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Evolutionary Money Management

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Evolutionary Money Management

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Abstract

This paper evolves trading strategies using genetic programming on high-frequency tick data of the USDEUR exchange rate covering the calendar year 2006. This paper proposes a novel quad tree structure for trading system design. The architecture consists of four trees each solving a separate task, but mutually dependent for overall performance. Specifically, the functions of the trees are related to initiating ("entry") and terminating ("exit") long and short positions. Thus, evaluation is contingent on the current market position. Using this architecture the paper investigates the effects of money management by comparing an entry-entry grammar with an entry-exit grammar. The former uses the same information set across all trees, while the latter has additional information about the current profit, drawdown and duration of a trade. Thus, money management is evolved endogenously instead of being superimposed as an additional layer.

Strategies are evolved with and without transaction costs, and under three different kinds of utility; risk neutrality, risk aversion and loss aversion. Under frictionless trading the strategies exploit the significant mean reversion effect in the underlying returns. This is a dominating strategy for all utility functions. Under market frictions and loss aversion the strategies spend more time in a neutral position and the characteristics of the strategy returns change.

Based on fitness measures it is not found that there is significant difference between the entry-entry and entry-exit strategies and from this perspective money management does not add value. Moreover, the out-of-sample performance seem to deteriorate over time – a finding that is consistent with the adaptive markets hypothesis. Significant profits are only generated in a single out-of-sample period and it can therefore be concluded that the high-frequency foreign exchange markets are highly efficient.

JEL classifications: C0, C45, C53, C63, G1, G11, F31

Keywords: Genetic programming, trading strategies, high-frequency data, foreign exchange and money management.

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1 Introduction

The foreign exchange (FX) market has always been the largest financial market in the world, but what is more impressive is its strong growth in recent years. In 2004 the average daily turnover was $1.9 trillion, but in 2007 it had increased more than 60% to $3.2 trillion. Around a quarter of this activity is related to the USDEUR currency pair [4]. Much of this volume is associated with transactions between dealers. It is common for FX dealers not to take overnight positions and consequently there is a lot of trading to reduce the risk associated with an accumulation of inventory [22]. In addition to the dealers, the FX markets are comprised of a plethora of agents, including institutional investors, hedge funds, retail investors and central banks.

In theory, exchange rates should be intimately linked to macro economic variables, such as interest rates, inflation, money supply and real income. However, in their seminal paper, Meese and Rogoff [25] found that these fundamentals are useless in forecasting exchange rate changes even at medium frequencies. This is known as the *determination puzzle of foreign exchange*. At shorter time horizons many traders tend to use *technical analysis* in decision making [1]. Technical analysis attempts to forecast future price changes based on historical observations. This broad definition covers a wide range of methods from visual pattern recognition to moving averages and more elaborate schemes. Traditionally, it has encountered much skepticism from academia since it clearly contradicts the *Efficient Market Hypothesis* (EMH) – one of the cornerstones of modern finance [12]. Trading on the basis of technical analysis has often been considered irrational behavior, but since it continues to be a widespread approach among practitioners this would have the paradoxical implication that the markets are not efficient to begin with [26]. A basic premise for efficient capital markets is the existence of *homo economicus*.

During the past few decades, there has been an increasing interest in technical analysis among financial economists and extensive literature has emerged on the subject. There is a general consensus that technical analysis on a daily frequency has been profitable in the past. Chang and Osler [6] find that the elaborate head-and-shoulders pattern is profitable even in the presence of transaction costs on a number of currencies *vis-a-vis* the dollar in the years between 1973 and 1994. However, they conclude that the rule is not efficient since it is dominated by simpler strategies such as moving average crossover, but it should be noted that, in this comparison, transaction costs are omitted. It has also been found that returns from following simple moving average rules have significantly different distributions from the underlying exchange rates. Specifically, the strategy returns are positively skewed, and the most negative returns are avoided [23]. LeBaron [20] finds that while moving average strategies were profitable in earlier periods, their performance significantly deteriorated throughout the nineties.

In contrast to the studies above, this paper uses high-frequency intraday tick data and considers both a frictionless environment and trading under market frictions through the quoted bid-ask spread. Instead of using a predetermined strategy as a moving average rule or a chart pattern, the computer evolves its own trading strategies using *genetic programming* (GP). GP has previously been applied to trading rule induction for foreign exchange markets. Using tick data from 1992 to 1993, GP was found to produce significant profits when trading in the presence of realistic transaction costs [16]. Bhattacharyya et al. [3] also generate excess returns using intraday data from 1987 to 1994. A more recent study also came to a positive conclusion using an assumed fixed spread of 1bp [19]. Dempster and Jones [11] have mixed success in combining traditional technical indicators using evolutionary computation on tick data from 1994 to 1997. Neely and Weller [27] use intraday data from 1996 and find that the FX market is efficient. However, Neely et al. [29] report significant positive excess return using GP on daily data in the period from 1981 to 1995 . Moreover, the returns generated are not compensation for bearing any systematic risk. Unfortunately, a follow-up study indicates that performance deteriorated substantially after 1995 [28].

As mentioned above, this paper considers high-frequency intraday tick data on the USDEUR exchange rate covering the full year of 2006. Besides being a much needed update on GP in this domain it adds to the existing literature in a number of ways. The standard approach of GP in trading rule induction is to use a single tree structure that makes buy or sell recommendations. This paper proposes a novel multiple tree structure consisting of four (quad) trees for ternary decision problems. Hence, strategies can take short, neutral and long positions. Which tree is evaluated is contingent on the current market position. Each of the four trees returns Boolean values and their functions can be characterized as long entry, long exit, short entry and short exit. The entry trees initiate either long or short positions, while the exit trees terminate those positions and revert to a neutral state. Using this division of entry and exit strategies, the benefits of money management are examined. Money management refers to certain measures that traders use to control risk and take profits, implying that closing of positions can be initiated by events other than "standard" buy/sell signals. To reflect this in an automated trading system, an extension to standard approaches needs to be made. Traditionally, one common rule for both positions is evaluated, and depending on the outcome, the signal is to enter (stay in) these positions or exit (stay out of) them, respectively. In money management, different rule sets, contingent on the current position, are used. Hence, a negative entry signal is not necessarily seen as an exit signal, but entirely different rules are evaluated to find exit signals. Furthermore, these exit signals can be based on other indicators or information. For example, stop losses are often placed to trigger an exit signal in order to limit downside risk. Since money management is a practitioner's way of controlling risk, the effects of evolving strategies under different utility functions are investigated.

The paper is structured as follows. Section 2 provides a brief introduction to genetic programming. Section 3 presents the data. The fitness function, model and parameter settings are described in Section 4. This is followed by empirical results in Section 5. Finally, Section 6 concludes and gives pointers to possible future research.

2 Genetic Programming

Genetic programming (GP) was pioneered by Koza [18] and is often seen as a derivative of genetic algorithms (GA). The GA was invented by Holland [14] in his ambitious quest to understand the principles of adaptive systems in a broad sense. An obvious inspiration came from biology, where the success of natural adaptive systems rests on competition and innovation in order to survive in changing and uncertain environments.

The GA is a population based search method, where the individuals are fixed-length binary strings, known as *genotypes* or *chromosomes* [35]. Generally, this representation requires an encoding which is problem specific. For example if the GA is used for real-valued parameter optimization, then it is necessary to discretize the search space, where the resolution is depends on the number of bits chosen to represent a given variable.

GAs work as follows. In generation zero, an initial population of M individuals is generated randomly. Hereafter, the fitness of each individual is calculated according to the pre-specified objective function. Then a new population is created by selecting between the operators, reproduction, crossover and mutation according to the probabilities p_r, p_c and p_m, respectively. Each of these operators select individuals from the parent population, such that better solutions are favored. A popular mechanism for doing this is *tournament selection*, in which a fixed number of individuals are chosen uniformly from the parent population, and the fittest individual wins the tournament and is selected. By controlling the tournament size it is possible to regulate the selection pressure. The reproduction operator simply copies the selected string to the new population. For the crossover operation, two individuals are selected from the parent population. Hereafter a position or index is uniformly selected within the bit-string and genetic material from the two parents is simply swapped around this point. The two resulting offspring are then inserted into the new population. The mutation operator simply selects a random element within an individual and negates the value, i.e., zero becomes one and vice versa. An advantage of the mutation operator is that it can introduce diversity into a population, but usually mutation is only invoked with a small probability. When the population size of the new population is equal to M, the algorithm has completed one generation and the process repeats itself until a termination criteria has been satisfied, e.g., until a maximum number of generations is reached

Genetic programming is basically a GA operating on hierarchical computer programs instead of binary strings. Any problem that is concerned with finding an optimal mapping from a set of inputs to a set of outputs, can be reformulated as a search for an optimal computer program. GP provides the means to search the space of possible programs. It is therefore a much more direct approach to problem solving than GAs, that are heavily dependent on problem encoding. In practice the individuals in GP are computer programs represented as tree structures. The programs are constructed from *functions* and *terminals*, where by definition the former take arguments and the latter do not. The sets of available functions and terminals for a given problem is known as the *function set* and *ter-*

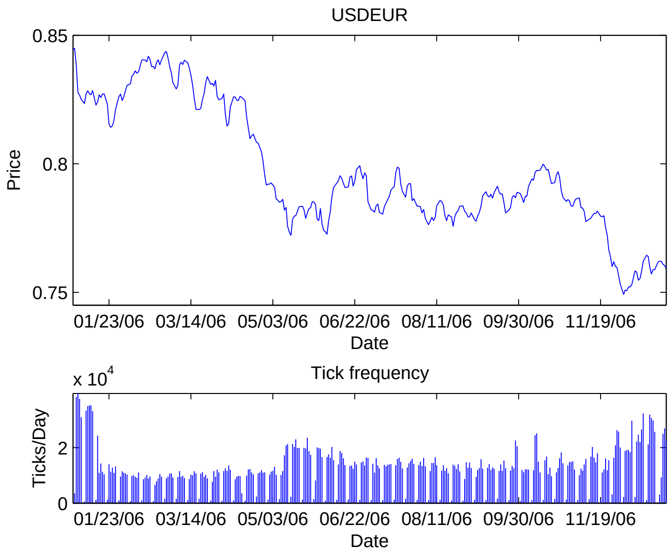


Figure 1: Average daily USDEUR exchange rate (top), and number of ticks per day (bottom).

minal set. For more details on GP and their application to automated trading, see Koza [18] and Saks and Maringer [32].

3 Data

The data is provided by OANDA FXticks, and comprises tick data on the USDEUR exchange rate covering the calendar year 2006. This constitutes a total of 3894525 bid-ask observations, and on average there are more than 12000 per trading day. Figure 1 shows the average daily prices, together with the number of ticks per day. Over the entire period the dollar depreciates from 0.8439 to 0.7577, corresponding to 10.77%. The daily tick frequency appears fairly constant for most of the year, but at the beginning and end of the year the activity is considerably higher. Moreover, during the month of May, there is also increased activity which coincides with a crisis in the equity markets.

Figure 2 depicts the unconditional distribution of waiting times between ticks. The high-frequency econometrics literature often models trading intensities using Poisson processes [2]. Under this assumption the waiting times between ticks are exponentially distributed. This is confirmed by the data, although there seems to be a slight bias for low waiting times.

Traditional time series analysis assumes that the data is *homogeneous*, i.e., equally spaced in time. As already demonstrated this is not the case for financial data, where waiting times between ticks are random, hence it is *inhomogeneous*. It is trivial to convert inhomogeneous data to homogeneous data, but it might not be sensible. Figure 3 shows a non-parametric kernel estimate of the average

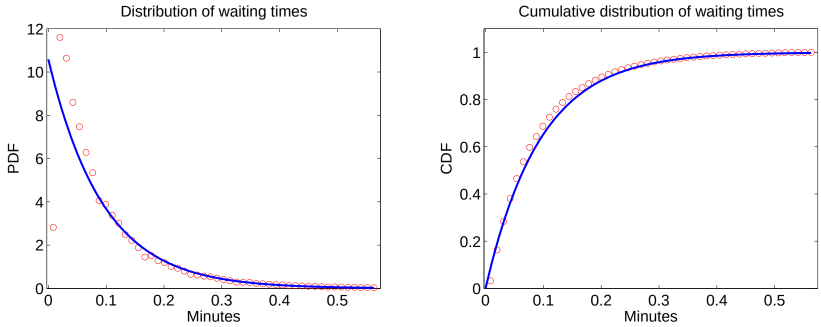


Figure 2: Unconditional probability (left) and cumulative probability density function (right) of waiting times between ticks. The circles are empirical estimates and the lines are fitted exponential distributions.

waiting times between ticks conditioned on the time of day [34]. Clearly, there is a strong intraday seasonality where the activity (inverse of waiting time) is higher during European working hours. Moreover, there are two distinct peaks around 08:00 and 14:00 with high activity, separated by lower activity at noon. By using an equidistant intraday sampling in calendar time this seasonality is neglected leading to disproportionately many samples during the night compared to daytime. Hence, important information might be dampened while unimportant information is amplified.

In this paper sampling is done in trading time. Specifically, the exchange rate is sampled every 10 ticks. On average this is close to 1 minute sampling in calendar time. The data comprises of bid-ask quotes, but in the following the statistical properties of the logarithmic middle prices are analyzed,

$$x_t = \frac{\ln\left(p_t^{\text{bid}}\right) + \ln\left(p_t^{\text{ask}}\right)}{2}. \tag{1}$$

An advantage of using logarithmic prices is that the inverse relationship between USDEUR and EURUSD is simply expressed by changing the sign of the series. The returns are the usual first-order difference of log-prices. The bid-ask spread is

$$s_t = \ln\left(p_t^{\text{ask}}\right) - \ln\left(p_t^{\text{bid}}\right). \tag{2}$$

Due to large spikes in the spreads, the top decile is winsorized. For the entire sample, the median spread is 1.1870 bp, with an interquartile range of 0.0707 bp. Table 1 contains summary statistics for the log-returns of the sampled USDEUR series. The series has negative skewness and significant excess kurtosis, thus strongly rejecting the null hypothesis of Gaussianity in a Kolmogorov-Smirnov test. At the 10-tick sampling frequency there is a significantly negative first order auto-correlation. This phenomenon has previously been reported in the literature using a one-minute sampling in calendar time [13]. Several explanations are

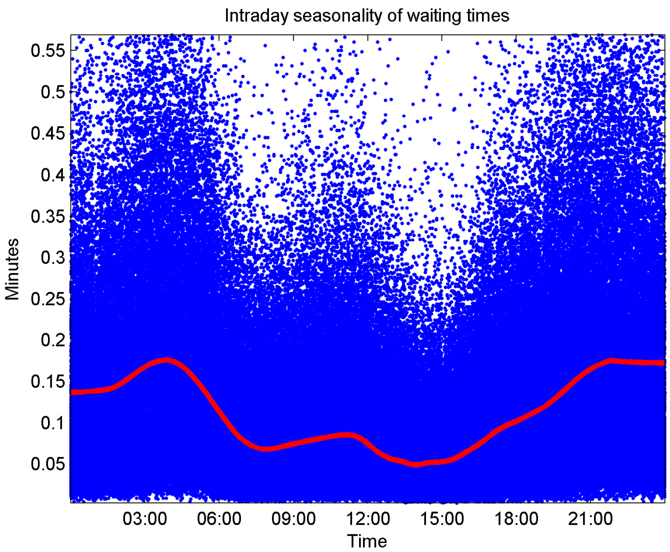


Figure 3: Non-parametric estimate of average waiting times between ticks conditioned on the time of day.

given for this short-term mean-reversion, such as when market makers have order imbalances they tend to skew the spread in a certain direction, or that some banks publish higher bid-ask spreads leading prices to bounce back and forth between them [10].

