the context of historical VaR. Also, Dacorogna et. al.

to both interpolate and extrapolate from the available maturities and obtain daily E-VaR values for any constant horizon from 1 day to m days and can be used regardless of the method for extracting the discrete RNDs. To test the robustness of our methodology we use the daily 90 day E-VaR reported by the Bank of England for the 95% confidence level to compare the performance of the GEV implied EE-VaR and also E-VaRs obtained from parametric RNDs for the Black Scholes and the Mixture of two Lognormals. We then proceed to report a 10 day EE-VaR which is easily done with our method regardless of the time horizon of the closest maturity option contracts. We analyse the performance of EE-VaR for different confidence levels, different time horizons, and for a large dataset, and compare it with the performance of historical VaR and the Black-Scholes E-VaR. In this paper we perform an in depth analysis of the daily EE-VaR performance for over 7 years, using daily closing index option prices on the FTSE 100 from 1997 to 2003. This is the first paper to do this and the empirical implementation and results are reported in Sections 3 and 4. Backtesting results, based on the FTSE 100 index from 1997 to 2003, show that EE-VaR has fewer violations than historical VaR. Note that statistical VaR is done for a 1 day return and then scaled by the square root of time rule. The 10 day S-VaR when corrected by a multiplication factor of 3, to satisfy the violation bound, requires substantially more risk capital than EE-VaR. This saving in risk capital with EE-VaR at high confidence levels of 99% arises because an implied VaR estimate responds quickly to market events and in some cases even anticipate them. In contrast, VaR estimates based on historical data reflect extreme losses in the past for longer.

2 Model and Methodology

2.1 Extraction of GEV based RND from option prices

A large number of methods have been proposed for extracting implied distributions from option prices since the seminal work of Breeden and Litzenberger (1978), (see Jackwerth (1999) for an extensive survey). In this paper we use the methodology proposed by Markose and Alentorn (2005) based on the Generalized Extreme Value (GEV) distribution.

Let S_t denote the underlying asset price at time t. The European call option

(2001) derived an extreme value based VaR scaling law for high frequency forex data. Here, we investigate the scaling relationship for implied VaR, rather than for historical VaR.

C_t is written on this asset with strike K and maturity T. We assume the interest rate r is constant. Following the Harrison and Pliska (1981) result on the arbitrage free European call option price, there exists a risk neutral density (RND) function, $g(S_T)$, such that the equilibrium call option price can bewritten as:

$$\begin{aligned} C_t(K) &= E_t^Q[e^{-r(T-t)} max(S_T - K, 0)] \\ &= e^{-r(T-t)} \int_K^\infty (S_T - K)\, g(S_T)\, dS_T. \end{aligned} \tag{1}$$

Also, the following martingale condition holds for the stock price

$$S_t = e^{-r(T-t)} E_t^Q[S_T]. \tag{2}$$

Here $E_t^Q[.]$ is the risk-neutral expectation operator, conditional on all information available at time t, and $g(S_T)$ is the risk-neutral density function of the underlying at maturity. Note that the GEV option pricing model in Markose and Alentorn (2005) is based on the assumption that negative returns, L_T, as defined in equation (3) below, follow a GEV distribution:

$$L_T = -R_T = -\frac{S_T - S_t}{S_t} = 1 - \frac{S_T}{S_t}. \tag{3}$$

The GEV distribution, in the form in von Mises (1936) (see, Reiss and Thomas, 2001, p. 16-17) which incorporates a location parameter μ, a scale parameter σ, and a tail shape parameter ξ, is defined by:

$$F_{\xi,\mu,\sigma}(x) = exp\left(-\left(1 + \frac{\xi}{\sigma}(x - \mu)\right)^{-1/\xi} \right), \xi \neq 0, \tag{4}$$

with

$$1 + \xi \frac{(x-\mu)}{\sigma} > 0,$$

and

$$F_{0,\mu,\sigma}(x) = exp\left(-exp\left(\frac{x-\mu}{\sigma} \right) \right), \xi = 0. \tag{5}$$

The tail shape parameter $\xi = 0$ yields thin tailed distributions with the so called tail index $1/\xi = \alpha$ being equal to infinity, implying that all moments of this class of distributions exist. When $\xi < 0$ the GEV distribution class is Weibull. The fat tailed Fréchet distributions arise when $\xi > 0$ and note $\xi > 25$

is sufficient to imply infinite kurtosis.

The RND function $g(S_T)$ in (1) for the underlying asset price given that L_T is assumed to satisfy the GEV density function (see, Reiss and Thomas, p. 16-17) is given by[8]:

$$g(S_T) = \frac{1}{S_t\sigma}\left(1+\frac{\xi\,(L_T-\mu)}{\sigma}\right)^{-1-1/\xi} exp\left(-\left(1+\frac{\xi\,(L_T-\mu)}{\sigma}\right)^{-1/\xi}\right), \quad (6)$$

with

$$1+\frac{\xi}{\sigma}(L_T-\mu) = 1+\frac{\xi}{\sigma}\left(1-\frac{S_T}{S_t}-\mu\right) > 0. \quad (7)$$

Note if the above condition in (7) is not satisfied, the GEV density function is not defined on the real line. When $\xi > 0$ and the distribution for L_t is fat tailed, condition (7) implies that the GEV density function for the price is truncated on the right, that is, the probability that the price will rise above this truncation value is zero. On the other hand, when the $\xi < 0$ and L_t is Weibull class, the GEV density function for S_T is truncated on the left implying that the price will not fall below the truncation value. Markose and Alentorn (2005) find that while this did affect the limits of integration for the option price equation in (1), the closed form solution for the call (and put) option for all cases of $\xi \neq 0$ is identical. Omitting the proof , which can be found in Markose and Alentorn (2005) the closed form GEV RND based call option price is given by

$$\begin{aligned} C_t\,(K) &= e^{-r(T-t)}\{\frac{-S_t\sigma}{\xi}\Gamma(1-\xi,H^{-1/\xi}) \\ &\quad -(S_t(1-\mu+\frac{\sigma}{\xi})-K)(-e^{-H^{-1/\xi}})\}, \end{aligned} \quad (8)$$

where $H = 1+\frac{\xi}{\sigma}\left(1-\frac{K}{S_t}-\mu\right)$ and $\Gamma(1-\xi,H^{-1/\xi}) = \int_{H^{-1/\xi}}^{\infty} z^{-\xi}e^{-z}dz$ is the incomplete Gamma function.

