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Finding Profit-Maximizing Strategies for the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket

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Abstract

In this paper we use the Generalised Population Based Incremental Learning (GPBIL) in order to find profit-maximizing strategies for the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). The artificial market has modeled explicitly a multidimensional consumers' and merchants' demand for payment instruments. Given the complex shape of the demand, we have found that the GPBIL effectively explores the areas of intersection between the price and the demand on both sides of the payment card market, in order to reach a price structure and a price level that maximize the profit of the payment card providers.
Keywords: competition in payment card markets. *JEL classification:*

1 Introduction

The payment cards, known as credit and debit cards, in the last two decades have become an important element of the modern economies. For instance, in 2000 the cards account for 35% of the consumers expenditures in United Kingdom, 30% in Australia and 25% in United States of America [1]. The growing importance of these electronic payment instruments is the reason

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why economists and policymakers have put a lot of efforts to understand the payment card market [2], [3], [4], [5]. This market is built over so called two-sided platform, in which for a successful transaction with an electronic instrument the consumers have to hold a card and the merchants have to accept it as a payment. Apart from this, the payment card provides an extra value to consumers and merchants by allowing them to perform a commercial transactions among each other.

Furthermore, the higher the number of establishments that accept a particular card, the higher the benefits to the card holders. Similarly, the merchants obtain higher benefits if the number of consumers using a particular card increases. These indirect externalities, which arise from the increase of the number of end-users on both sides, are crucial element of the competition among payment card purveyors.

For a long time, the main focus of the literature has been on the fee structure of payment cards, with the emphasis laid on the interchange fee. The interchange fee is the amount that the bank of a merchant pays to the bank of the consumer for each transaction with a card. The research in the field can be divided into models studying the problematic of a single card [2], [3], [6], [7] and [8], and models that allow the competition between payment methods [9], [10] and [11].

Nevertheless, given the intellectual challenge of incorporating the complex market dynamics into an analytical model, the results of the theoretical studies strongly depends on the assumptions on the relationships among the market participants. In addition, these models are unable to incorporate the heterogeneity in the preferences of consumers and merchants, neither to model explicitly the complex shape of two-sided demand for electronic payment methods.

In this context, in order to gain better understanding of the market insights, we have developed the first to our knowledge $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). This is an agent-based model, which simulates the individual interactions at the point of sale among consumers and merchants. An instantiation of the model explicitly reproduces the demand of payment cards on both sides, given a specific price and cost of publicity, determined by the card providers in the artificial market. Each competitor decides his own price level and structure[1] as well as the amount to spend in publicity. Those variables form the payment card purveyor's strategy.

In this paper we present the application of the Generalised Population Based Incremental Learning (GPBIL) algorithm in the $\mathcal{APCM}$, in order to find a

[1] The price structure consists in variable and fixed fees on both sides of the market, but each competitor decides which particular fees to apply

profit maximizing strategy by exploring the areas of intersections between the consumers' demand and the consumers' price together with the areas of intersections between the merchants' demand and the merchants' price. Those areas are constrained between each other.
The paper is organized as following: in the next section we briefly introduce the elements and the setting of the $\mathcal{APCM}$. Following, in section 3 we present the application of the GPBIL in finding a profit maximizing strategy, given multiple number of competitors; then in section 4 we present the setting of the experiment and the results obtained by the search. We finalize the paper with our conclusions in section 5.

2 $\mathcal{APCM}$

In this section we present the elements and the setting of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. The model simulates commercial transactions among consumers and merchants in order to reproduce the demand of payment cards at the point of sale. Furthermore, it is aimed to study the competition among card issuers. To that end, in this section we introduce formally the sets of consumers and merchants with their corresponding set of decisions.

2.1 Merchants

2.1.1 Definition

Suppose we have a set of merchants $\mathcal{M}$ with $|\mathcal{M}| = N_{\mathcal{M}}$, who are offering a homogeneous good at a common price and face marginal cost of production lower than this price. In other words we eliminate the price competition among merchants in order to concentrate our analysis on the competition among payment cards. The merchants are located at random intersections of a $N \times N$ lattice, where $N^2 \gg N_{\mathcal{M}}$. Let the top and bottom edges as well as the right and left edges of this lattice be connected.