Correlation only captures the linear dependency between variables, while a *copula* can provide a complete picture of the dependency structure between variables. *Sklar's theorem* dictates that any multivariate distribution can be decomposed into a set of marginals and a copula function [8]. Instead of imposing a specific dependency structure using a parametric copula, a non-parametric kernel-based model is considered [7]. Figure 4 depicts the copula densities for successive 10-tick returns during the months of January, April, July and October.

	10 Ticks
Sample size	389452
Mean ($\cdot 10^{-6}$)	-0.277
Standard deviation ($\cdot 10^{-4}$)	1.323
Skewness	-0.215
Kurtosis	18.823
Auto-correlation (lag-1)	-0.048
Auto-correlation (lag-2)	0.002
Kolmogorov-Smirnov (p)	0.000

Table 1: Summary statistics of log-returns of sampled USDEUR series.

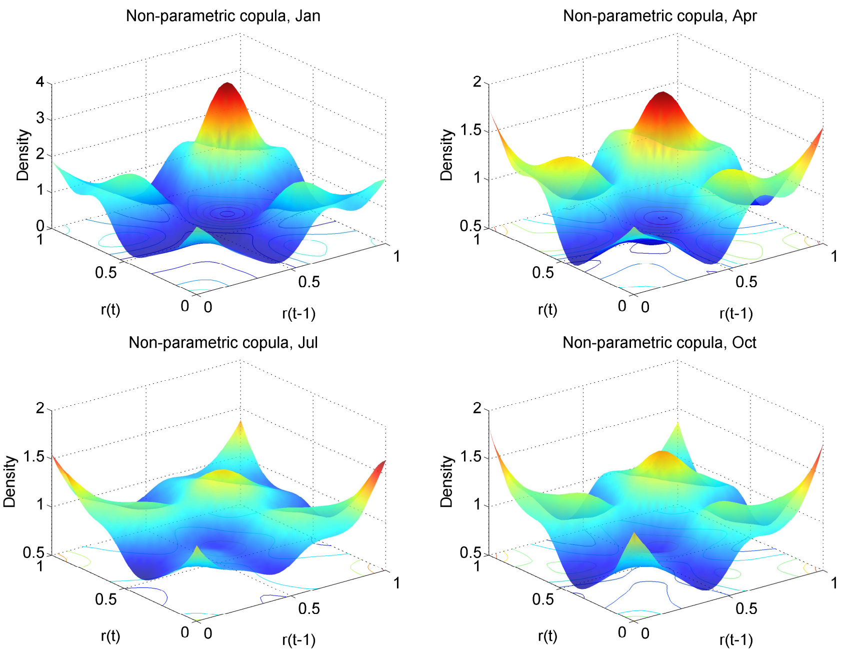


Figure 4: Non-parametric copula densities for successive returns during the months of January, April, July and October.

For all four months the negative correlation is present, but in addition there are some momentum effects when the returns are slightly above the median value of zero. That is a small dollar appreciation over the previous period is likely to be followed by a small appreciation during the next period. Moreover, this response is asymmetric and does not hold for dollar depreciations. The momentum effect seems to deteriorate over time, but the pattern clearly persists.

4 Framework

4.1 Objectives and Fitness Function

Choosing an appropriate objective function is essential in evolutionary computation. Since this paper is concerned with trading system design and ultimately generation of wealth, it is natural to seek inspiration in *economic utility theory*. In classical utility theory, the sole objective of agents is to maximize expected utility of wealth, where more wealth is always preferred to less [9]. However, cognitive psychology has revealed that, in evaluating different outcomes, reference dependence plays a crucial role [17]. In the context of financial investments the reference point is determined by how myopic an agent is, i.e., how frequently wealth is evaluated [33]. This implies that both the long-term level and short-term changes in wealth are important factors in determining overall happiness for speculators. Hence, happiness is a path-dependent function of the evolution of wealth. To capture this path dependency, the period over which the trading

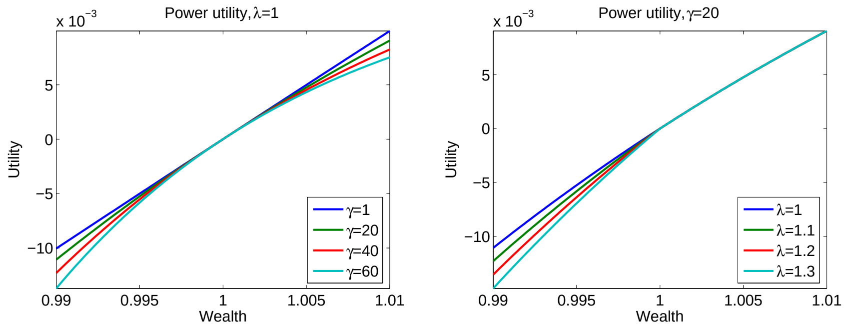


Figure 5: Power utilities for different values of γ (left) and λ (right).

rule is optimized is divided into I sub-intervals and a utility is evaluated for each interval. Let the return within an interval i be defined as,

$$r_i = \frac{w_t - w_{t-k}}{w_{t-k}} \tag{3}$$

where k is the length of the interval. A modified terminal interval wealth is then introduced,

$$\hat{v}_i = \begin{cases} v_0 \cdot (1 + r_i) & \text{if } r_i \geqslant 0 \\ v_0 \cdot (1 + r_i)^{\lambda} & \text{if } r_i < 0 \text{ with } \lambda \geq 1 \end{cases} \tag{4}$$

where $\lambda > 1$ implies loss aversion, i.e., a greater sensitivity to decreases in wealth, while equality models no additional disutility of losses beyond risk aversion; see also [24]. The initial endowment at the beginning of the interval is v_0, but in this paper a unit investor is considered such that $v_0 = 1$. Assuming a power utility function, the utility of a modified terminal interval wealth is,

$$U(\hat{v}_i) = \begin{cases} \frac{\hat{v}_i^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} & \text{if } \gamma > 1 \\ \ln(\hat{v}_i) & \text{if } \gamma = 1 \end{cases} \tag{5}$$

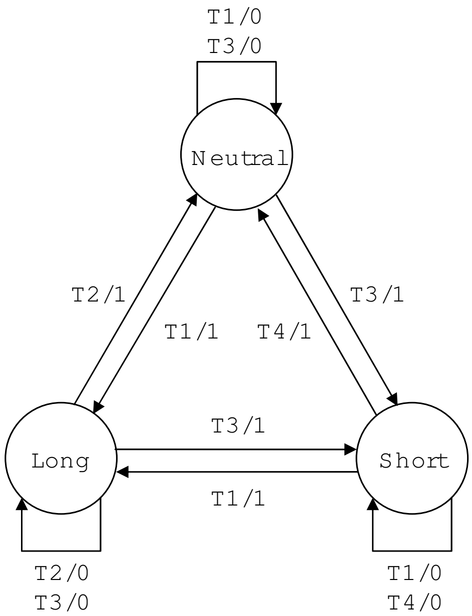
For a risk averse trader, $\gamma > 1$, whereas $\gamma = 1$ implies risk neutrality. From this the objective function simply follows as the average interval utility,

$$F = \frac{1}{I} \sum_{i=1}^{I} U(\hat{v}_i). \tag{6}$$

The power utility function is shown for different values of γ and λ in Figure 5. γ controls the concavity of the utility function, and λ regulates the loss aversion by decreasing utilities for losses while leaving utilities for gains unchanged.

4.2 Model

The purpose of this paper is to evolve trading models using genetic programming. The traditional approach in the context of FX forecasting is to evolve binary decision rules where outputs correspond to either long or short positions



	current position											
	neutral				long				short			
T1	0	0	1	1	.	.	.	.	0	0	1	1
T2	.	.	.	.	0	0	1	1	.	.	.	.
T3	0	1	0	1	0	1	0	1	.	.	.	.
T4	.	.	.	.	.	.	.	.	0	1	0	1
new position	N	S	L	N	L	S	N	S	S	N	L	L

Figure 6: State diagram of the quad tree structure consisting of long entry (T1), long exit (T2), short entry (T3) and short exit (T4), going from the current position to neutral (N), short (S), long (L) position. 0 = FALSE and 1 = TRUE

in a given currency. Using this representation a trading model is forced to take a directional view and cannot remain neutral. To overcome this problem in a single tree framework, it is possible to construct programs that return a trinary Boolean variable instead of the normal binary Boolean variable [3].

In the context of binary trading models it has previously been found that using a dual tree structure instead of the traditional single tree model has significant impact on performance, especially when market frictions are taken into account [31]. Capitalizing on these findings this paper proposes a unique quad tree structure. The four trees consist of a long entry (T1), long exit (T2), short entry (T3) and short exit (T4). Unlike a stock, an exchange rate does not have a distinct up and down. The inverse relationship of exchange rates dictates that up for one currency is down for the other and vice versa. In this paper the positions relate to dollar. Thus, when a long (short) position is initiated we expect an appreciation (depreciation) of the dollar relative to the euro.

The workings of the four trees is illustrated in Figure 6. Each tree returns a Boolean variable, but which tree is evaluated depends on the current market position. For example if the current position is neutral, then either a long or a short position can be initiated. This happens when either T1 or T3 is true. If both T1 and T3 are true, the signal is ambiguous and a neutral position is maintained. When the current position is long and T2 is true, a transition is made to a neutral position. If T3 is true a short position is initiated. In the short state, T1 initiates a long position and T4 results in a neutral position. In order to resolve conflicting decisions, the strongest views are given precedence such that directional views trump neutrality.

One objective of this paper is to examine the effects of money management on trading strategies. This is done by comparing strategies with special grammars for the exit strategies (T2 and T4), to strategies where the grammar is the same across all the trees. In addition to type constraints the trees have semantic restrictions, which improves the search efficiency significantly, since computational resources are not wasted on nonsensical solutions [3]. The function set for the entry strategies (T1 and T3) consists of numeric comparators, Boolean operators and addition. Furthermore, three special functions have been introduced. `BTWN` takes three arguments and evaluates if the first is between the second and third. `HASINC` (`HASDEC`) returns true if the second argument has increased (decreased) over the lag period given by the first argument. The terminals include the variables price and moving averages thereof (`price`) and the time of day (`time`). Special constants are available for conditioning on time (`timeConst`), and the difference between price indicators (`pConst`). The entry strategy grammar is documented in Table 2.

In practice, traders employ various exit strategies for money management, such as stop losses and profit targets. A stop loss automatically exits the strategy when the current profit is below a certain level. Likewise a profit target closes out a position when a given profit is obtained. A more elaborate scheme is a trailing stop, which ensures that the drawdown does not exceed a given value. Simple stop losses and profit targets might be augmented with time exits such that the duration of a trade is constrained. To capture these ideas the exit strategy grammar contains information about the current profit, drawdown and duration of a trade. The exit strategy grammar is an extension to the entry strategy grammar and their difference is documented in Table 3.

4.3 Parameter Settings

In the following computational experiments a population of 500 individuals is initialized using the *ramped half-and-half* method. It evolves for a maximum of 50 generations, but is stopped after 20 generations if no new elitist (*best-so-far*) individual has been found. A normal tournament selection is used with a size of 5, and the crossover and mutation probabilities are 0.9 and 0.05, respectively. Moreover, the probability of selecting a function node during reproduction is 0.5, and each of the trees in the programs are constrained to a maximum complexity of 25 nodes. Again, this constraint is imposed to minimize the risk of overfitting,

Function	Arguments	Return Type
+	(price, pConst)	priceNew
<=, >=	(price, price)	bool
<=, >=	(price, priceNew)	bool
<=, >=	(time, timeConst)	bool
BTWN	(price, price, price)	bool
BTWN	(price, priceNew, priceNew)	bool
BTWN	(time, timeConst, timeConst)	bool
HASDEC, HASINC	(lag, price)	bool
AND, OR, XOR	(bool, bool)	bool
NOT	(bool)	bool

Table 2: Entry strategy grammar. BTWN checks if the first argument is *between* the second and third. HASINC (HASDEC) returns true if the price has increased (decreased) over the last period.

Function	Arguments	Return Type
<=, >=	(duration, durationConst)	bool
<=, >=	(profit, pConst)	bool
>=	(drawdown, drawdownConst)	bool
BTWN	(duration, durationConst, durationConst)	bool
BTWN	(profit, pConst, pConst)	bool

Table 3: Additional grammar for exit strategy. The complete exit grammar is composed of the entry strategy grammar and the functions in this table.

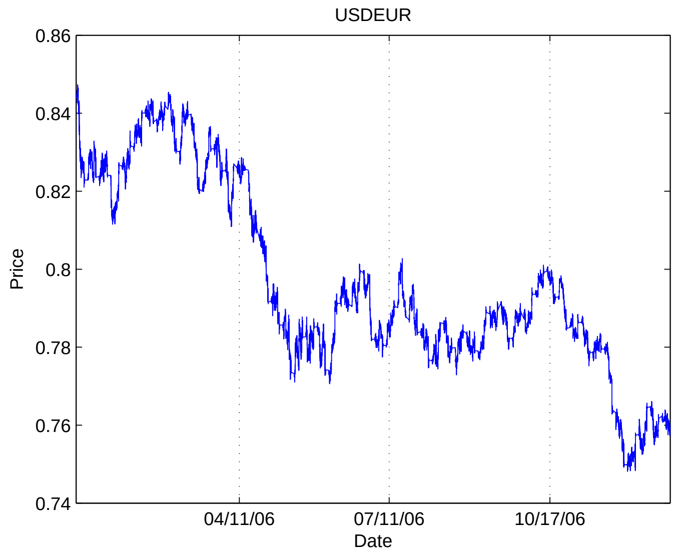


Figure 7: Intraday USDEUR exchange rate split into four equal-size blocks in trading time.

but also to facilitate interpretability. If the models lose tractability, it defies the purpose of genetic programming as a knowledge discovery tool.