The structural GEV parameters ξ, μ and σ can be estimated by minimizing the sum of squared errors (SSE) between the analytical solution of the GEV option pricing equations in (8) and the observed traded option prices with strikes K_i, as given in (9) below:

$$SSE(t) = \underset{\xi,\mu,\sigma}{min}\left\{\sum_{i=1}^{N}\left(C_t\,(K_i)-\widetilde{C_t}((K_i)\right)^2\right\}. \quad (9)$$

[8]Note the relationship between the density function for L_t, $f(L_t)$, and that for the underlying, $g(S_T)$, is given by the general formula $g(S_T) = f(L_T)|\frac{\partial L_T}{\partial S_T}| = f(L_T)\frac{1}{S_t}$.

For purposes of comparison, we use the above method to back out the respective implied parameters for the Black-Scholes model and also the RND from the Mixture of two Lognormals (MLN) first constructed by Ritchey (1990).

At the estimation stage, we use the data on the index futures contract with the same maturity as the options and as the futures price at maturity yields, $F_T = S_T$, the no arbitrage martingale condition in (2) enables us to substitute out $E^Q(S_T)$ by using $F_{t,T} = E^Q(S_T)$, This also vitiates the need for data on the dividend yield rate. The optimization problem in (10), was performed using the non-linear least squares algorithm from the Optimization toolbox in MatLab.[9]

2.2 EE-VaR calculation from GEV RND

The quantile for the GEV distribution i.e. the VaR value associated with a given confidence level q, is given as a function of the three GEV parameters (see, Dowd 2004: pp. 274):

$$VaR = \mu - \frac{\sigma}{\xi}\left[1 - (-\log(q))^{-\xi}\right], \xi \neq 0, \tag{10}$$

and

$$VaR = \mu - \sigma \log\left[\log(1/q)\right], \xi = 0. \tag{11}$$

On substituting the implied GEV parameters from daily traded option prices for a given maturity horizon, the extreme economic value at risk (EE-VaR) is calculated from (10) and (11).

The results obtained using EE-VaR will be compared with E-VaR values under the Gaussian assumptions of the Black-Scholes model and that of the mixture of two lognormals. The quantile of the normal distribution is used to calculate the E-VaR values for the Gaussian case using the Black-Scholes implied volatility. The MLN method models the RND as a weighted sum of two lognormals, and is given by:

$$f(S_T) = ph(S_T \mid \mu_1 T, \sigma_1\sqrt{T}) + (1-p)h(S_T \mid \mu_2, \sigma_2\sqrt{T}). \tag{12}$$

The MLN RND has been extensively used in the literature, given that it is

[9]For a more detailed analysis of the estimation results, including time series of implied parameters, pricing performance and comparison of results of the GEV model with other parametric models, can be found in Alentorn and Markose (2006) and Alentorn (2007). As already noted in the Introduction, the daily implied tail shape parameters ξ, for the sample period ranged between -0.2 and +0.22.

very flexible, and allows the modelling of different levels of skewness, as well as bimodal densities. However, compared to the GEV RND it has are five unknown parameters $\theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, p\}$, the means of each lognormal function $\mu_1$and μ_2 , the standard deviations $\sigma_1$and σ_2 , and the weighting coefficient p. We obtain the set of implied parameters $\hat{\theta}$ by the method in (9). Then, E-VaR is calculated as the quantile of the MLN density, which consists of a weighted sum of the two inverse cumulative distribution functions, H , and given by:

$$E - VaR(q, k) = \hat{p}H^{-1}\left(q \mid \hat{\mu}_1, \hat{\sigma}_1, T\right) + (1 - \hat{p})\, H^{-1}\left(q \mid \hat{\mu}_2, \hat{\sigma}_2, T\right). \qquad (13)$$

Some authors, such as Shiratsuka (2001) and Melick (1999), argue that the values for the higher quantiles of implied RNDs are very sensitive to the choice of RND estimation technique, since the range of strike prices that are actually traded is very limited and the tails of the estimated implied RND vary depending on the procedure employed. Table 1 below shows the percentage number of days between 1997 and 2003 with traded put options with strike below each of the confidence levels.

Table 1: Percentage number of days with put option prices with strikes below each of the confidence levels FTSE-100 Traded Options (1997-2003)

Confidenc level	***Percentage number of days***
70%	94%
80%	86%
90%	68%
95%	51%
99%	22%

Hence, we will also compare the quantile values obtained from the parametric RND models with those at the highest confidence level of 95% reported by the Bank of England which uses the semi-parametric RND method discussed earlier.

3 Data description

The data used in this study are the daily settlement prices of the FTSE 100 index call and put options published by the London International Financial Futures and Options Exchange (LIFFE). These settlement prices are based on quotes and transactions during the day and are used to mark options and futures positions to market. Options are listed at expiry dates for the nearest three months and for the nearest March, June, September and December. FTSE 100 options expire on the third Friday of the expiry month. The FTSE 100 option strikes are in intervals of 50 or 100 points depending on time-to-expiry, and the minimum tick size is 0.5.

The period of study was from 1997 to 2003, so there were 28 expiration dates (7 years with 4 contracts per year). This period includes some events, such as the Asian crisis, the LTCM crisis and the 9/11 attacks, which resulted in a sudden fall of the underlying FTSE 100 index, and will be useful to analyze the performance of the methods under extreme events. The average number of maturities available with more than 3 options traded in our sample (1997-2003) is displayed in Table 2 below . In average across all years, we have 5.33 different maturities each day.