2.1.2 Decision

During the transactions at the point of sale, the merchants take one decision. After certain period of interactions[2] they decide to which new cards to subscribe and which old subscriptions to keep. In order to do so, they initially have certain number of cards assigned. Then for each commercial transaction the establishments keep track of the cards presented to them

[2] The number of interactions is different across merchants and it is determined by poisson distribution specific for each m

by the consumers. Every time a card $p \in \mathcal{P}$ is presented to the merchant $m \in \mathcal{M}$ and he has a subscription to this card $p \in \mathcal{P}_m$ with $|\mathcal{P}_m| = N_{\mathcal{P}_m}$, he increases the score of the card $\theta^-_{m,p}$ by one. Here, $\theta^-_{m,p}$ is an element of the vector defined as

$$\left(\theta^-_{m,1}, \ldots, \theta^-_{m,N_{\mathcal{P}_m}}\right).$$

On the other hand, if the merchant does not have subscription to the card, i.e $p \in \mathcal{P}^-_m$, the score of the card $\theta^+_{m,p}$ is increased by one, given that $\theta^+_{m,p}$ is an element of the vector:

$$\left(\theta^+_{m,1}, \ldots, \theta^+_{m,N_{\mathcal{P}^-_m}}\right).$$

The merchant decides to cancel the subscription of a card with probability[3]

$$\pi^-_{m,p} = \frac{x^-_m q}{x^-_m q + exp\left(\frac{\theta^-_{m,p}}{\theta_m}\right)} \qquad (1)$$

where θ_m denotes the number of cards presented. Similarly he decides to subscribe to a new card with probability

$$\pi^+_{m,p} = \frac{exp\left(\frac{\theta^+_{m,p}}{\theta_m}\right)}{x^+_m q + exp\left(\frac{\theta^+_{m,p}}{\theta_m}\right)} \qquad (2)$$

where $x^-_m q$ and $x^+_m q$ represent the inertia to add or drop a card; $q = \left(1 + \mathit{\Gamma}_p + N_{\mathcal{P}_m} + \frac{\varepsilon}{\beta_p}\right)$, whereas x^-_m and x^+_m are constants.

2.2 Consumers

2.2.1 Definition

Consumers occupy all remaining intersections of the $N \times N$ lattice. The set of consumers is denoted $\mathcal{C}$ with $|\mathcal{C}| = N_{\mathcal{C}}$, where $N_{\mathcal{C}} \gg N_{\mathcal{M}}$ and $N^2 = N_{\mathcal{C}} + N_{\mathcal{M}}$. Each consumer has a budget constraint that allows him to buy exactly one unit of the good offered by the merchants, in a single interaction. The utility gained from the consumption of this good is bigger than its price. In order to obtain the good any consumer $c \in \mathcal{C}$ has to travel to a merchant $m \in \mathcal{M}$. The distance is measured by the Manhattan distance $d_{c,m}$ between the locations on the lattice and it imposes travel costs on consumers. The

[3]The probabilities defined in equations 1 and 2 are affected by the publicity applied by each payment card provider.

longitude between two adjacent intersections is normalized to unity. Let $\mathcal{M}_c$ denotes the set of merchants a consumer considers to go to, given that we restrict him to the nearest merchants[4]

2.2.2 Decisions

In order for the commercial transaction to occur, the consumers need to take three decisions: which merchant to visit, which card to use and similar to the decision of the merchants, to which card to subscribe?

Regarding the first decision, we assume that when deciding which merchant to visit, the consumer has not yet decided which of the cards he holds will be used. Suppose $\mathcal{P}_{c,m}$ is the set of cards the consumers and merchants have in common. Given that $|\mathcal{P}_{c,m}| = N_{\mathcal{P}_{c,m}}$, we assume that the more common payment cards the merchant m and the consumer c have, the more attractive a merchant becomes. This, due to the fact that the consumer always carries all his cards with him and he decides which card to use at the moment of the payment. Additionally the smaller the distance $d_{c,m}$ between the consumer and the merchant, the higher the possibility for this merchant to be chosen by the consumer. From these deliberations we propose to use a preference function for consumer to visit merchant:

$$\mathbf{v}_{c,m} = \frac{\frac{N_{\mathcal{P}_{c,m}}}{d_{c,m}}}{\sum_{m' \in \mathcal{M}_c} \frac{N_{\mathcal{P}_{c,m'}}}{d_{c,m'}}} \tag{3}$$

Each consumer $c \in \mathcal{C}$ chooses a merchant $m \in \mathcal{M}$ with probability $\mathbf{v}_{c,m}$ as defined in 3. The consumers will continuously update their beliefs regarding the number of common payment cards for all merchants they may visit.