The entire data set is split into four equal-size blocks of 97,364 samples each. Henceforth the blocks are denoted; I, II, III and IV. Since the blocks have been constructed in trading time, their durations in calendar time differ. Block I covers the period from 01-Jan-2006 to 11-Apr-2006, Block II continues to 11-Jul-2006, Block III ends 17-Oct-2006, and Block IV is the remainder until 29-Dec-2006. Figure 7 shows the USDEUR exchange rate and the four blocks. The returns in each of the blocks are -2.37%, -4.75%, 1.40% and -5.06%, respectively. In the following experiments, rolling window estimation is made on blocks I-III and successive out-of-sample tests are made on blocks II-IV.

As fitness functions, three different utility functions are considered: risk neutral ($\gamma = 1$, $\lambda = 1$), risk averse ($\gamma = 35$, $\lambda = 1$) and loss averse ($\gamma = 35$, $\lambda = 1.15$). Each Block is divided into a number of sub-intervals, each consisting of 1000 samples. This implies that, on average, wealth is evaluated between one and two times per day, which does not seem unreasonable for a high-frequency trader. As mentioned in Section 4.1, the fitness of an individual trading strategy is the average utility obtained within each sub-interval. Due to the high-frequency domain considered in this paper the overnight interest rates are neglected when calculating the returns of the strategies.

5 Empirical Results

5.1 Performance

In this section the results from rolling window estimation and testing on blocks I to IV are presented. For statistical inferences ten independent runs are considered for both the entry-entry and entry-exit grammar strategies. Moreover, strategies are evolved both under the assumption of frictionless trading and in the presence of market frictions, i.e., trading occurs on middle prices or through the bid-ask spread, respectively.

Table 7 reports fitness values and interval statistics for entry-entry grammar strategies evolved under frictionless trading and risk neutrality. The interval statistics give information about the moments of the interval return distributions, i.e., the return generated within each 1000 sample sub-interval used in the fitness evaluation.

The median in-sample fitness value is 60.44 for Block I, and out-of-sample it decreases to 39.05 on Block II. However, when Block II is used for training, the median fitness is identical, which indicates that the strategies have captured the underlying dynamics of the price process perfectly. For the remainder of the blocks the performance is comparable. Overall the interval returns appear relatively normal, albeit with some excess kurtosis in some of the blocks. To test the null hypothesis that the evolved strategies have uncovered significant regularities, their fitness values are compared to that of 1000 randomly initialized strategies. Figure 8 show the empirical fitness distributions for the random strategies under different utility functions with and without market frictions. Under frictionless trading and risk-neutrality trading without an edge is a fair gamble. However, as risk aversion is introduced the expected utility from random trading is negative, and loss aversion only exacerbates this effect. Introducing market frictions causes the expected utility to become substantially negative even for risk neutral speculators. This suggests that real speculation in the considered currency markets requires either a strong risk-seeking behavior or a significant edge.

To gain further understandings of the actual trading, Table 14 in the Appendix shows relevant statistics such as the number of trades (NT), the ratio of long versus total number of trades (LSR), proportion of time spent in a neutral position (NR), percentage of profitable trades (PP), average return of trade (AT) and maximum drawdown of the strategy (MDD). In a frictionless environment the median number of trades is in excess of 40,000 per block and is balanced evenly between long and short positions. The strategies are generally neutral less than 6% of the time, and the proportion of winning trades is approximately 70%. Given the high turnover it is not surprising that the average return of trades are tiny ($\leqslant 0.1$ bp).

Under frictionless trading the evolved strategies simply capture the mean-reversion effect inherent in the data as documented in Section 3. Exploiting this microstructural effect is a winning strategy regardless of the utility functions considered in this paper. This is true for both the entry-entry and entry-exit

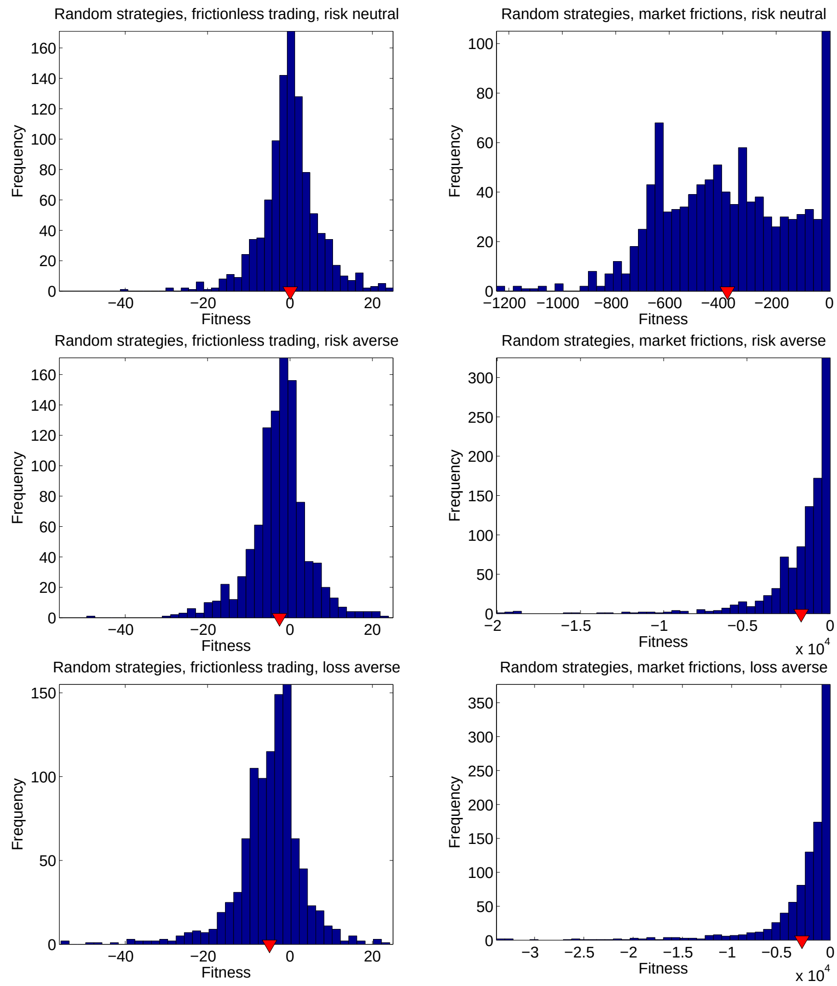


Figure 8: Histograms of the performance of 1000 random strategies for different utility functions under frictionless trading (left column) and market frictions (right column). The triangle marks the average utility.

grammar strategies. Consequently, for the sake of brevity the detailed results for other utility functions and grammars are omitted, and the rest of the paper will focus mainly on trading under market frictions.

Appendixes A and B contain detailed results on interval and trade statistics, for both the entry-entry and entry-exit grammar strategies under various utility functions. Naturally, the introduction of market frictions has an adverse effect on performance. Under risk neutrality the in-sample median fitness in Block I drops to 7.78 and 8.08 for the entry-entry and entry-exit grammar strategies, respectively. What is more interesting is that the out-of-sample median fitnesses in Block II are positive for both grammars (4.51 and 4.37). Unfortunately, this pattern does not repeat itself during the later periods. This holds for all utility functions. However, it should be noted that negative utility need not imply unprofitability – the out-of-sample results in Block III for the entry-exit grammar strategies under loss aversion illustrate this point (Table 13). When risk aversion and loss aversion are introduced it has an adverse effect on in-sample performance for both grammars as expected. Finally, it should be noted that most of the strategies significantly outperform the random strategies both in-sample and out-of-sample. This suggests that high-frequency currency speculation is extremely difficult and a sensible strategy is simply to stay neutral in the absence of any edge.

As mentioned previously, a main objective of this paper is to examine the effects of money management for different types of speculators. To test formally whether money management, i.e., an extended exit grammar, makes a difference, the Wilcoxon rank sum test is employed for each block and for each utility function. Table 4 lists the p-values for the null hypothesis of identical median fitnesses for the two grammars. Only Block IV out-of-sample under risk neutrality is significant at the usual level. In the context of the other results, this can clearly be treated as a spurious rejection and does not lead to an overall rejection of the null hypothesis. It must therefore be concluded that money management has a detrimental effect on utility, since the evolved strategies do not make use of it. [30] offer a possible explanation for this result: in the foreign exchange markets, stop and limit orders tend to be clustered around round numbers giving rise to distinct support and resistance levels, where trend reversals are more likely to occur. This has not been taken into account in this chapter. Having concluded that money management does not add significant value, a more detailed analysis of the entry-entry grammar results is provided in the following.

Figures 9 to 12 show boxplots of the moments of the interval return distributions under risk neutrality, risk aversion and loss aversion. To determine if medians differ across utility functions, the Kruskal-Wallis test is employed. Table 5 reports the p-values for the null hypothesis of equal medians. For the mean interval returns the different utility functions have the same median values in-sample on Block I and II, but for Block III the median under loss aversion is significantly lower. Out-of-sample on Block II, loss aversion produces lower means, while on Block IV the opposite holds.

Strategies evolved under loss aversion have significantly smaller standard deviations of interval returns both in-sample and out-of-sample. In accordance

Utility function	In-sample I	In-sample II	In-sample III	Out-of-sample II	Out-of-sample III	Out-of-sample IV
Risk neutral	0.3847	0.6232	0.2730	0.5708	0.9097	0.0312
Risk averse	0.6776	0.7913	0.3447	0.0757	0.4727	0.9097
Loss averse	0.7337	0.4274	0.2413	0.5452	0.9097	0.2413

Table 4: Rank sum test p-values for the null hypothesis of equal median fitnesses of the entry-entry and entry-exit grammar strategies.

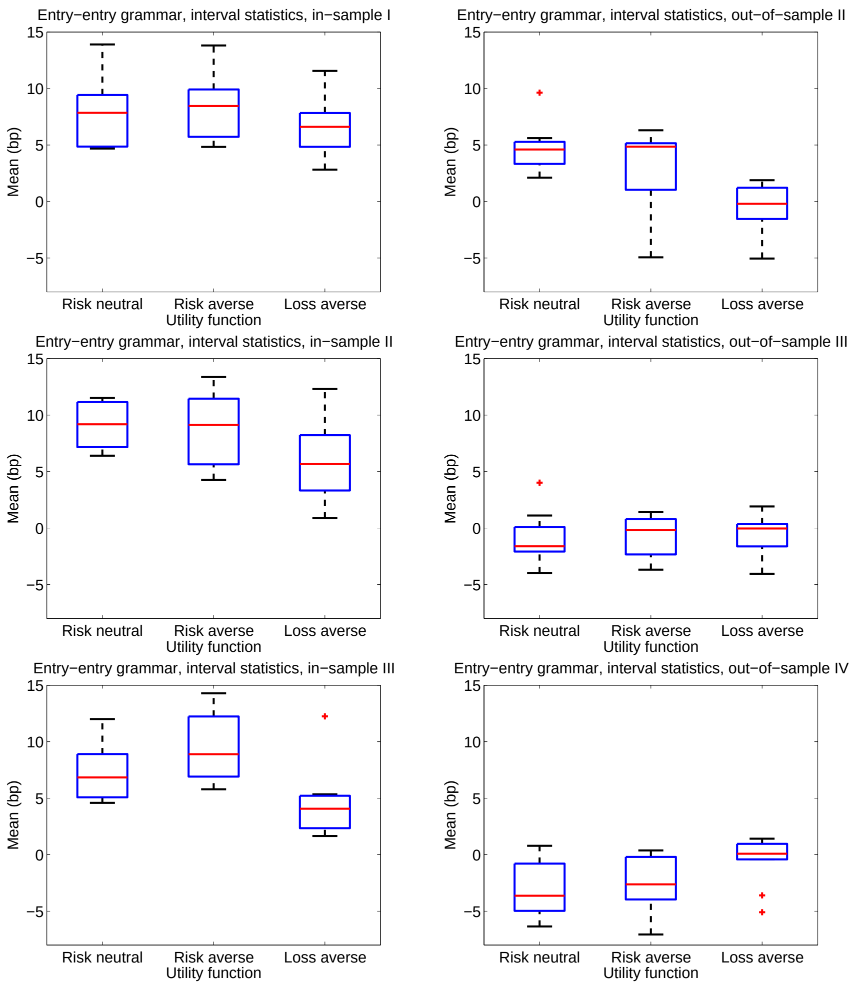


Figure 9: Boxplots of the average of interval returns across the 10 runs for each block and different utility functions.

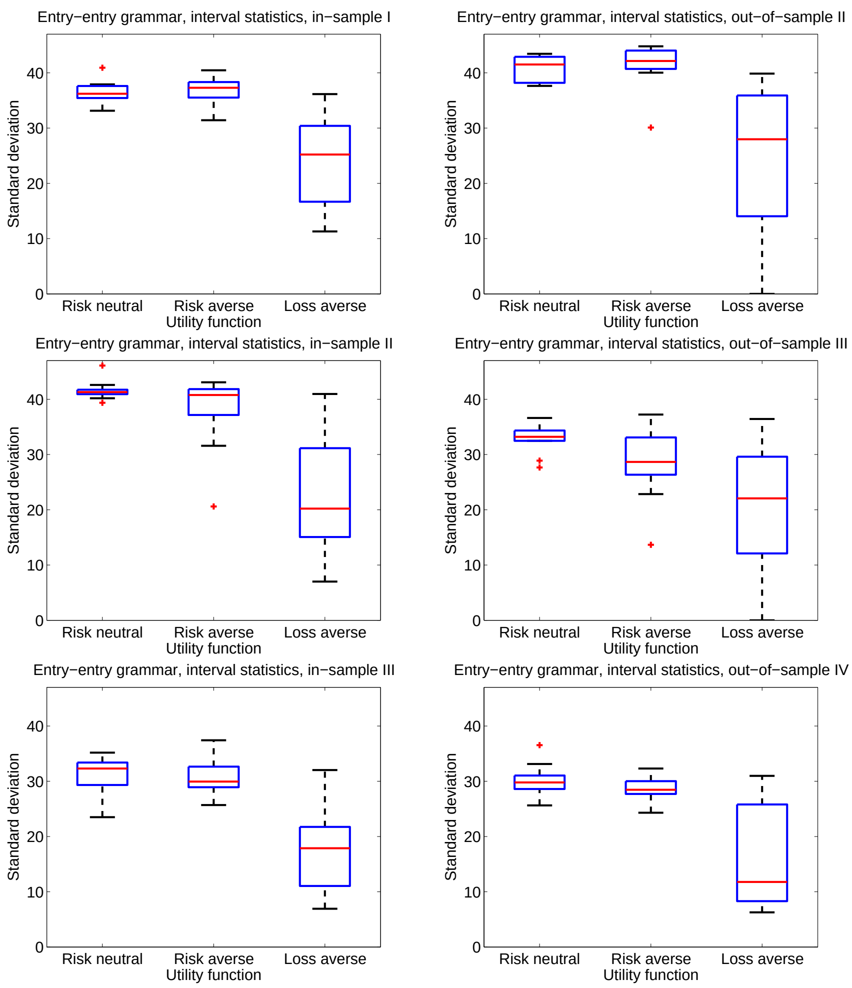


Figure 10: Boxplots of the standard deviation of interval returns across the 10 runs for each block and different utility functions.