Table 2: Average number of maturities available FTSE-100 Traded Options (1997-2003)

Year	**Average number of maturities available**
1997	3.96
1998	4.57
1999	5.19
2000	5.49
2001	5.84
2002	6.19
2003	6.09
Average	**5.33**

The LIFFE exchange quotes settlement prices for a wide range of options, even though some of them may have not been traded on a given day. In this study we only consider prices of traded options, that is, options that have a non-zero volume. The data were also filtered to exclude days when the cross-sections of options had less than three option strikes, since a minimum of three strikes is required to estimate the three parameters of the GEV model.[10] Also, options whose prices were quoted as zero or that had less than 5 days to expiry were eliminated. Finally, option prices were checked for violations of the monotonicity condition.[11]

[10] The number of option prices needed to extract the RND must be at least equal than the number of degrees of freedom for the parametric method used. The number of degrees of freedom is equal to the number of parameters that need to be estimated minus the number of constraints. For example, the GEV model has three parameters while the mixture of lognormals have five parameters.

[11] Monotonicity requires that the call (put) prices are strictly decreasing (increasing) with respect to the exercise price.

The risk-free rates used are the British Bankers Association's 11 a.m. fixings of the 3-month Short Sterling London InterBank Offer Rate (LIBOR) rates from the website www.bba.org.uk. Even though the 3-month LIBOR market does not provide a maturity-matched interest rate, it has the advantages of liquidity and of approximating the actual market borrowing and lending rates faced by option market participants (Bliss and Panigirtzoglou, 2004).

4 Empirical Modelling and Results

4.1 Term structure of RNDs

To calculate the EE-VaR, ideally, one would use a RND implied by options with time to maturity exactly equal to the time horizon we are interested. That is, to calculate the 10 day EE-VaR we would use prices from options that mature in 10 days to obtain an implied RND, and calculate the quantile of that density at the confidence level required. However, in practice, we only have options that expire every month during the next three months, and also, options that expire in March, June, September and December. In the original study of Markose and Alentorn (2005), at each trading day, only the RND implied by the closest to maturity contracts for which futures contracts were available (March, June, September and December) was extracted. Here, we propose, on a daily basis, the extraction of an RND for each of the maturities with a sufficient number of traded option prices. Then, using this discrete set of RNDs, each with a different maturity, we can construct what we call a term structure of implied RNDs. This term structure can be visualized as a 3 dimensional chart that displays, for a given day, how the implied RNDs vary across different maturities. For purposes of illustration, Figure 1 below displays the implied RND term structure for a typical day, 21 August 2001, using the GEV model. Note from Figure 1 that the main feature of the term structure, which is independent of the RND extraction method used, is that the peakedness of the RNDs decreases as the time horizon increases. This term structure of implied RNDs will be used in the following section to obtain constant time horizon E-VaRs.

Table 3 below displays the actual EE-VaR values. As one would expect, the EE-VaR values increase both with confidence level and with time horizon. Also, note how the number of options prices available decreases as time to maturity increases, that is, the options with the closest to maturity dates are the ones that have the widest range of traded strikes.

Figure 1: Term Structure of GEV based implied RNDs on 21 August 01

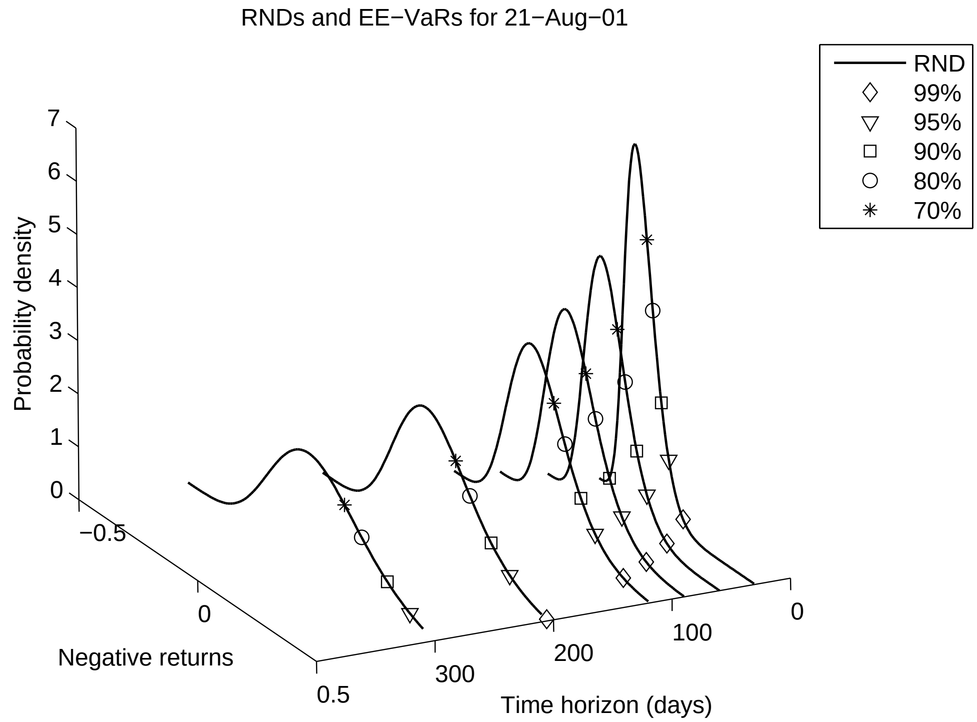


Table 3: EE-VaR values for each available maturity and at different confidence levels on 21 August 01

Expiry month	Days to maturity	Number options	70%	EE 80%	– 90%	VaR 95%	99%
Sep-01	31	44	2.4%	4.4%	7.4%	10.1%	15.6%
Oct-01	59	31	3.1%	5.9%	10.2%	14.0%	21.7%
Nov-01	87	13	3.7%	7.2%	12.7%	17.5%	27.4%
Dec-01	122	16	4.2%	8.5%	15.0%	20.8%	32.6%
Mar-02	213	13	5.7%	11.4%	20.1%	27.7%	42.8%
Jun-02	304	10	6.9%	13.7%	23.8%	32.4%	49.0%