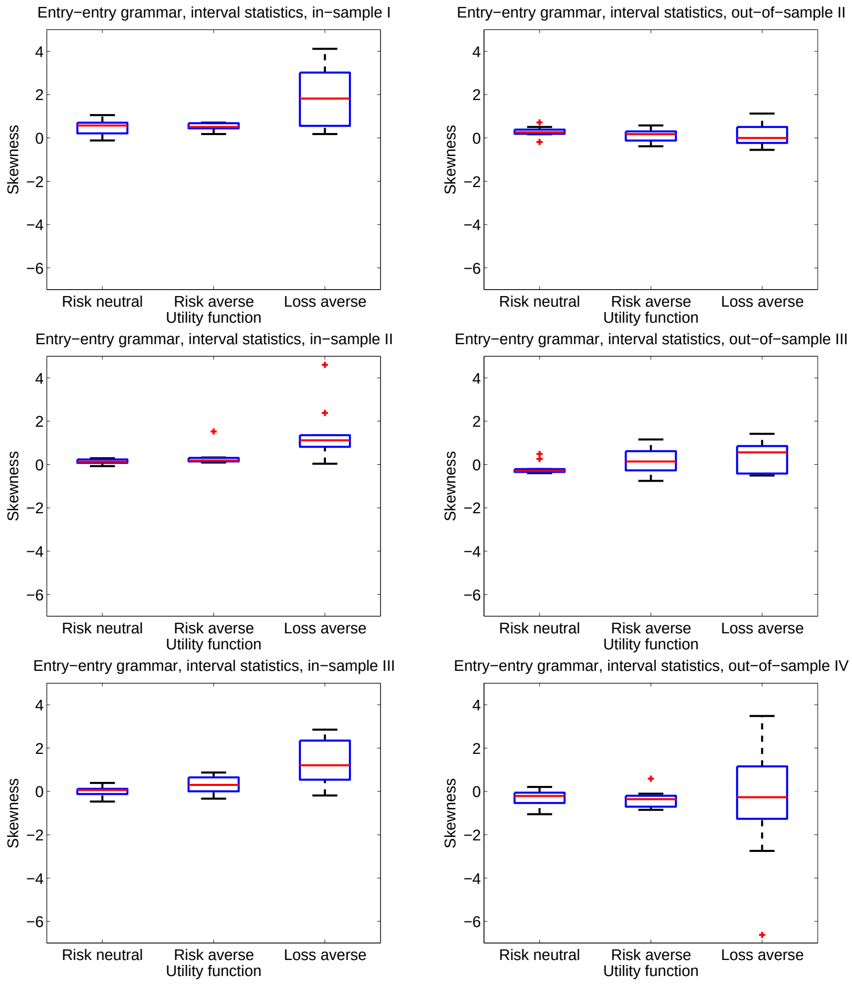


Figure 11: Boxplots of the skewness of interval returns across the 10 runs for each block and different utility functions.

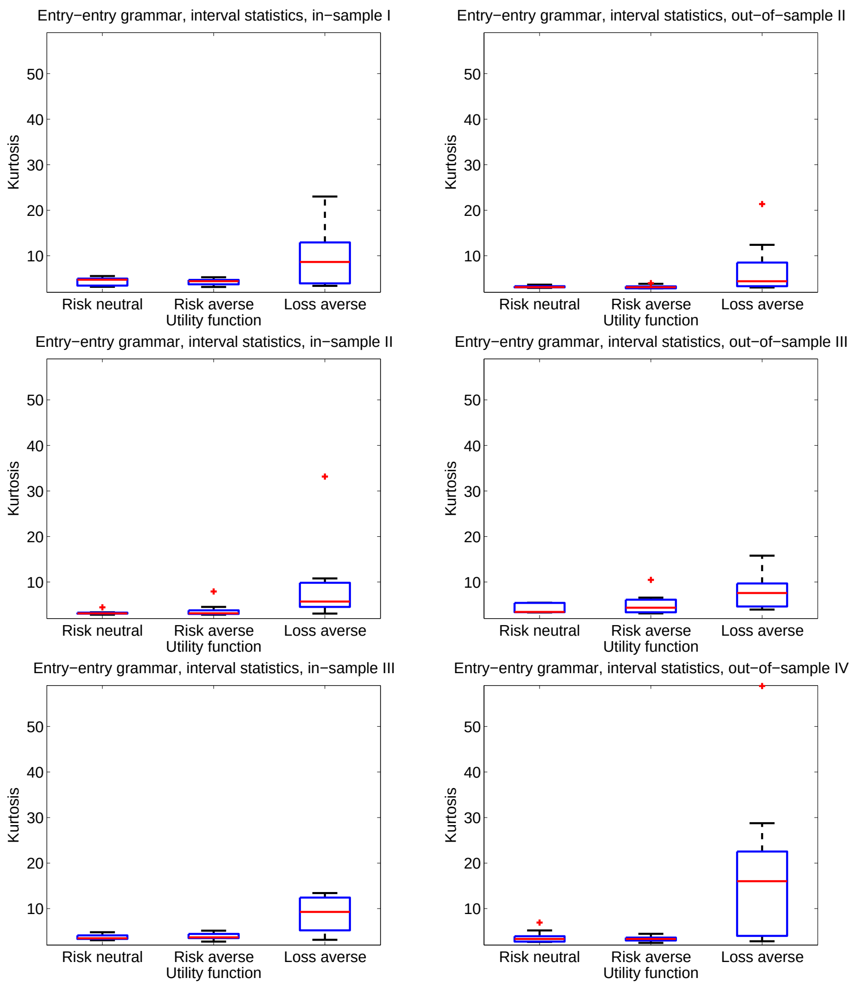


Figure 12: Boxplots of the kurtosis of interval returns across the 10 runs for each block and different utility functions.

Mean

	In-sample			Out-of-sample		
Grammar	I	II	III	II	III	IV
Entry-entry	0.2058	0.1756	0.0025	0.0013	0.6755	0.0649
Entry-exit	0.1194	0.8973	0.0038	0.0097	0.0969	0.0087

Standard deviation

	In-sample			Out-of-sample		
Grammar	I	II	III	II	III	IV
Entry-entry	0.0003	0.0004	0.0004	0.0004	0.0099	0.0013
Entry-exit	0.0001	0.0110	0.0306	0.0026	0.0363	0.1756

Skewness

	In-sample			Out-of-sample		
Grammar	I	II	III	II	III	IV
Entry-entry	0.0466	0.0019	0.0039	0.2359	0.5693	0.6941
Entry-exit	0.0373	0.4569	0.3352	0.0985	0.2838	0.2360

Kurtosis

	In-sample			Out-of-sample		
Grammar	I	II	III	II	III	IV
Entry-entry	0.0451	0.0022	0.0013	0.0087	0.0203	0.0032
Entry-exit	0.0164	0.0118	0.0965	0.0226	0.0049	0.2441

Table 5: Kruskal-Wallis test p-values for the null hypothesis of equal median moments of the interval return distributions across utility functions.

with [24], the skewness of the interval returns is generally also higher in-sample under loss aversion, but it does not seem to generalize out-of-sample. Finally, the kurtoses are significantly higher across all blocks under loss aversion.

To understand these results it is instructive to consider the trade statistics in Appendix B. Under risk neutrality and risk aversion, the strategies generally have neutral exposure less than 10% of the time, but when loss aversion is introduced it increases significantly to around 70%. Having a neutral position results in zero return. Thus, by increasing the time with neutral exposure the effect is that more zero interval returns are introduced. Naturally, this decreases the standard deviation whilst increasing the kurtosis, *ceteris paribus*.

As the random strategies in Figure 8 indicate, the required risk premium to enter a market position grows significantly under loss aversion, and as a result the number of opportunities in strategy space decreases. The interval returns under loss have higher skewness in-sample, but do not generalize out-of-sample. This suggests that loss aversion can lead to a higher degree of overfitting. However, loss aversion is not necessarily a bad thing. As mentioned previously the mean interval returns are smaller out-of-sample for Block II and larger for Block

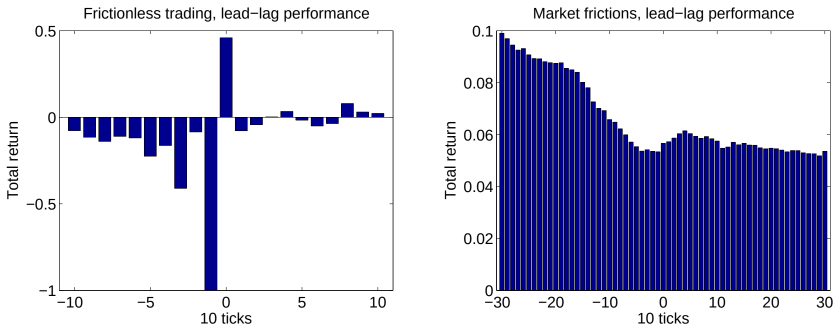


Figure 13: Out-of-sample (Block II) lead-lag performance of the aggregated entry-entry grammar strategies under risk neutrality. Frictionless trading (left) and with market frictions (right).

IV under loss aversion. The main difference between these two blocks is that during the former there is significant generalization from the in-sample results, while in the latter there is none. Consequently, if persistent patterns exist in the data then loss aversion is problematic because it limits the space for viable strategies. However, if persistent patterns have not been uncovered, then it is beneficial because it encourages conservative trading under market frictions and therefore minimizes losses.

5.2 Stability, Decay and Timing

The previous section briefly mentioned, that under frictionless trading, the evolved strategies exploit a microstructural mean-reversion effect, which presents itself as a significant negative first order autocorrelation in the USDEUR return series. Figure 13 shows the lead-lag performance during of the aggregated entry-entry grammar strategies under risk-neutrality.

The lead-lag performance provides a measure for the temporal robustness of the strategies by shifting their positions relative to the USDEUR returns and evaluating the performance. This example considers the total out-of-sample return generated in Block II. Under frictionless trading the strategies generate a substantial positive return close to 50% at lag zero, but when the positions are lagged just one period the return becomes slightly negative. If instead the positions are shifted one period forward then everything is lost. This pattern is a typical signature of mean-reverting strategies [32].

Under market frictions the situation changes. At lag zero the return is around 5.5%, but when positions are shifted forward in time the return improves which is typical for momentum strategies. For example if the dollar has depreciated against the euro recently, and a short position is taken under the expectation that the depreciation will continue, then having made this decision earlier would increase the performance. This example is important because it shows that the quad tree structure can capture two of the main effects in financial time series [15, 5].

Entry-entry grammar

Utility function	Out-of-sample II	III	IV	Aggregate
Risk neutral	2.729	5.765	-2.062	3.714
Risk averse	3.541	4.395	-3.286	2.658
Loss averse	-1.845	-1.840	1.437	-1.391
Aggregate	2.711	5.092	-2.448	

Entry-exit grammar

Utility function	Out-of-sample II	III	IV	Aggregate
Risk neutral	2.813	1.558	-2.567	1.042
Risk averse	1.413	-0.378	-1.059	-0.011
Loss averse	0.885	3.705	0.309	2.663
Aggregate	2.953	2.601	-1.912	

Table 6: t-statistics for the time coefficient in the logistic regression between the probability of a negative interval return and time.

Until this stage the out-of-sample performances of the strategies have been evaluated over the entire duration of a block. For experimental considerations the blocks in this paper have a fixed length corresponding to a quarter of the overall observations. Accepting the notion that the financial markets are complex adaptive systems, then any fixed strategy is bound to become obsolete in the limit. The question is whether that limit is reached within the chosen block size, i.e., do the performances of the strategies decay over time. Let r_i be the return within interval i, then the probability of a loss is fitted using a logit model,

$$\text{prob}(r_i < 0) = \text{logsig}(\alpha_0 + \alpha_1 \cdot i) + \epsilon_i \tag{7}$$

where ϵ_i is an error term and $\text{logsig}(x) = 1/(1+\exp(-x))$. Table 6 shows the t-statistics of the time coefficient (α_1) conditioned on utility function and out-of-sample period.

On an aggregate level the probability of loss significantly increases with time for the out-of-sample periods II and III. For Block IV the reverse is the case, which can be interpreted as poor generalization from the in-sample period. Comparing Blocks III and IV in Figure 7 the difference is striking – in Block III the price is moving sideways most of the time with a slight appreciation in the dollar, and in Block IV it depreciates substantially, but toward the end the price movement appears more similar to that in Block III. If the latter part of Block IV better resembles the in-sample period in Block III, then it is not surprising that the strategies perform less poorly over time.

Aggregating across blocks for the different utility functions, another interesting finding emerges. For loss aversion there is a significant difference between the entry-entry and entry-exit grammar strategies. Previously, the null hypothesis of equal median fitnesses between the two grammars could not be rejected, but the decay analysis seems to indicate that there is more to strategy dynamics than is captured in the fitness measure. Comparing the interval statistics in Appendix A with the trade statistics in Appendix B shows that this is indeed the case. There are plenty of examples of strategies that have similar fitness values, but they differ greatly in number of trades and proportions of long trades etc. While the decay analysis give mixed results, it does appear that strategy performance generally deteriorates over time – a finding which would be consistent with the Adaptive Markets Hypothesis [21].

6 Conclusion

This paper evolves trading strategies using genetic programming (GP) on high-frequency tick FX data, an area that has been widely neglected in the literature so far. Furthermore, this paper proposes a novel quad tree structure for trading system design to allow for a money management system where exit rules can be based on additional indicators and triggers than rules for entering positions.

In practice traders often use so-called money management that builds on a different information set when deciding on whether to exit a trade. For example, a stop loss is a measure to control downside risk and exits a position when a loss has exceeded a given threshold. This paper investigates the potential use of money management by comparing strategies composed of two different grammars. The first is an entry-entry grammar, where the information set is the same for both entry and exit trees. The second is an entry-exit grammar that has a larger information set including variables such as current profit, drawdown and duration of a trade. Evolving money management as an endogenous feature has not previously been attempted in the literature.

The quad tree architecture consists of four trees each solving a separate task, but mutually dependent for overall performance. Specifically, the functions of the trees are; long entry, long exit, short entry and short exit. Thus, evaluation is contingent on the current market position. For example if the current position is neutral it is possible to go either long or short, but if the current position is long, then the long exit and short entry are evaluated. Making this distinction provides a more accurate description of the decision problem facing real traders.

The trading strategies are evolved using a fitness measure based on the power utility function, where three different kinds of behavior are investigated: risk neutral, risk averse and loss averse. Evolution is done with and without accounting for transaction costs. In a frictionless environment the strategies exploit the significant mean reverting properties of the returns series. This is a dominating strategy regardless of the utility function, and it proves that the framework is capable of capturing a well-known regularity.

The empirical investigation uses the USDEUR exchange rate covering the calendar year 2006, sampled at 10 tick intervals. Under market frictions and loss aversion, the strategies spend considerably more time in a neutral position since there are fewer satisfying opportunities. The downside is that generalization can suffer as a result. However, it cannot be concluded that loss aversion always has an adverse effect on performance; it depends on the quality of the patterns discovered in-sample. If the strategies have overfitted the data, or the patterns cease to exist out-of-sample, then loss aversion is beneficial because it promotes cautious trading that limits transaction costs.

When comparing the entry-entry and entry-exit grammar strategies, the null hypothesis of identical median performance is not rejected, neither in-sample nor out-of-sample. Hence, the results are not significantly different. This suggests that money management has a detrimental effect on utility, and raises the question as to why it is extensively used by practitioners. A possible explanation is that stop orders and limit orders, in the foreign exchange market, tend to be clustered around round numbers, thus giving rise to distinct support and resistance levels where trend reversals are more likely to occur [30]. This has not been taken into account in this chapter, but could be an interesting avenue for future research. The second part of the chapter examines whether the out-of-sample performance decays over time. The results are mixed but it does often seem to be the case – a finding which is inline with the Adaptive Markets Hypothesis [21]. Moreover, the response from the entry-entry and entry-exit grammar strategies differs, which suggests that there is more to strategy dynamics than is captured in a single fitness measure. Treating strategy evolution as a multi-objective problem is a possibility.

It is only in one out-of-sample period that there is significantly positive performance, and it can therefore be concluded that the high-frequency foreign exchange market is very efficient.

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A Interval Statistics

A.1 Entry-Entry Grammar, Frictionless Trading

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	60.37	0.000	60.70	55.72	0.24	4.14	37.79	0.000	38.04	59.57	-0.22	3.28
2.	64.29	0.000	64.64	53.64	0.19	4.00	40.26	0.000	40.51	60.06	0.01	3.01
3.	60.33	0.000	60.66	54.76	0.19	4.13	39.41	0.000	39.67	60.43	-0.18	3.26
4.	61.29	0.000	61.64	57.06	0.27	4.22	38.40	0.000	38.65	60.47	-0.19	3.27
5.	60.38	0.000	60.70	54.22	0.22	4.15	39.26	0.000	39.52	60.35	-0.19	3.26
6.	58.67	0.000	58.98	53.32	0.07	3.74	35.99	0.000	36.23	59.50	-0.26	3.20
7.	60.76	0.000	61.09	54.01	0.26	4.16	38.76	0.000	39.02	60.95	-0.23	3.47
8.	60.50	0.000	60.83	54.69	0.20	4.14	39.09	0.000	39.34	60.26	-0.17	3.29
9.	60.36	0.000	60.70	56.84	0.28	4.14	39.02	0.000	39.27	59.88	-0.12	3.33
10.	60.73	0.000	61.07	56.23	0.25	4.36	40.14	0.000	40.40	60.87	0.07	3.49
Avg	60.44	0.000	60.77	54.73	0.23	4.14	39.05	0.000	39.31	60.30	-0.18	3.27

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	40.44	0.000	40.68	57.86	0.40	3.21	32.72	0.001	32.87	45.21	0.06	2.58
2.	39.87	0.000	40.13	59.57	-0.04	3.07	42.27	0.000	42.46	43.21	-0.31	3.29
3.	46.74	0.000	46.98	52.28	0.59	3.68	31.84	0.001	31.97	39.23	-0.15	2.39
4.	44.20	0.000	44.48	61.17	-0.27	3.23	38.76	0.000	38.92	42.92	-0.40	2.77
5.	39.02	0.000	39.26	58.20	-0.09	3.19	44.83	0.000	45.03	43.52	-0.37	3.19
6.	39.07	0.000	39.32	59.34	-0.09	3.38	45.85	0.000	46.06	45.43	-0.34	3.17
7.	39.32	0.000	39.56	58.59	-0.18	3.19	44.90	0.000	45.09	44.09	-0.32	3.29
8.	41.17	0.000	41.44	60.23	-0.11	2.93	42.35	0.000	42.53	42.45	-0.35	3.16
9.	35.13	0.000	35.32	51.31	-0.05	3.10	35.31	0.001	35.47	44.36	0.15	2.53
10.	39.44	0.000	39.68	58.26	-0.07	3.43	46.08	0.000	46.28	43.43	-0.25	2.84
Avg	39.66	0.000	39.91	58.42	-0.08	3.20	42.31	0.000	42.49	43.47	-0.31	3.00

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	48.57	0.000	48.79	44.61	-0.19	3.28	40.46	0.000	40.63	42.96	-0.54	4.07
2.	47.45	0.000	47.66	44.63	-0.31	3.18	40.74	0.000	40.92	43.58	-0.55	4.21
3.	46.27	0.000	46.47	44.98	-0.37	2.87	34.15	0.000	34.29	40.76	0.07	3.53
4.	48.05	0.000	48.26	44.25	-0.34	3.11	39.91	0.000	40.09	43.16	-0.56	4.33
5.	48.06	0.000	48.27	43.93	-0.26	3.19	40.77	0.000	40.95	43.70	-0.49	4.05
6.	48.03	0.000	48.24	43.50	-0.31	3.09	40.19	0.000	40.36	42.24	-0.33	3.81
7.	48.36	0.000	48.58	44.43	-0.26	3.13	40.02	0.000	40.19	42.49	-0.49	4.16
8.	47.99	0.000	48.21	44.66	-0.29	3.16	40.67	0.000	40.85	43.15	-0.56	4.11
9.	47.52	0.000	47.73	44.65	-0.28	3.19	40.55	0.000	40.73	43.68	-0.56	4.23
10.	48.39	0.000	48.61	44.48	-0.27	2.98	40.41	0.000	40.59	43.73	-0.64	4.24
Avg	48.04	0.000	48.25	44.55	-0.29	3.15	40.44	0.000	40.61	43.16	-0.55	4.14

Table 7: Interval statistics of entry-entry grammar strategies under frictionless trading and risk neutrality.

A.2 Entry-Entry Grammar, Market Frictions

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	7.16	0.000	7.22	36.04	0.70	4.98	5.09	0.002	5.16	37.64	0.37	3.42
2.	4.79	0.000	4.86	37.91	0.55	4.71	4.75	0.009	4.84	43.44	0.17	2.95
3.	7.66	0.000	7.72	35.61	1.06	5.11	4.28	0.009	4.36	40.93	0.71	3.08
4.	4.60	0.000	4.69	40.92	-0.12	3.19	2.02	0.013	2.11	41.86	0.25	3.06
5.	9.74	0.000	9.80	33.54	0.21	3.46	3.25	0.011	3.33	38.19	-0.19	3.14
6.	4.74	0.000	4.81	37.55	0.63	4.78	5.52	0.000	5.61	42.03	0.27	3.05
7.	9.36	0.000	9.43	36.40	0.70	4.83	3.45	0.011	3.55	43.11	0.24	3.09
8.	13.84	0.000	13.90	33.13	0.17	3.34	3.00	0.011	3.07	37.87	0.51	3.66
9.	8.01	0.000	8.08	35.44	0.57	5.54	9.54	0.000	9.63	41.17	0.19	3.32
10.	7.90	0.000	7.97	37.61	0.58	4.70	5.18	0.002	5.27	42.90	0.39	3.27
Med	7.78	0.000	7.85	36.22	0.58	4.74	4.51	0.009	4.60	41.51	0.26	3.12

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	11.43	0.000	11.52	41.11	0.08	3.06	1.05	0.011	1.11	36.61	0.49	5.41
2.	7.08	0.000	7.17	41.47	0.12	3.25	0.03	0.016	0.08	32.73	-0.28	3.39
3.	10.72	0.000	10.81	41.09	0.12	3.10	-0.23	0.051	-0.18	32.89	-0.26	3.32
4.	9.87	0.000	9.95	39.36	-0.01	4.45	-1.98	0.065	-1.93	32.47	-0.29	3.40
5.	6.62	0.000	6.71	42.59	0.18	3.11	-1.36	0.053	-1.31	34.23	-0.30	3.38
6.	6.32	0.000	6.41	41.72	0.24	2.83	-3.12	0.068	-3.06	35.27	-0.35	3.38
7.	8.34	0.000	8.42	40.18	0.24	3.27	-1.98	0.065	-1.94	28.89	-0.21	5.14
8.	11.27	0.000	11.36	40.92	-0.07	3.36	3.96	0.000	4.02	33.51	-0.34	3.48
9.	7.34	0.000	7.45	46.10	0.29	3.02	-4.02	0.069	-3.97	34.34	0.26	5.47
10.	11.06	0.000	11.15	41.48	0.14	3.11	-2.11	0.066	-2.08	27.65	-0.40	5.43
Med	9.10	0.000	9.19	41.29	0.13	3.11	-1.67	0.059	-1.62	33.20	-0.29	3.44

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	4.89	0.000	4.95	33.05	0.08	3.06	-5.02	0.054	-4.97	31.04	-0.06	2.76
2.	7.92	0.000	7.96	29.33	0.39	4.40	-4.45	0.054	-4.41	28.60	-0.23	3.95
3.	5.04	0.000	5.07	23.50	-0.47	4.12	-4.01	0.052	-3.97	26.79	-0.53	3.32
4.	10.81	0.000	10.86	32.10	0.12	3.38	-6.40	0.069	-6.35	29.50	-0.20	2.74
5.	4.54	0.000	4.59	30.53	-0.12	3.54	0.74	0.018	0.79	30.24	-0.22	3.60
6.	8.85	0.000	8.91	32.51	0.38	3.44	-3.34	0.051	-3.30	28.85	0.21	3.36
7.	11.93	0.000	12.00	35.17	0.05	3.33	-1.36	0.051	-1.29	36.53	-1.05	6.93
8.	7.96	0.000	8.01	33.38	0.08	3.89	-5.44	0.067	-5.39	30.07	0.05	2.68
9.	5.64	0.000	5.70	34.71	0.05	3.17	-0.85	0.049	-0.80	33.12	-0.09	2.80
10.	5.46	0.000	5.50	29.05	-0.38	4.81	0.59	0.018	0.62	25.64	-0.75	5.21
Med	6.78	0.000	6.83	32.30	0.06	3.49	-3.67	0.051	-3.63	29.79	-0.21	3.34

Table 8: Interval statistics of entry-entry grammar strategies under market frictions and risk neutrality.

Risk Aversion

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	11.19	0.000	13.81	37.62	0.49	4.32	2.54	0.000	5.29	40.02	0.23	3.28
2.	6.17	0.000	8.56	36.96	0.49	5.28	1.82	0.008	5.16	44.04	0.17	2.83
3.	6.06	0.000	8.25	35.52	0.68	5.10	0.37	0.011	3.60	42.76	-0.38	2.85
4.	3.71	0.000	5.41	31.42	0.58	3.53	-3.05	0.046	-1.50	30.09	0.58	3.85
5.	2.38	0.000	4.83	38.03	0.69	4.44	1.89	0.008	5.01	42.68	0.30	3.04
6.	2.89	0.000	5.72	40.46	0.18	4.66	1.52	0.008	4.85	44.03	0.18	2.94
7.	6.90	0.000	8.78	32.39	0.24	3.15	-2.48	0.044	1.03	44.80	-0.12	4.00
8.	5.79	0.000	8.34	38.32	0.44	4.41	1.98	0.000	4.86	40.71	0.01	2.86
9.	8.22	0.000	10.50	35.87	0.71	4.70	3.34	0.000	6.31	41.61	0.33	3.16
10.	7.08	0.000	9.92	40.32	0.53	3.74	-7.97	0.051	-4.94	40.71	-0.30	3.17
Med	6.11	0.000	8.45	37.29	0.51	4.42	1.67	0.008	4.85	42.14	0.17	3.10

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	7.96	0.000	11.01	41.38	0.14	3.13	-3.62	0.047	-2.34	26.58	-0.76	5.14
2.	8.57	0.000	11.46	40.15	0.19	2.84	-4.06	0.055	-2.79	26.35	-0.75	5.23
3.	8.63	0.000	11.83	42.29	0.16	3.01	-0.90	0.044	1.43	37.24	0.72	6.58
4.	7.98	0.000	11.04	41.39	0.14	3.13	-2.27	0.045	-0.35	33.10	-0.26	3.27
5.	2.54	0.000	4.28	31.57	0.09	4.54	-1.21	0.045	-0.31	22.83	0.23	3.59
6.	4.86	0.000	5.61	20.60	1.53	7.94	0.47	0.001	0.79	13.66	1.16	10.48
7.	3.95	0.000	6.57	38.92	0.30	3.63	-0.20	0.028	1.32	29.69	0.11	3.12
8.	4.87	0.000	7.27	37.15	0.32	3.80	-0.84	0.044	0.47	27.62	0.16	3.35
9.	2.45	0.000	5.64	43.07	0.18	3.03	-1.91	0.045	-0.03	32.78	-0.27	3.36
10.	10.18	0.000	13.37	41.84	0.19	2.85	-6.02	0.065	-3.68	36.69	0.62	6.13
Med	6.41	0.000	9.14	40.77	0.18	3.13	-1.56	0.045	-0.17	28.65	0.14	4.36

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	5.32	0.000	6.91	30.19	0.65	3.68	-5.38	0.058	-3.96	28.39	0.59	3.41
2.	10.60	0.000	12.23	28.92	0.33	2.76	-4.56	0.056	-3.16	27.71	-0.70	3.49
3.	3.34	0.000	5.78	37.42	0.00	3.52	-3.93	0.055	-2.05	32.30	-0.62	3.08
4.	6.19	0.000	8.09	32.64	0.22	3.44	-8.62	0.080	-7.07	28.52	-0.20	3.00
5.	5.17	0.000	6.76	29.71	-0.34	5.15	-4.91	0.056	-3.56	27.51	-0.10	2.48
6.	8.43	0.000	9.69	25.70	0.39	4.67	-1.21	0.048	0.37	30.04	-0.37	3.65
7.	11.82	0.000	14.28	36.32	0.87	4.44	-3.50	0.054	-2.10	28.04	-0.34	3.25
8.	10.86	0.000	12.87	32.07	-0.25	3.66	-1.81	0.052	-0.19	30.02	-0.81	3.76
9.	5.71	0.000	7.22	29.08	0.27	3.73	-7.15	0.076	-5.54	29.50	-0.23	3.00
10.	8.59	0.000	9.94	26.98	0.87	4.26	-1.17	0.048	-0.12	24.30	-0.85	4.47
Med	7.31	0.000	8.89	29.95	0.30	3.70	-4.24	0.056	-2.63	28.46	-0.36	3.33

Table 9: Interval statistics of entry-entry grammar strategies under market frictions and risk aversion.