4.2 Empirical Scaling of EE-VaR

One of the requirements of the Basel accord is that banks should report the daily 10 day VaR at 99% confidence level of their portfolios. However, there are some difficulties with estimating the 10 day VaR, due to the need for a long time series in order to compute the 10 day returns, and then, calculate the quantiles of their distribution. In practice, the square root of time scaling rule is widely used to scale up the 1 day VaR to the 10 day VaR. This scaling rule is only appropriate for time series that have Gaussian properties, but it has been well established in the literature for a long time (see, Fama (1965) and Mandelbrot (1967)), that financial data is non-Gaussian. Following the wide spread use of VaR as a risk measure and reporting requirement, there have been several recent studies that looked at the problem of scaling VaR, such as McNeil and Frey (2000), Hauksson et. al. (2001), Kaufmann and Patie (2003), Danielsson and Zigrand (2004), Menkens (2004) and Provizionatou et al (2005).

In this study we are faced with a similar problem, but instead of having to scale up the 1 day E-VaR, we need to scale down from the maturities available, to 10 day and 1 day E-VaR. Without resorting to scaling, we would only be able to calculate the 10 day VaR for only one day each month, the day when there are exactly 10 days to maturity for the closest to maturity contract (in the case of FTSE 100 data, it would be around the first Friday of each month, since contracts mature in the third Friday of the month). Following a similar approach as in Hauksson et. al. (2001) and Menkens (2004), we have identified an empirical scaling law for EE-VaR against time horizon that is linear in a log-log scale.

$$\log\left(EEVaR\left(k,q\right)\right) = b\left(q\right)\log\left(k\right) + c\left(q\right), \tag{14}$$

where k is the number of days, $c(k)$ is the 1-day EE-VaR value (given that $\log(1) = 0$), and the slope $b(q)$ is the EE-VaR scaling parameter for a given confidence level q. Once we estimate the parameters $b(q)$ and $c(q)$ for a given day and for a given confidence level q, we can obtain the k-day EE-VaR value as follows:

$$EEVaR\left(k,q\right) = 10^{\hat{b}(q)\log(k)+\hat{c}(q)}. \tag{15}$$

Figure 2 below displays the EE-VaR values obtained from the RNDs in Figure 1 above, using the linear regression line from equation (14).

Figure 2: log-log plot for 21 August 01, with the estimated linear scaling rule for each confidence level.

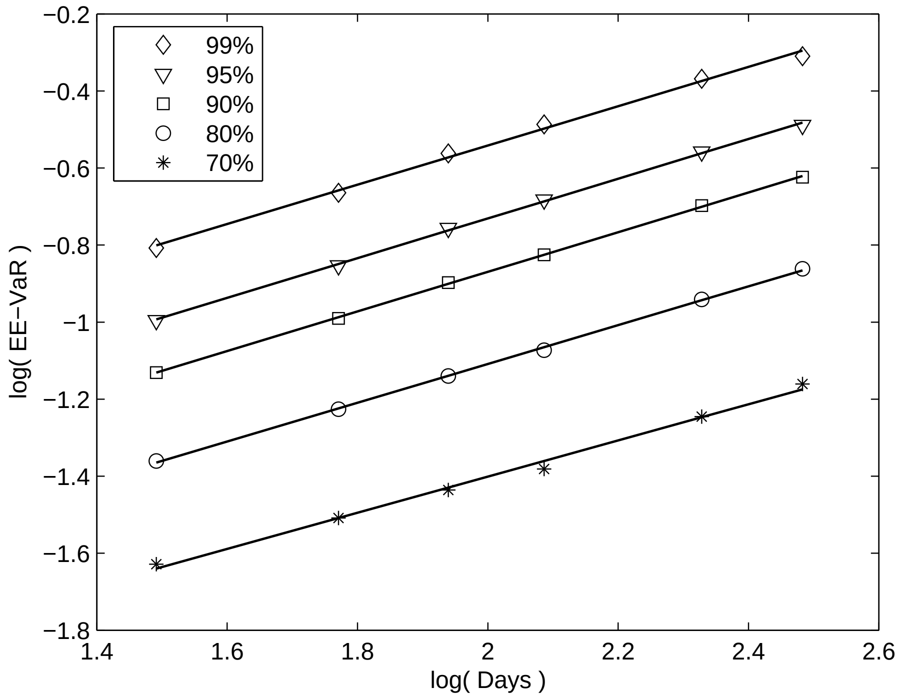


Table 4: Regression coefficients b and c for 21 August 2001.

Confidence level	b	c
70%	0.47	-2.18
80%	0.50	-2.09
90%	0.51	-1.91
95%	0.52	-1.82
99%	0.51	-1.73
Average	**0.50**	**-1.95**

While we report the average of the full set of daily scaling coefficients, b, implied from the term structure of EE-VaR for the sample period, what Table 4 indicates is how the E-VaR based scaling coefficients differ from scaling in historical VaR. Hauksson et. al. (2001) report these to be around .43 (though it is not clear what confidence level this is for) while Provitinatou et. al report scaling coefficients which range from .47 to .45 for the 70% and 99% confidence levels, respectively. As will be seen, in general the market implied VaR scales more vigorously with time at higher quantiles. However, the size of the scaling coefficients in Tables 4 and 5 should not be confused with implying unbounded second and higher moments of the RND functions as the implied tail parameters ξ at all times for the sample period showed that up to 4 moments exist.

4.3 Improving the estimation of the linear scaling law by using WLS

The linear regression estimated to obtain the time scaling for EE-VaR can be affected by EE-VaR values calculated from an RND constructed from very few option prices. The EE-VaR estimates in such cases will have very wide confidence intervals. As an example, take the data and regression for 12 Nov 97, shown in Figure 3 below. The R^2 of the OLS regression was 64.8%, a very poor fit. The EE-VaR value furthest away from maturity was obtained from an RND estimated using only 4 option prices, and thus the confidence intervals of the EE-VaR estimate are much wider than the EE-VaR values obtained for closer maturities, which are based on RNDs extracted using around 25 contracts.