Loss Aversion

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	5.17	0.000	7.74	27.89	0.18	3.39	-4.33	0.055	0.15	34.84	0.13	3.19
2.	4.38	0.000	5.04	16.67	2.55	10.10	-0.78	0.047	-0.41	8.83	1.13	21.36
3.	4.02	0.000	6.63	30.40	0.73	5.54	-10.30	0.060	-5.05	35.90	-0.55	4.11
4.	7.17	0.000	8.39	23.20	2.15	7.71	-3.37	0.052	-0.42	28.23	-0.28	5.37
5.	3.87	0.000	4.12	11.31	3.15	12.93	0.53	0.001	1.22	14.04	1.07	12.40
6.	2.46	0.000	2.82	13.04	4.11	23.01	0.00	0.031	0.00	0.00	NaN	NaN
7.	4.65	0.000	6.59	27.21	1.50	9.58	-0.97	0.049	1.70	27.73	0.32	4.41
8.	4.09	0.000	7.83	36.16	0.38	3.94	-7.66	0.058	-1.77	39.86	-0.03	3.04
9.	4.06	0.000	4.84	19.02	3.02	16.06	-3.47	0.054	-1.55	21.73	-0.21	7.20
10.	8.90	0.000	11.56	30.96	0.56	3.82	-3.37	0.052	1.88	39.61	-0.00	3.37
Med	4.24	0.000	6.61	25.21	1.82	8.65	-3.37	0.052	-0.20	27.98	-0.00	4.41

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	3.78	0.000	4.90	19.10	1.12	7.32	0.40	0.000	0.89	10.93	0.82	15.79
2.	0.75	0.001	0.88	7.00	4.60	33.16	0.00	0.033	0.00	0.00	NaN	NaN
3.	1.58	0.000	2.11	12.95	1.35	9.83	-3.61	0.054	-1.86	20.66	-0.37	7.59
4.	3.70	0.000	6.44	28.84	0.72	5.82	-3.13	0.053	0.37	29.83	-0.51	3.96
5.	2.59	0.000	3.32	15.07	0.92	5.63	-0.60	0.034	0.04	12.11	0.56	11.51
6.	6.94	0.000	8.21	21.33	1.26	5.47	0.49	0.000	1.91	21.32	1.42	8.85
7.	3.67	0.000	4.28	15.38	2.38	10.80	-2.85	0.051	-0.70	24.05	-0.39	9.08
8.	9.53	0.000	12.31	32.31	1.12	4.52	-3.84	0.054	-1.62	22.81	0.95	4.59
9.	6.14	0.000	11.21	40.95	0.04	3.08	-9.06	0.077	-4.04	36.42	0.70	6.85
10.	4.42	0.000	7.48	31.15	0.82	4.53	-3.55	0.053	-0.07	29.61	-0.51	4.63
Med	3.74	0.000	5.67	20.22	1.12	5.72	-2.99	0.052	-0.04	22.07	0.56	7.59

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	4.59	0.000	5.21	14.44	1.20	5.64	-1.19	0.044	1.42	26.13	-0.04	3.70
2.	2.10	0.000	2.49	11.05	2.35	11.73	-0.11	0.040	0.22	8.30	3.48	28.76
3.	1.25	0.000	1.66	10.50	-0.19	12.43	-0.73	0.044	-0.43	7.81	-6.62	58.87
4.	2.46	0.000	3.13	16.69	2.67	13.42	-8.36	0.049	-5.10	25.80	-0.35	4.02
5.	3.92	0.000	5.34	20.78	0.69	5.23	-0.66	0.044	-0.05	11.85	-2.75	22.54
6.	3.12	0.000	5.00	25.28	0.54	5.09	-0.14	0.041	0.32	11.10	1.16	19.12
7.	1.12	0.000	2.34	19.06	1.21	8.98	0.44	0.015	0.96	11.69	-1.26	20.11
8.	9.04	0.000	12.24	32.03	0.19	3.15	-7.81	0.049	-3.60	30.98	-0.18	2.84
9.	3.73	0.000	5.08	21.74	2.05	9.57	-1.26	0.044	-0.30	14.32	-0.44	7.39
10.	1.88	0.000	2.01	6.93	2.85	12.77	0.85	0.015	1.01	6.29	2.19	12.93
Med	2.79	0.000	4.07	17.87	1.21	9.28	-0.69	0.044	0.09	11.77	-0.27	16.03

Table 10: Interval statistics of entry-entry grammar strategies under market frictions and loss aversion.

A.3 Entry-Exit Grammar, Market Frictions

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	7.96	0.000	8.03	38.06	0.41	4.93	4.05	0.016	4.15	44.17	0.18	2.91
2.	7.10	0.000	7.17	36.01	0.99	4.62	3.12	0.022	3.17	31.79	1.01	5.51
3.	9.75	0.000	9.82	36.83	0.63	4.57	1.97	0.025	2.06	41.78	0.30	3.17
4.	7.49	0.000	7.56	37.73	0.21	4.01	4.06	0.016	4.15	41.69	-0.08	2.88
5.	8.78	0.000	8.87	40.85	0.21	5.85	6.13	0.000	6.22	41.61	0.24	2.93
6.	5.85	0.000	5.93	38.61	0.36	4.74	4.68	0.013	4.78	43.84	0.14	2.97
7.	8.17	0.000	8.24	38.08	0.47	5.13	4.72	0.012	4.83	47.29	-0.10	3.46
8.	11.10	0.000	11.16	33.35	0.21	2.89	-0.20	0.064	-0.15	32.16	0.38	3.07
9.	9.51	0.000	9.57	35.29	0.77	4.94	4.91	0.011	5.00	41.63	0.22	3.25
10.	7.99	0.000	8.06	36.37	0.77	5.23	5.11	0.003	5.20	42.76	0.20	3.08
Med	8.08	0.000	8.15	37.28	0.44	4.84	4.37	0.014	4.46	41.73	0.21	3.08

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	6.19	0.000	6.26	37.96	0.48	3.32	-4.05	0.076	-4.00	31.39	0.18	2.59
2.	8.94	0.000	9.04	42.87	-0.29	3.16	-1.94	0.058	-1.88	34.30	-0.34	3.16
3.	7.52	0.000	7.58	33.83	0.16	3.45	-1.60	0.046	-1.57	25.94	-0.04	3.61
4.	8.39	0.000	8.45	36.50	0.38	3.79	-2.07	0.059	-2.03	28.12	0.20	3.01
5.	7.31	0.000	7.38	36.46	0.74	3.99	-1.19	0.044	-1.14	30.96	0.44	2.91
6.	7.35	0.000	7.44	41.58	-0.05	2.98	-1.27	0.044	-1.21	34.62	-0.32	3.13
7.	10.80	0.000	10.87	35.39	0.39	3.98	1.47	0.007	1.50	26.30	-0.22	4.64
8.	8.93	0.000	9.00	36.21	0.09	3.74	0.39	0.014	0.43	29.23	-0.24	2.89
9.	6.29	0.000	6.39	42.49	0.09	3.13	-1.51	0.045	-1.46	32.76	-0.17	2.85
10.	12.01	0.000	12.11	41.73	0.10	3.06	-3.66	0.075	-3.59	36.79	0.64	6.24
Med	7.95	0.000	8.02	37.23	0.13	3.38	-1.56	0.045	-1.51	31.18	-0.11	3.07

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	6.25	0.000	6.30	31.21	0.47	3.57	-9.26	0.109	-9.20	34.22	-2.15	15.26
2.	6.25	0.000	6.31	32.15	-0.12	2.72	-4.96	0.079	-4.91	29.79	-0.24	2.83
3.	4.64	0.000	4.69	30.39	0.28	3.53	-5.26	0.085	-5.21	31.07	0.21	3.47
4.	6.84	0.000	6.89	31.49	0.31	3.61	-6.54	0.095	-6.51	24.37	-0.14	3.68
5.	8.83	0.000	8.87	27.66	-0.59	3.54	-5.57	0.090	-5.53	28.63	-0.22	2.66
6.	4.74	0.000	4.79	31.61	0.21	3.23	-5.48	0.090	-5.43	29.59	0.34	3.29
7.	4.75	0.000	4.78	23.93	0.53	5.24	-4.93	0.078	-4.88	31.14	0.18	3.43
8.	4.67	0.000	4.71	28.11	0.88	4.38	-2.16	0.070	-2.14	21.90	0.65	3.20
9.	5.60	0.000	5.65	31.46	0.39	3.67	-4.97	0.079	-4.93	25.63	0.13	2.78
10.	5.66	0.000	5.69	22.99	0.98	7.14	-5.22	0.083	-5.17	29.84	0.22	3.76
Med	5.63	0.000	5.67	30.80	0.35	3.59	-5.24	0.084	-5.19	29.69	0.16	3.36

Table 11: Interval statistics of entry-exit grammar strategies under market frictions and risk neutrality.

Risk Aversion

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	5.52	0.000	7.96	37.10	-0.06	3.63	-8.87	0.091	-5.56	42.50	-0.29	3.35
2.	10.49	0.000	13.22	38.69	0.46	3.78	0.86	0.027	3.80	40.80	-0.33	3.48
3.	3.99	0.000	6.36	36.98	0.34	4.57	1.35	0.023	4.86	45.04	0.03	3.56
4.	7.57	0.000	9.75	35.21	0.90	4.95	-0.10	0.053	2.02	35.23	0.36	5.16
5.	3.45	0.000	6.02	38.82	0.56	4.33	-10.87	0.105	-7.42	43.35	0.04	2.91
6.	3.66	0.000	5.93	36.65	0.83	4.50	-2.51	0.061	-0.26	36.31	0.48	4.84
7.	3.72	0.000	5.97	36.36	0.72	4.69	2.37	0.003	4.78	37.62	0.50	3.50
8.	5.73	0.000	7.34	30.65	1.31	5.93	-4.36	0.063	-2.05	36.70	0.63	5.21
9.	5.68	0.000	7.96	36.50	0.84	3.65	-4.78	0.064	-2.70	34.37	0.09	2.91
10.	7.10	0.000	9.32	35.43	0.40	3.69	-0.57	0.056	2.14	39.50	-0.06	3.20
Med	5.60	0.000	7.65	36.58	0.64	4.41	-1.54	0.058	0.88	38.56	0.06	3.49

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	8.01	0.000	10.76	39.08	0.13	3.95	-0.91	0.056	0.63	29.80	0.14	3.26
2.	5.08	0.000	7.02	32.74	-0.52	3.43	-4.67	0.102	-2.87	31.62	-0.45	4.06
3.	5.19	0.000	7.59	37.29	0.60	3.39	-5.90	0.109	-4.14	31.41	0.15	3.10
4.	6.22	0.000	8.35	34.45	-0.05	4.09	-3.64	0.076	-1.54	34.38	-0.35	3.15
5.	4.44	0.000	5.32	22.35	1.04	8.71	1.69	0.000	1.92	11.53	2.29	13.51
6.	8.29	0.000	9.52	25.56	0.67	5.75	-0.97	0.057	-0.30	19.80	0.65	7.49
7.	3.58	0.000	6.82	43.13	0.04	2.85	-4.00	0.089	-1.94	33.95	-0.38	3.23
8.	7.17	0.000	9.00	31.84	0.40	4.09	-1.38	0.061	0.10	28.87	-0.78	4.80
9.	7.02	0.000	8.25	25.72	-0.03	3.87	-3.24	0.071	-1.86	27.72	-0.65	5.70
10.	10.94	0.000	12.35	27.02	1.84	7.50	-2.14	0.067	-1.15	23.66	-0.36	7.86
Med	6.62	0.000	8.30	32.29	0.27	4.02	-2.69	0.069	-1.34	29.33	-0.36	4.43

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	8.30	0.000	9.50	25.43	1.10	6.52	-3.01	0.067	-2.22	21.16	-0.04	6.64
2.	9.00	0.000	10.77	30.41	-0.44	3.69	0.57	0.032	2.04	29.09	0.05	4.14
3.	4.55	0.000	6.84	35.98	-0.15	2.75	-4.00	0.070	-2.07	32.70	-0.60	2.96
4.	8.48	0.000	10.31	31.46	0.17	4.34	-3.34	0.068	-1.46	32.49	-0.46	3.84
5.	5.87	0.000	8.70	39.87	-0.18	2.70	2.03	0.026	3.53	29.07	-0.60	3.69
6.	4.82	0.000	6.81	34.23	0.97	4.75	-6.40	0.076	-4.73	30.56	0.36	3.40
7.	5.48	0.000	7.00	29.46	0.66	3.81	-5.78	0.075	-4.20	29.80	0.30	2.98
8.	5.84	0.000	6.45	17.97	0.23	8.76	-6.41	0.076	-5.13	26.29	-0.47	3.78
9.	5.06	0.000	5.90	21.78	1.05	7.63	-5.15	0.073	-3.81	27.15	-0.41	3.58
10.	8.16	0.000	9.53	26.77	-0.32	2.76	-9.29	0.104	-7.28	32.50	-0.42	3.05
Med	5.85	0.000	7.85	29.94	0.20	4.07	-4.57	0.072	-3.01	29.45	-0.41	3.64

Table 12: Interval statistics of entry-exit grammar strategies under market frictions and risk aversion.