One method to solve this issue is to use a *Weighted Linear Squares* (WLS) regression, using the number of option prices available at each maturity relative to total prices as weights for the EE-VaR values.

$$WeightedR^2 = 1 - \frac{\sum_{i=T_1}^{T_N} w_i \left(y_i - \hat{y}_i\right)^2}{\sum_{i=T_1}^{T_N} w_i \left(y_i - \overline{y}_i\right)^2}, with \sum_{i=T_1}^{T_N} w_i = 1, \tag{16}$$

$$w_i = \frac{NumberOfPriceAtMaturity_i}{TotalNumberOfPrices} \tag{17}$$

Table 5: Average R^2 for different quantiles, and number of days with different ranges of R^2

Confidence level	**70%**	**80%**	**90%**	**95%**	**99%**
Average R^2	87.9%	97.9%	98.8%	98.7%	97.9%
Number of days with $R^2 > 99\%$	429	1152	1410	1320	901
$99\% \geq R^2 > 90\%$	860	510	282	375	742
$R^2 \leq 90\%$	444	71	41	38	90

Table 6 below shows the average weighted R^2 at each confidence level. Note how the fitting performance increases with confidence level, while it is lowest at 96.8% for the lowest quantile of 70%.

Figure 3: Example of linear regression using OLS vs. WLS for a day when there are some maturities with very few option prices available.

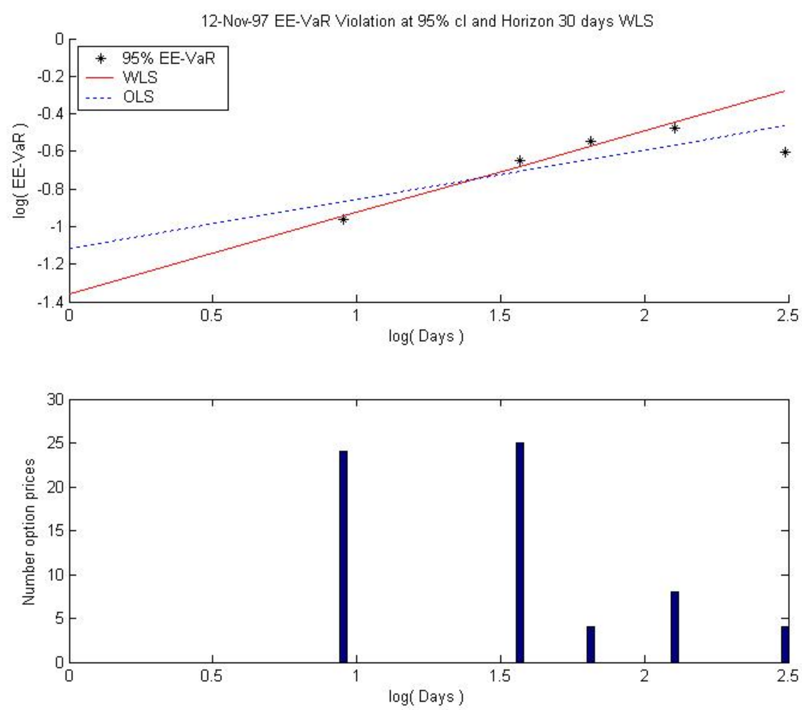


Table 6: Average weighted R^2 at each quantile

Confidence level	**70%**	**80%**	**90%**	**95%**	**99%**
Weighted R^2	96.8%	99.4%	99.6%	99.5%	99.3%

4.4 Full Regression results for scaling in EE-VaR

The average regression coefficients b and c across the 1733 days in our sample period are displayed in Table 7 below. We can see that the slope b increases with the confidence interval, and that intercept, i.e. the 1 day EE-VaR also increases with confidence level, as one would expect. The standard deviation of the estimates at different confidence levels is fairly constant. We also report the percentage number of days where b was found to be statistically significantly different from one half.[12] On average, irrespective of the confidence level, we

[12]Newey-West (1987) heterosckedasticity and autocorrelation consistent standard error was

found that in around 50% of the days the scaling was significantly different than 0.5, the scaling implied by the square root of time rule.

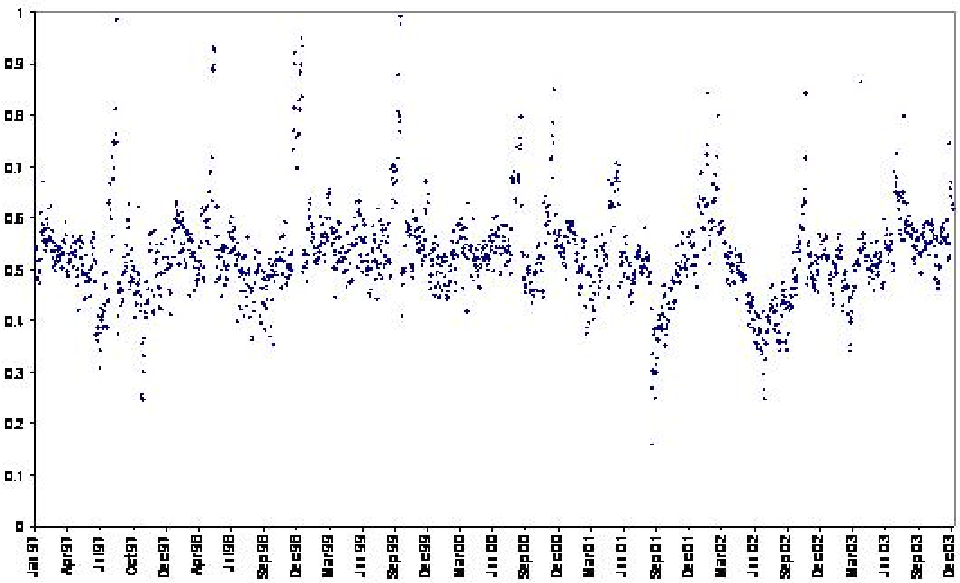
Table 7: Average regression coefficients b,c and exp(c) across the 1733 days in the sample, for the GEV case. #: The percentage number of days where b was statistically significantly different from 0.5. †: Standard deviation of b

Confidence level q	**b(q)**	**c**	**EVaR(1,q) = exp(c)**
	0.41	-2.24	
70%	(51.3%)#	(0.26)†	0.6%
	(0.12)†		**level**
	0.48	-2.04	
80%	(51.1%)#	(0.23)†	0.9%
	(0.09)†		
	0.51	-1.84	
90%	(52.4%)#	(0.23)†	1.4%
	(0.09)†		
	0.53	-1.73	
95%	(52.1%)#	(0.23)†	1.9%
	(0.09)†		
	0.56	-1.57	
99%	(52.6%)#	(0.25)†	2.7%
	(0.11)†		
		-1.88	

Figure 4 displays the time series of the b estimates (the slope of the scaling law) for the GEV case at the 95% confidence level. Even though the average value of b at this confidence level is 0.53 (see Table 7), it appears to be time varying and takes values that range from 0.2 to 1.

used to test the null hypothesis H0 :$b = 0.5$. We employed this methodology when testing the statistical significance of the estimated slope b, because E-VaR estimates are for overlapping horizons, and therefore are auto-correlated. The Newey-West lag adjustment used was n - 1.

Figure 4: Time series of b estimates for the GEV model at 95% confidence



4.5 Comparison of EE-VaR with other models for E-VaR

Figure 5 below displays the time series of 90 day EVaR estimates at the 95% confidence level for each of the three parametric methods for RND extraction (combined with their respective empirical scaling regression results) together with the estimates from the Bank of England non-parametric method. The 90 day FTSE returns are also displayed in Figure 5.

Table 8 below shows the sample mean and standard deviation of each of the four EVaR time series. If we use the BoE values as the benchmark, we can see that on average, the mixture of lognormals method overestimates EVaR, while the Black-Scholes method underestimates it. Among the three parametric methods, the GEV method yields the time series of EVaRs closest to the BoE one. This can be seen both from Table 8 and also in the Figure 5, where the BoE time series practically overlaps the GEV E-VaR time series. This confirms that the GEV based VaR calculation is equivalent to the semi-parametric one used by the Bank of England but also has the added advantage, as we will see in the next section, of being capable of providing 10 day E-VaRs at the industry standard 99% confidence level.

Figure 5: BoE EVaR vs. GEV EVaR vs. Mixture lognormals for 90 days at 95% confidence level

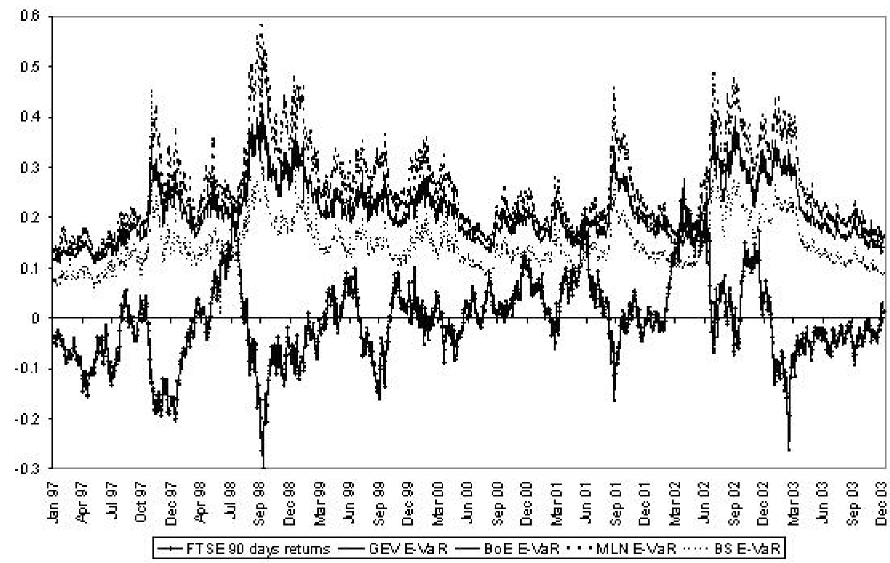


Table 8: BoE EVaR vs. GEV EVaR vs. Mixture lognormals for 90 days at 95% confidence level

Method	Sample mean	Sample standard deviation
BoE method	0.217	5.9%
GEV model	0.212	6.0%
Mixture lognormals	0.259	8.5%
Black Scholes	0.147	4.7%

4.6 EE-VaR in Risk Management : Backtesting results

The performance of a VaR methodology is usually assessed in terms of its backtesting performance. Here we compare the backtesting performance of EE-VaR against statistical VaR (S-VaR) estimated using the historical method and scaled using the square root of time, and also against the E-VaR estimated under the Black-Scholes assumptions.

The historical 1 day VaR at, say 99% confidence level, is obtained by taking the third highest loss in the time window of the previous 250 trading days, that is, in the previous year. Then, to obtain the 10 day VaR , the most commonly used technique is to scale up the 1 day VaR using the square root of time rule. The percentage number of violations at each confidence level q needs to be below the benchmark (1-q). For example, at the 99% confidence level, the percentage of days where the predicted VaR is exceeded by the market should be 1%. If there are fewer violations than the benchmark 1%, it means that the VaR estimate is too conservative, and it could impose an excessive capital requirement for the banks. On the other hand, if there are more violations than the benchmark 1%, the VaR estimates are too low, and the capital set aside by the bank would be insufficient.

We use our method to calculate the GEV and Black-Scholes based E-VaR values at constant time horizons of 1, 10, 30, 60, and 90 days. Tables 9, 10 and 11 show the results in terms of percentage number of violations for EE-VaR, S-VaR, and Black-Scholes E-VaR, respectively. The values highlighted in bold indicate that the benchmark percentage number of violations has been exceeded. On the other hand, the non highlighted values indicate that the benchmark percentage number of violations at the given confidence level has not been exceeded. The last row of each table displays the average percentage number of violations across maturities for each confidence level. In the row labelled "Benchmark" we can see the target percentage number of observations at each confidence level.