Loss Aversion

	In-sample I						Out-of-sample II					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	5.01	0.000	6.87	28.29	2.21	12.55	-3.99	0.076	1.72	41.13	0.03	3.34
2.	7.56	0.000	8.57	20.16	2.66	14.91	0.29	0.002	1.99	21.41	0.48	7.40
3.	1.77	0.000	2.04	11.09	4.22	30.31	0.00	0.038	0.00	0.00	NaN	NaN
4.	6.81	0.000	8.56	25.44	0.97	4.54	-10.66	0.090	-4.13	41.46	-0.06	3.05
5.	3.13	0.000	3.50	12.51	3.05	17.91	-3.91	0.076	-1.52	24.67	-1.29	9.98
6.	7.42	0.000	10.22	30.43	0.38	3.23	-6.25	0.080	-1.60	35.01	-0.45	4.26
7.	4.13	0.000	6.32	25.82	-0.57	5.74	0.36	0.002	3.73	32.43	0.09	5.42
8.	4.71	0.000	5.74	19.71	1.53	6.46	1.35	0.000	4.74	32.16	0.02	3.64
9.	3.65	0.000	4.16	14.32	2.49	9.98	-2.17	0.063	-0.69	20.17	-0.67	16.62
10.	5.84	0.000	7.00	19.80	0.70	4.75	-0.61	0.044	1.74	24.69	0.13	3.27
Med	4.86	0.000	6.60	19.98	1.87	8.22	-1.39	0.053	0.86	28.43	0.02	4.26

	In-sample II						Out-of-sample III					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	6.16	0.000	9.51	33.59	0.23	5.63	-0.13	0.032	1.42	20.18	-1.05	10.12
2.	3.14	0.000	7.42	37.07	0.06	3.59	-3.50	0.057	-0.05	29.79	-0.45	4.16
3.	6.98	0.000	8.83	25.98	1.60	10.93	-6.78	0.091	-2.14	34.08	-0.33	3.22
4.	5.74	0.000	7.68	25.96	0.29	5.30	-4.70	0.064	-1.38	28.42	-0.58	4.40
5.	3.25	0.000	4.05	17.54	1.80	8.19	-0.20	0.032	0.21	9.47	0.16	15.98
6.	3.81	0.000	7.63	34.93	-0.08	4.07	-1.85	0.035	1.06	27.77	-0.31	4.23
7.	6.47	0.000	8.07	25.52	1.52	5.77	-0.84	0.033	1.38	25.28	0.08	5.07
8.	1.48	0.000	2.12	13.98	1.04	7.58	-0.79	0.033	-0.02	13.33	0.55	9.74
9.	6.78	0.000	11.88	41.46	0.14	3.06	-1.11	0.033	-0.38	15.32	-0.63	39.94
10.	6.35	0.000	8.96	29.15	0.25	4.99	0.21	0.001	1.84	21.65	0.28	5.60
Med	5.95	0.000	7.87	27.57	0.27	5.46	-0.98	0.033	0.09	23.46	-0.32	5.34

	In-sample III						Out-of-sample IV					
Run	F	p	Mean	Std	Skew	Kurt	F	p	Mean	Std	Skew	Kurt
1.	3.70	0.000	4.74	17.97	0.38	5.23	-0.41	0.059	2.37	27.12	-0.38	4.17
2.	2.68	0.000	2.92	9.59	0.94	8.72	1.85	0.000	2.38	13.49	1.14	15.72
3.	5.97	0.000	6.93	16.14	0.48	3.92	-4.70	0.086	-2.42	20.94	-0.45	3.76
4.	3.33	0.000	5.63	25.78	0.64	5.28	-9.16	0.102	-5.47	27.05	-0.15	2.87
5.	3.20	0.000	3.80	14.14	1.15	9.06	0.87	0.016	2.33	19.30	-0.05	4.60
6.	2.89	0.000	4.25	22.20	1.47	8.61	-2.96	0.080	-0.86	21.89	-0.04	4.34
7.	5.03	0.000	8.18	31.75	0.26	3.32	-5.66	0.088	-1.48	32.58	0.18	3.23
8.	3.68	0.000	6.97	31.75	0.25	4.06	-9.03	0.101	-4.84	30.50	0.23	3.32
9.	2.73	0.000	4.44	20.93	0.15	4.36	-9.52	0.113	-5.18	31.07	0.21	3.47
10.	4.86	0.000	5.80	17.60	0.82	5.00	-1.61	0.069	0.75	23.91	-0.23	3.64
Med	3.50	0.000	5.19	19.45	0.56	5.12	-3.83	0.083	-1.17	25.48	-0.04	3.70

Table 13: Interval statistics of entry-exit grammar strategies under market frictions and loss aversion.

B Trade Statistics

Abbreviations used in the following tables.

NT	total number of trades
LSR	ratio of long and total number of trades
NR	proportion of time in neutral position
PP	percentage of profitable trades
AT	average return of trades
MDD	maximum drawdown

B.1 Entry-Entry Grammar, Frictionless Trading

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	38992	51.49	2.82	71.76	0.15	165	45207	51.52	3.15	68.51	0.08	281
2.	43895	47.63	4.86	71.62	0.14	131	49665	47.65	5.44	66.67	0.08	323
3.	40125	52.83	6.51	72.31	0.15	162	46209	52.50	5.66	68.03	0.08	300
4.	38795	51.24	2.31	72.02	0.15	168	44904	51.19	2.40	68.82	0.08	320
5.	40207	52.73	6.70	72.24	0.15	162	46231	52.48	5.71	67.98	0.08	311
6.	40333	53.10	7.20	71.76	0.14	154	46448	52.81	6.45	67.54	0.07	345
7.	40241	52.68	6.81	72.22	0.15	162	46255	52.45	5.80	67.95	0.08	331
8.	40195	52.74	6.68	72.28	0.15	164	46254	52.45	5.80	67.98	0.08	317
9.	38855	51.30	2.49	72.04	0.15	166	45067	51.29	2.72	68.68	0.08	282
10.	43824	50.16	0.41	71.58	0.14	142	50032	50.12	0.34	67.19	0.08	261
Med	40201	52.09	5.69	72.03	0.15	162	46243	51.99	5.55	67.98	0.08	314

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	46094	50.00	0.00	65.17	0.08	195	46372	50.00	0.00	64.52	0.07	114
2.	52299	51.49	6.77	67.02	0.07	252	53216	51.84	6.81	67.05	0.08	145
3.	43204	54.85	25.97	63.79	0.10	143	42327	54.54	27.30	63.44	0.07	115
4.	47139	50.00	0.13	66.40	0.09	213	48092	50.00	0.00	65.76	0.08	138
5.	48637	52.30	11.59	65.94	0.08	271	49873	52.14	11.37	66.36	0.09	162
6.	46066	48.65	5.38	67.90	0.08	259	47142	48.57	4.88	68.44	0.09	155
7.	48207	52.77	10.42	66.85	0.08	282	49441	52.59	10.25	67.32	0.09	167
8.	51727	51.76	5.50	67.76	0.08	239	52568	51.65	5.28	67.77	0.08	140
9.	52539	49.42	1.05	63.61	0.06	183	55127	49.45	1.07	63.73	0.06	140
10.	46621	47.52	6.78	67.53	0.08	253	47812	47.58	6.49	67.93	0.09	134
Med	47673	50.74	6.13	66.63	0.08	245	48767	50.83	5.89	66.70	0.08	140

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	45828	49.58	1.42	69.97	0.10	135	45113	49.50	1.47	70.76	0.09	216
2.	45194	50.00	0.00	70.21	0.10	155	44547	50.00	0.00	71.06	0.09	229
3.	57750	48.40	3.44	68.25	0.08	142	57623	48.48	3.47	69.22	0.06	190
4.	46001	50.88	1.87	70.02	0.10	159	45265	50.79	1.86	70.85	0.09	228
5.	45446	50.04	0.54	69.98	0.10	155	44690	49.98	0.33	70.93	0.09	218
6.	45718	50.41	1.25	69.74	0.10	153	44912	50.35	0.96	70.74	0.09	201
7.	44645	50.61	2.19	70.08	0.11	156	43889	50.75	2.06	70.96	0.09	214
8.	45306	50.09	0.22	70.13	0.10	155	44613	50.05	0.14	70.98	0.09	225
9.	45218	49.97	0.06	70.20	0.10	155	44561	49.98	0.04	71.05	0.09	229
10.	45641	49.71	1.02	69.89	0.10	154	44946	49.68	0.95	70.62	0.09	232
Med	45544	50.02	1.13	70.00	0.10	155	44801	49.99	0.96	70.89	0.09	222

Table 14: Trade statistics of entry-entry grammar strategies under frictionless trading and risk neutrality.

B.2 Entry-Entry Grammar Market Frictions

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	63	50.79	9.42	49.21	11.67	271	87	56.32	23.90	45.98	5.53	223
2.	12	0.00	4.50	66.67	42.16	305	9	0.00	2.07	55.56	49.81	398
3.	18	11.11	23.19	61.11	43.56	247	21	33.33	17.60	66.67	19.20	355
4.	99	50.51	0.12	64.65	4.93	296	121	49.59	0.11	61.16	1.52	515
5.	55	43.64	6.97	54.55	17.92	240	115	42.61	22.27	34.78	2.54	245
6.	45	2.22	5.78	42.22	11.13	297	33	0.00	7.31	42.42	15.85	373
7.	23	47.83	0.93	82.61	41.26	158	16	50.00	1.61	50.00	20.17	428
8.	22	40.91	24.55	90.91	62.88	185	23	39.13	21.73	52.17	12.11	231
9.	10	10.00	21.32	80.00	77.71	194	13	30.77	11.28	76.92	70.24	264
10.	27	48.15	0.35	62.96	29.90	231	20	50.00	1.27	60.00	24.52	298
Med	25	42.27	6.38	63.80	35.58	244	22	40.87	9.29	53.86	17.52	326

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	6	50.00	1.29	100.00	186.32	214	3	33.33	5.35	66.67	38.77	357
2.	4	0.00	10.21	75.00	169.17	333	1	0.00	6.38	100.00	17.90	357
3.	4	50.00	1.33	100.00	262.25	214	1	0.00	5.41	0.00	-7.70	357
4.	10	40.00	25.17	70.00	95.01	157	5	40.00	10.43	40.00	-35.48	357
5.	7	42.86	2.09	57.14	90.29	451	3	33.33	1.91	66.67	-39.20	357
6.	23	30.43	3.95	60.87	26.20	363	12	33.33	0.00	25.00	-23.98	480
7.	3	0.00	11.36	100.00	269.70	214	2	0.00	26.38	50.00	-88.75	357
8.	13	38.46	4.78	69.23	83.30	195	13	46.15	1.34	61.54	30.70	179
9.	55	49.09	2.39	65.45	12.76	329	17	47.06	1.34	29.41	-22.09	457
10.	4	50.00	1.34	100.00	270.43	214	1	0.00	30.06	0.00	-190.30	357
Med	7	41.43	3.17	72.50	132.09	214	3	33.33	5.38	45.00	-23.03	357

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	35	57.14	3.24	71.43	13.12	327	16	56.25	0.42	43.75	-30.65	695
2.	66	80.30	29.60	65.15	11.41	152	52	88.46	23.85	44.23	-8.36	649
3.	32	100.00	54.21	75.00	14.82	128	19	100.00	28.74	52.63	-20.64	562
4.	79	50.63	2.41	62.03	13.07	209	42	54.76	6.41	40.48	-14.73	824
5.	142	50.00	10.91	55.63	3.00	320	109	50.46	10.44	54.13	0.67	389
6.	37	67.57	3.50	64.86	22.79	218	24	79.17	8.45	37.50	-13.65	483
7.	65	53.85	5.82	75.38	17.57	208	130	87.69	8.49	35.38	-1.04	496
8.	146	52.74	8.21	37.67	5.18	337	53	58.49	10.11	32.08	-10.02	706
9.	57	50.88	0.07	71.93	9.33	219	29	51.72	0.00	58.62	-2.97	600
10.	140	50.00	39.49	54.29	3.67	376	107	49.53	30.61	55.14	0.55	247
Med	66	53.29	7.02	65.01	12.24	218	47	57.37	9.30	43.99	-9.19	581

Table 15: Trade statistics of entry-entry grammar strategies under market frictions and risk neutrality.

Risk Aversion

Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
	In-sample I						Out-of-sample II					
1.	118	49.15	0.23	51.69	11.64	136	178	48.88	0.40	32.58	2.77	289
2.	33	48.48	0.19	69.70	26.19	169	30	50.00	1.25	50.00	15.95	369
3.	71	50.70	1.30	50.70	11.77	204	70	52.86	3.16	41.43	4.68	526
4.	90	32.22	29.36	51.11	6.23	175	114	57.02	50.85	44.74	-1.42	523
5.	17	0.00	12.02	58.82	29.55	310	53	0.00	19.29	39.62	8.77	380
6.	23	47.83	2.96	65.22	25.57	302	27	48.15	2.69	29.63	16.61	382
7.	31	48.39	5.80	64.52	27.15	217	33	63.64	8.29	45.45	2.38	476
8.	40	35.00	1.95	60.00	21.08	217	30	36.67	6.32	60.00	15.03	371
9.	35	48.57	0.04	60.00	30.09	198	36	50.00	1.29	38.89	16.42	350
10.	65	44.62	5.68	64.62	15.31	266	70	44.29	8.75	48.57	-7.14	800
Med	38	48.11	2.46	60.00	23.32	210	45	49.44	4.74	43.08	6.73	381

Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
	In-sample II						Out-of-sample III					
1.	4	50.00	1.28	100.00	267.13	214	1	0.00	29.60	0.00	-215.40	357
2.	6	50.00	7.26	100.00	185.40	214	1	0.00	29.53	0.00	-259.50	357
3.	4	50.00	1.31	100.00	286.80	214	3	33.33	5.39	66.67	49.10	357
4.	4	50.00	1.27	100.00	267.75	214	1	0.00	5.34	0.00	-24.20	357
5.	67	1.49	55.54	59.70	5.91	273	71	0.00	57.30	42.25	-0.28	283
6.	13	7.69	70.88	84.62	41.72	85	9	0.00	85.24	66.67	8.40	119
7.	66	0.00	33.79	59.09	9.31	314	71	0.00	35.73	47.89	1.94	226
8.	74	5.41	47.19	59.46	9.25	214	71	0.00	49.28	42.25	0.86	294
9.	1	0.00	1.30	100.00	528.10	391	1	0.00	5.46	100.00	7.20	357
10.	6	50.00	1.27	83.33	216.08	214	3	33.33	5.37	33.33	-116.30	417
Med	6	28.85	4.29	92.31	200.74	214	3	0.00	29.57	42.25	0.29	357

Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
	In-sample III						Out-of-sample IV					
1.	24	87.50	15.80	75.00	27.11	146	26	88.46	18.55	26.92	-15.08	560
2.	119	49.58	24.38	64.71	9.80	122	97	50.52	31.43	51.55	-3.20	531
3.	140	50.00	2.13	57.14	3.85	453	107	49.53	1.79	51.40	-1.95	493
4.	37	51.35	1.24	64.86	20.65	201	33	51.52	1.13	27.27	-21.01	758
5.	32	75.00	37.07	65.63	19.89	172	24	66.67	11.13	41.67	-14.67	512
6.	100	33.00	36.37	64.00	9.36	140	71	35.21	24.13	57.75	0.37	374
7.	75	58.67	5.04	70.67	18.37	163	34	61.76	9.27	55.88	-6.04	438
8.	141	51.06	10.83	65.25	8.71	151	124	56.45	11.90	49.19	-0.17	306
9.	39	61.54	10.31	58.97	17.46	201	27	55.56	4.11	37.04	-20.19	608
10.	129	36.43	37.56	57.36	7.41	130	155	41.29	32.26	40.65	-0.08	265
Med	88	51.21	13.32	64.79	13.63	157	53	53.54	11.51	45.43	-4.62	503

Table 16: Trade statistics of entry-entry grammar strategies under market frictions and risk aversion.