We can see that of all the three methods, EE-VaR yields the least number of cases where the benchmark percentage violations is exceeded. However, it exceeds the benchmark at all confidence levels for the 1 day horizon. The S-VaR also exceeds the benchmark at all confidence levels for the 1 day horizon, but additionally, it exceeds it for the 10 day horizon at 97% and 99% confidence levels, and for the 30 day horizon at almost all confidence levels. The Black-Scholes based E-VaR is the worst of all methods, exceeding the benchmark in 14 out of the 25 cases.

Looking at the averages across horizons for the three methods, we see that EE-VaR and S-VaR yield similar results at the higher quantiles (98% and 99%), but EE-VaR appears further away from the benchmark than S-VaR at the other confidence levels, indicating that it may be too conservative. On average, the Black-Scholes based E-VaR exceeds the benchmark percentage number of violations at all but the lowest confidence level, which indicates that it substantially underestimates the probability of downward movements at high confidence levels.

Table 9: Percentage violations of EE-VaR

Horizon		**Confidence**	**level**		
(days)	95%	96%	97%	98%	99%
Benchmark	5%	4%	3%	2%	1%
1	**6.2%**	**5.5%**	**4.4%**	**3.8%**	**3.1%**
10	2.4%	1.8%	1.5%	1.3%	0.6%
30	2.8%	2.0%	1.3%	0.5%	0.1%
60	2.5%	2.2%	1.3%	0.7%	0.0%
90	2.9%	2.3%	1.5%	0.8%	0.0%
Average	3.4%	2.8%	2.0%	1.4%	0.8%

Table 10: Percentage violations of Statistical VaR (S-VaR)

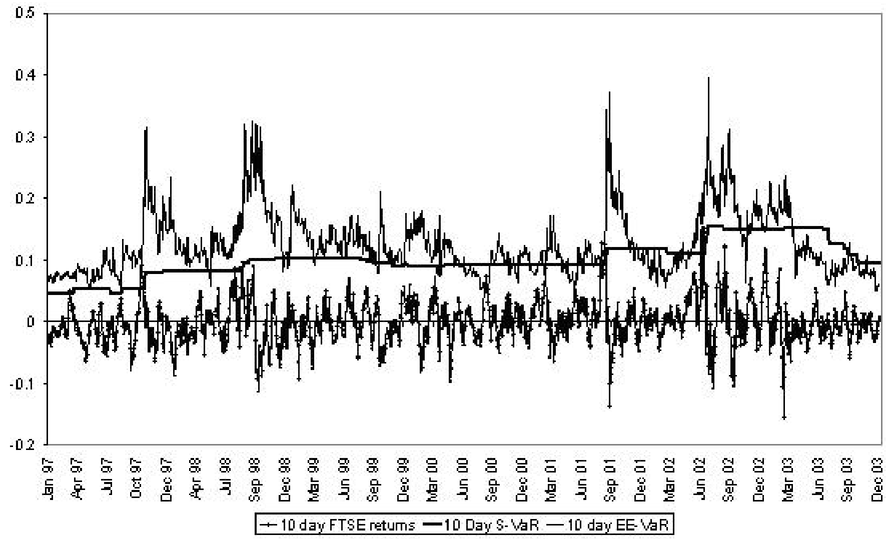
Horizon		**Confidence**	**level**		
(days)	95%	96%	97%	98%	99%
Benchmark	5%	4%	3%	2%	1%
1	**5.9%**	**4.7%**	**3.8%**	**2.5%**	**1.5%**
10	4.6%	4.0%	**3.1%**	1.7%	**1.4%**
30	**5.3%**	**4.2%**	**3.3%**	**2.1%**	0.8%
60	4.3%	3.6%	2.8%	1.1%	0.3%
90	4.5%	2.8%	1.9%	0.7%	0.0%
Average	4.9%	3.9%	3.0%	1.6%	0.8%

Table 11: Percentage violations of Black-Scholes based E-VaR

Horizon		**Confidence**	**level**		
(days)	95%	96%	97%	98%	99%
Benchmark	5%	4%	3%	2%	1%
1	**6.8%**	**5.9%**	**5.1%**	**4.4%**	**3.5%**
10	3.6%	3.2%	2.3%	2.0%	1.0%
30	4.0%	3.8%	**3.3%**	**2.4%**	**1.3%**
60	3.9%	3.3%	2.9%	**2.4%**	**1.7%**
90	4.6%	**4.1%**	**3.7%**	**3.0%**	**2.0%**
Average	4.6%	**4.1%**	**3.5%**	**2.8%**	**1.9%**

Figure 6 below shows the time series of 10 day FTSE returns, 10 day EE-VaR and 10 day S-VaR, both at 99% confidence level. We have chosen to plot the VaR of these particular set of (q, k) values as the 10 day VaR at 99% confidence level is one of most relevant VaR measures for practitioners, given the regulatory reporting requirements. We can see how the S-VaR is violated more times (25 times, or 1.4% of the time) than the EE-VaR (10 times, or 0.6%) by the 10 day FTSE return.

Figure 6: Time series of the 10 day FTSE returns, 10 day EE-VaR and 10 day S-VaR, both at a 99% confidence level.



4.7 E-VaR vs. Historic VaR x3

The Basle Committee on Banking Supervision explains in the "Overview of the Amendment to the Capital Accord to Incorporate Market Risks" (1996) that the multiplication factor, ranging from 3 to 4 depending on the backtesting results of a bank's internal model, is needed to translate the daily value-at-risk estimate into a capital charge that provides a sufficient cushion for cumulative losses arising from adverse market conditions over an extended period of time. But it is also designed to account for potential weaknesses in the modelling process. Such weaknesses exist because:

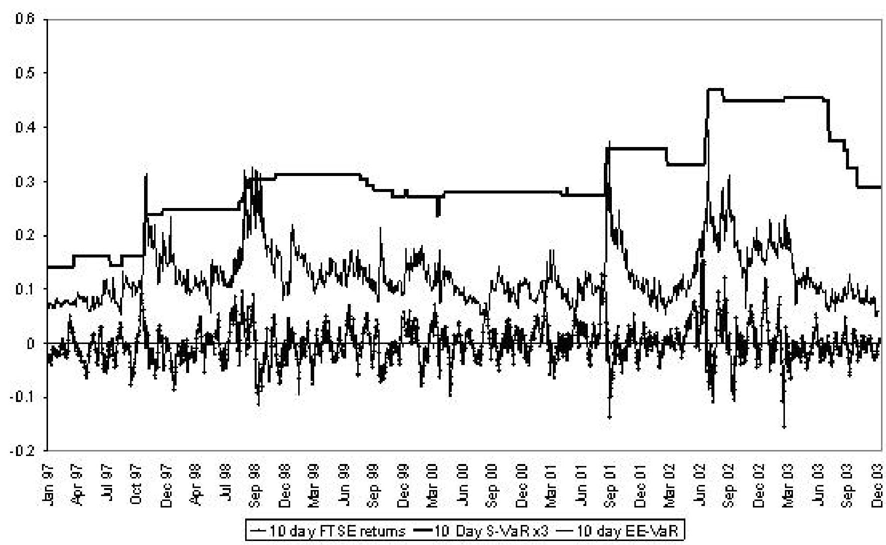
- Market price movements often display patterns (such as "fat tails") that differ from the statistical simplifications used in modelling (such as the assumption of a "normal distribution").
- The past is not always a good approximation of the future (for example

volatilities and correlations can change abruptly).

- VaR estimates are typically based on end-of-day positions and generally do not take account of intra-day trading risk.
- Models cannot adequately capture event risk arising from exceptional market circumstances.

It is interesting to note from Figure 7 that the E-VaR values in periods of market turbulence (Asian Crisis, LTCM crisis and 9/11) are similar to the historical S-VaR values when multiplied by a factor of 3. Thus, the multiplication factor 3 for a 10 day S-VaR appears to justify the reasoning that it will cover extreme events. In contrast, as we are modelling extreme events explicitly under EE-VaR such ad hoc multiplication factors are unnecessary.

Figure 7: 10 day at 99% E-VaR vs. Statistical VaR multiplied by a factor of 3



4.8 Capital requirements

The average capital requirement based on the EE-VaR, S-VaR and S-VaRx3 for the 10 day VaR at different confidence level is displayed in Table 12 below.

Here, the capital requirement is calculated as a percentage of the value of a portfolio that replicates the FTSE 100 index. What is very clear is that EE-VaR when compared to S-VaRx3 shows substantial savings in risk capital, needing only on average 1.7% more than S-VaR to give cover for extreme events. S-VaRx3 needs over 2.36 times as much capital for risk cover at 99% level. Figure 7 clearly gives the main draw back of historically derived VaR estimates where the impact of large losses in the past result in high VaR for some 250 days at a time. The EE-VaR estimates are more adept at incorporating market data information contemporaneously.

Table 12: Average daily capital requirement based on a 10 day horizon at different confidence levels, with standard deviations in brackets.

		Confidence	**level**			**Average**
	95%	**96%**	**97%**	**98%**	**99%**	
S-VaR	6.5%	7.3%	7.8%	9.1%	10.1%	**8.2%**
	(1.9%)	(2.3%)	(2.4%)	(2.9%)	(2.8%)	**(2.5%)**
S-VaR ×3	19.5%	21.9%	23.4%	27.3%	30.3%	**24.48%**
	(5.7%)	(6.9%)	(7.2%)	(8.7%)	(8.4%)	**(7.38%)**
EE-VaR	8.1%	8.7%	9.6%	10.8%	12.7%	**9.9%**
	(3.1%)	(3.3%)	(3.7%)	(4.2%)	(5.0%)	**(3.9%)**

5 Conclusions

We propose a new risk measure, Extreme Economic Value at Risk (EE-VaR), which is calculated from an implied risk neutral density that is based on the Generalized Extreme Value (GEV) distribution. In order to overcome the problem of maturity effect, arising from the fixed expiration of options, we have developed a new methodology to estimate a constant time horizon EE-VaR by deriving an empirical scaling law in the quantile space based on a term structure of RNDs. Remarkably, the Bank of England semi-parametric method for RND extraction and the constant horizon implied quantile values estimated daily for a 90 day horizon coincides closely with the EE-VaR values at 95% confidence level showing that GEV model is flexible enough to avoid model error displayed by the Black-Scholes and Mixture of Lognormal models. The main difference between the Bank's method and the one that relies on an empirical linear scaling law of the E-VaR based on a daily term structure of the GEV RND is that shorter than 1 month E-VaRs and in particular daily 10 day E-VaR can be reported in our framework. This generally remains problematic in the RND extraction method based on the implied volatility surface in delta space as it is non-linear in time to maturity and also it cannot reliably report E-VaR for high confidence levels of 99%.

Based on the backtesting and capital requirement results, it is clear there is a trade-off between the frequency of benchmark violations of the VaR value, and the amount of capital required. The 10 day EE-VaR gives fewer cases of benchmark violations, but yields higher capital requirements compared to the 10 day S-VaR. However, when the latter is corrected by a multiplication factor of 3, to satisfy the violation bound, the risk capital needed is more that 2.3 times as much as EE-VaR for the same cover at extreme events. This saving in risk capital with EE-VaR at high confidence levels of 99% arises because an implied VaR estimate responds quickly to market events and in some cases even anticipate them.

While the power of such a market implied risk measure is clear, both as an additional tool for risk management to estimate the likelihood of extreme outcomes and for maintaining adequate risk cover, the EE-VaR needs further testing against other market implied parametric models as well as S-VaR methods. This will be undertaken in future work.

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