Loss Aversion

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	52	26.92	18.45	67.31	15.02	148	91	36.26	30.60	50.55	-0.03	516
2.	16	93.75	80.76	62.50	30.44	64	13	100.00	96.45	30.77	-3.08	105
3.	38	60.53	41.29	65.79	16.79	221	30	80.00	22.65	30.00	-16.16	608
4.	7	71.43	65.64	100.00	109.77	102	3	100.00	53.20	33.33	-11.27	291
5.	21	47.62	92.12	76.19	18.98	47	27	25.93	91.57	40.74	4.33	90
6.	2	0.00	86.06	100.00	136.50	64	NaN	NaN	NaN	NaN	NaN	NaN
7.	31	41.94	60.91	61.29	20.49	181	44	38.64	65.17	52.27	3.67	211
8.	4	50.00	28.17	100.00	177.68	209	5	60.00	3.72	40.00	-33.74	597
9.	22	59.09	85.87	77.27	20.45	127	21	61.90	72.00	47.62	-7.28	287
10.	22	54.55	28.72	68.18	50.72	150	18	61.11	18.18	61.11	9.72	337
Med	22	52.27	63.27	72.19	25.47	137	21	61.11	53.20	40.74	-3.08	291

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	63	60.32	79.16	50.79	7.52	93	22	45.45	87.86	59.09	3.88	91
2.	6	100.00	98.06	83.33	14.27	54	NaN	NaN	NaN	NaN	NaN	NaN
3.	9	22.22	85.36	66.67	23.68	108	6	16.67	73.71	50.00	-30.45	303
4.	59	0.00	34.29	57.63	10.56	149	51	0.00	26.74	52.94	0.62	227
5.	10	0.00	78.50	90.00	32.12	125	4	0.00	79.84	75.00	4.45	106
6.	16	0.00	60.62	81.25	49.64	113	6	0.00	70.19	66.67	30.55	122
7.	27	0.00	83.93	66.67	15.33	80	9	0.00	59.97	44.44	-6.19	296
8.	25	12.00	46.65	80.00	47.91	116	22	0.00	61.31	31.82	-6.60	344
9.	6	50.00	1.27	100.00	181.23	214	3	33.33	5.37	0.00	-127.83	436
10.	56	5.36	33.43	55.36	13.08	169	55	1.82	30.60	52.73	-0.20	216
Med	21	8.68	69.56	73.33	19.51	115	9	0.00	61.31	52.73	-0.20	227

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	14	28.57	68.88	92.86	34.95	110	10	50.00	27.70	60.00	13.42	241
2.	22	0.00	92.28	68.18	10.95	57	20	0.00	94.60	45.00	1.07	66
3.	18	0.00	89.84	83.33	8.89	59	8	0.00	94.22	62.50	-5.20	103
4.	8	100.00	85.71	100.00	37.84	67	4	100.00	27.82	0.00	-125.28	543
5.	25	0.00	50.76	72.00	20.62	98	15	0.00	84.69	53.33	-0.39	113
6.	6	66.67	61.15	100.00	80.38	145	3	66.67	85.90	66.67	10.23	143
7.	33	0.00	72.00	51.52	6.84	112	9	0.00	83.00	66.67	10.28	116
8.	37	51.35	0.78	75.68	31.52	195	15	53.33	0.83	60.00	-23.84	561
9.	12	33.33	55.29	100.00	39.57	113	12	16.67	72.44	58.33	-2.50	169
10.	11	0.00	92.93	90.91	17.68	29	11	0.00	93.55	72.73	8.92	55
Med	16	14.29	70.44	87.12	26.07	104	11	8.33	83.84	60.00	0.34	130

Table 17: Trade statistics of entry-entry grammar strategies under market frictions and loss aversion.

B.3 Entry-Exit Grammar, Market Frictions

Risk Neutrality

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	17	47.06	1.45	88.24	47.83	223	13	46.15	1.54	46.15	29.30	406
2.	47	4.26	28.66	48.94	15.54	267	48	0.00	36.00	47.92	6.31	291
3.	27	48.15	0.22	77.78	36.56	161	22	40.91	4.28	40.91	8.14	423
4.	64	42.19	5.57	53.13	12.00	269	55	56.36	8.35	30.91	6.95	382
5.	31	41.94	6.06	64.52	28.81	202	41	36.59	8.81	51.22	14.37	316
6.	21	47.62	1.00	71.43	29.00	264	23	47.83	1.08	34.78	19.22	379
7.	44	27.27	3.00	68.18	18.94	238	64	39.06	2.86	50.00	6.96	494
8.	55	30.91	20.50	56.36	18.81	172	63	26.98	28.46	39.68	-0.50	329
9.	33	33.33	8.40	66.67	29.19	213	33	24.24	13.46	48.48	14.07	304
10.	41	39.02	0.49	63.41	19.90	207	30	23.33	2.03	46.67	16.11	363
Med	37	40.48	4.28	65.59	24.36	218	37	37.82	6.32	46.41	11.11	371

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	25	0.00	12.75	60.00	23.60	234	27	0.00	10.67	33.33	-14.01	445
2.	61	3.28	15.87	95.08	14.05	337	6	0.00	2.04	83.33	-28.92	336
3.	107	0.00	46.01	56.07	6.61	281	113	0.00	42.85	46.02	-1.24	368
4.	50	38.00	15.41	52.00	16.45	206	39	28.21	25.51	43.59	-5.15	446
5.	24	4.17	32.17	62.50	29.12	322	26	0.00	24.89	34.62	-3.85	390
6.	22	0.00	14.37	95.45	31.94	380	5	0.00	2.63	80.00	-21.70	326
7.	14	14.29	18.93	100.00	75.49	176	7	0.00	38.37	85.71	22.49	216
8.	39	0.00	26.13	97.44	21.96	282	14	0.00	19.71	92.86	3.73	315
9.	39	0.00	12.76	97.44	15.39	380	8	0.00	3.35	87.50	-16.49	325
10.	6	50.00	1.27	100.00	195.73	214	3	33.33	5.38	33.33	-113.37	415
Med	32	1.64	15.64	95.27	22.78	282	11	0.00	15.19	63.01	-9.58	352

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	34	70.59	5.36	61.76	17.39	159	148	56.76	18.31	22.97	-6.09	957
2.	86	54.65	6.26	52.33	6.88	242	25	64.00	3.36	36.00	-19.38	674
3.	17	100.00	13.26	94.12	25.59	189	1	100.00	0.69	0.00	-513.70	667
4.	40	57.50	11.22	57.50	16.21	185	44	63.64	25.35	27.27	-14.50	694
5.	152	62.50	35.70	43.42	5.54	243	94	57.45	20.17	39.36	-5.79	666
6.	18	83.33	4.33	83.33	24.72	205	12	58.33	0.74	16.67	-44.57	605
7.	61	100.00	44.82	100.00	7.55	157	3	100.00	2.09	66.67	-160.50	625
8.	43	100.00	35.69	51.16	10.19	172	38	100.00	42.74	42.11	-5.52	354
9.	34	70.59	9.39	55.88	15.54	202	53	67.92	25.87	35.85	-9.16	568
10.	16	100.00	52.93	93.75	33.36	149	1	100.00	5.43	0.00	-509.60	649
Med	37	76.96	12.24	59.63	15.87	187	32	65.96	11.87	31.56	-16.94	657

Table 18: Trade statistics of entry-exit grammar strategies under market frictions and risk neutrality.

Risk Aversion

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	49	59.18	10.77	51.02	16.46	198	64	56.25	17.83	35.94	-8.75	756
2.	110	46.36	13.13	46.36	11.96	160	205	48.78	11.85	31.22	1.69	433
3.	48	64.58	5.77	81.25	13.17	281	41	56.10	4.04	68.29	10.97	407
4.	48	25.00	25.74	60.42	20.42	163	57	15.79	32.48	42.11	3.33	312
5.	857	48.07	0.04	63.71	0.71	313	880	46.93	0.17	57.73	-0.84	1059
6.	133	39.10	22.05	38.35	4.59	358	133	39.10	31.47	31.58	-0.33	378
7.	32	12.50	14.74	53.13	19.18	271	39	23.08	19.94	51.28	11.41	212
8.	87	39.08	48.82	59.77	8.19	242	130	38.46	42.32	38.46	-1.58	451
9.	84	33.33	29.74	51.19	9.61	248	107	36.45	35.49	46.73	-2.61	467
10.	16	37.50	8.99	81.25	56.28	243	24	37.50	19.01	58.33	7.82	324
Med	67	39.09	13.93	56.45	12.57	246	86	38.78	19.48	44.42	0.68	420

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	87	4.60	36.61	64.37	11.69	212	74	0.00	40.72	45.95	0.95	303
2.	66	28.79	40.98	78.79	10.08	172	45	31.11	18.77	64.44	-5.96	446
3.	28	0.00	24.84	60.71	25.42	199	31	0.00	23.47	32.26	-12.64	498
4.	36	0.00	32.21	97.22	22.06	380	6	0.00	2.46	83.33	-23.30	337
5.	19	57.89	68.71	52.63	27.02	199	12	50.00	83.01	58.33	15.48	85
6.	26	7.69	55.60	88.46	35.37	132	20	15.00	64.42	50.00	-1.55	358
7.	16	0.00	5.55	93.75	40.13	391	2	0.00	0.70	50.00	-89.60	357
8.	23	34.78	29.63	82.61	37.27	178	10	10.00	26.47	80.00	2.09	222
9.	89	48.31	48.74	66.29	8.84	175	47	27.66	36.75	44.68	-3.92	283
10.	46	58.70	63.68	82.61	25.94	77	14	35.71	62.58	50.00	-8.15	376
Med	32	18.24	38.79	80.70	25.68	188	17	12.50	31.61	50.00	-4.94	347

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	64	76.56	56.90	67.19	14.14	173	54	85.19	66.90	50.00	-4.02	470
2.	107	35.51	17.40	73.83	9.57	174	84	35.71	23.69	63.10	2.34	256
3.	143	48.95	3.26	58.04	4.49	400	135	39.26	2.73	47.41	-1.57	525
4.	119	48.74	13.33	68.07	8.36	229	65	52.31	17.56	50.77	-2.31	319
5.	138	50.00	0.17	57.97	5.95	486	105	49.52	0.13	60.00	3.24	230
6.	61	63.93	8.71	55.74	10.49	199	71	64.79	9.03	35.21	-6.75	642
7.	83	59.04	5.60	60.24	7.94	183	57	66.67	3.47	40.35	-7.28	568
8.	77	81.82	65.65	88.31	8.17	129	43	48.84	26.36	41.86	-11.73	556
9.	74	94.59	53.75	100.00	7.77	114	32	81.25	22.22	81.25	-11.77	530
10.	62	80.65	35.41	79.03	14.60	151	71	25.35	11.87	40.85	-10.07	877
Med	80	61.49	15.36	67.63	8.26	178	68	50.92	14.72	48.70	-5.38	527

Table 19: Trade statistics of entry-exit grammar strategies under market frictions and risk aversion.

Loss Aversion

	In-sample I						Out-of-sample II					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	8	0.00	59.11	100.00	82.85	132	6	0.00	9.87	83.33	24.48	399
2.	60	28.33	78.28	83.33	14.50	82	51	27.45	74.68	58.82	3.73	167
3.	2	0.00	91.11	100.00	98.65	54	NaN	NaN	NaN	NaN	NaN	NaN
4.	11	90.91	58.19	100.00	75.18	95	1	100.00	3.64	0.00	-398.20	603
5.	10	90.00	87.62	100.00	33.86	63	7	100.00	64.84	85.71	-19.86	304
6.	46	65.22	29.87	65.22	21.43	141	60	63.33	24.93	46.67	-2.68	497
7.	41	0.00	52.67	100.00	14.87	168	27	0.00	48.03	96.30	12.75	187
8.	14	14.29	71.61	100.00	36.61	114	22	13.64	38.11	90.91	20.69	174
9.	39	58.97	84.35	71.79	10.32	114	16	56.25	82.72	68.75	-4.28	302
10.	37	78.38	71.71	81.08	18.29	92	49	75.51	53.67	59.18	3.39	205
Med	26	43.65	71.66	100.00	27.65	105	22	56.25	48.03	68.75	3.39	302

	In-sample II						Out-of-sample III					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	23	13.04	33.06	95.65	40.25	176	7	0.00	60.09	85.71	21.51	169
2.	10	0.00	19.09	90.00	70.24	214	4	0.00	16.56	75.00	1.43	357
3.	33	6.06	64.06	96.97	25.53	139	4	0.00	3.44	75.00	-49.70	357
4.	11	9.09	56.95	100.00	68.21	146	4	25.00	21.19	75.00	-30.80	357
5.	9	0.00	83.29	88.89	43.43	128	3	0.00	83.76	66.67	11.60	130
6.	13	15.38	29.51	100.00	57.17	176	7	0.00	29.40	85.71	16.33	222
7.	27	51.85	66.19	70.37	29.19	133	5	20.00	55.59	60.00	26.22	200
8.	7	0.00	83.60	85.71	29.26	125	5	0.00	76.63	60.00	2.36	121
9.	14	14.29	3.23	71.43	82.29	204	11	0.00	95.97	45.45	-3.43	154
10.	16	12.50	41.65	100.00	54.63	176	11	18.18	64.84	72.73	17.33	188
Med	14	10.80	49.30	92.83	49.03	161	5	0.00	57.84	73.86	6.98	194

	In-sample III						Out-of-sample IV					
Run	NT	LSR	NR	PP	AT	MDD	NT	LSR	NR	PP	AT	MDD
1.	23	0.00	73.58	100.00	19.94	98	17	0.00	24.24	94.12	13.42	236
2.	11	0.00	89.88	100.00	25.68	110	9	0.00	82.57	100.00	25.53	99
3.	51	68.63	60.58	88.24	13.44	103	35	60.00	45.17	57.14	-6.88	320
4.	68	91.18	33.11	98.53	8.21	152	49	71.43	12.86	57.14	-10.98	601
5.	28	0.00	84.87	100.00	13.14	98	35	17.14	50.20	74.29	6.45	179
6.	4	75.00	67.67	100.00	102.47	130	5	60.00	39.78	40.00	-17.86	271
7.	19	47.37	7.48	68.42	42.23	195	6	50.00	2.78	66.67	-24.50	587
8.	37	51.35	5.60	67.57	17.72	159	38	50.00	4.35	34.21	-12.58	633
9.	49	100.00	50.62	100.00	9.01	156	2	100.00	1.00	50.00	-255.60	649
10.	20	25.00	71.84	100.00	28.05	110	18	11.11	27.70	77.78	4.02	197
Med	26	49.36	64.13	100.00	18.83	120	18	50.00	25.97	61.90	-8.93	296

Table 20: Trade statistics of entry-exit grammar strategies under market frictions and loss aversion.