With respect to the second decision, the consumer decides which payment card he wants to use at the merchant he has chosen. We assume a *preferred card choice*, given that $\mathcal{P}_{c,m}$ is the set of common cards the consumer and the merchant have. There are three possible scenarios. In the first, there are more then one common cards in $\mathcal{P}_{c,m}$. In this case the consumer chooses the card with the higher benefits b_p. In the second scenario, there is only one element in the set $\mathcal{P}_{c,m}$, then the common card is used. Finally, in the case the merchant does not accept any of the consumers' cards the transaction is settled using cash payment.

Finally, regarding the subscription of the consumers, after certain periods of

[4]We have modeled three types of *network connections* among consumers and merchants $nc \in \mathcal{NC} = \{l, sw, r\}$, whereas l stands for local, sw for small world and r for random. In the present paper we present the case of local connections

interactions[5] these agents decide which new cards to subscribe to and which old subscriptions to keep.

Similarly to the merchants, initially the consumers have certain number of cards $\mathcal{P}_c$ with $|\mathcal{P}_c| = N_{\mathcal{P}_c}$. Every consumer $c \in \mathcal{C}$ keeps track whether the cards he owns are accepted by the merchant or not. If card $p \in \mathcal{P}_c$ is accepted by the visited merchant $m \in \mathcal{M}_c$, the consumer increases the score of the card $\omega^-_{c,p}$ by one. Here $\omega^-_{c,p}$ is an element of the vector specified as

$$\left(\omega^-_{c,1}, \ldots, \omega^-_{c,N_{\mathcal{P}_c}}\right).$$

Assume that he cancels his subscription with probability[6] defined in 4, given that the number of merchants visited is ω_c.

$$\pi^-_{c,p} = \frac{x^-_c k}{x^-_c k + exp\left(\frac{\omega^-_{c,p}}{\omega_c}\right)} \tag{4}$$

Here $x^-_c k$ accounts for the inertia of the consumer to change cards; $k = \left(1 + F_p + N_{\mathcal{P}_c} + \frac{\varepsilon}{b_p}\right)$, whereas ε and x^-_c are constants.

At the other hand, let $\mathcal{P}^-_c$ with $|\mathcal{P}^-_c| = N_{\mathcal{P}^-_c}$ be the set of payment cards, to which the consumer does not have subscription. Let the consumer c visits a merchant m. Suppose that they do not have cards in common, i.e. $\mathcal{P}_{c,m}= \{\emptyset\}$, and the set of cards the merchant accepts $\mathcal{P}_m \neq \{\emptyset\}$. In that case the consumer increases the score $\omega^+_{c,p}$ by one $\forall p \in \mathcal{P}_m \subset \mathcal{P}^-_c$. Here $\omega^+_{c,p}$ is an element of the vector, which is defined as

$$\left(\omega^+_{c,1}, \ldots, \omega^+_{c,N_{\mathcal{P}^-_c}}\right).$$

Given that x^+_c is a constant, the probability of subscribing to these cards is then determined by

$$\pi^+_{c,p} = \frac{exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)}{x^+_c k + exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)} \tag{5}$$

2.3 Payment Cards

2.3.1 Definition

There exists a set of payment cards $\mathcal{P}$ with $|\mathcal{P}| = N_{\mathcal{P}}+1$ and $N_{\mathcal{P}} \ll N_{\mathcal{M}}$. All payment forms are card payments, with the exception of the first payment

[5] The number of interactions is different across consumers and is defined by individual poisson distribution

[6] The probabilities defined in equations 4 and 5 are affected by the publicity applied by each payment card provider.

method, which is the benchmark and can be interpreted as cash payment. The cash is used by all consumers and is accepted by all merchants. Furthermore, in order for a card payment to occur, the consumer as well as the merchant must have a subscription to the card in question. We have explained above that consumers prefer card payments over cash payments.
A fixed subscription fee of $F_p \geq 0$ could be charged per each interaction to the consumer whereas $\Gamma_p \geq 0$ could be charged per each interaction to the merchant. The domains of those fees, $\mathbb{D}_{F_p}$ and $\mathbb{D}_{\Gamma_p}$ are subsets of real numbers. Cash payments do not attract any fees.
For each unit of goods sold using a payment card $p \in \mathcal{P}_m$, a merchant $m \in \mathcal{M}$ receives net benefits of β_p. Such benefits may include reduced costs from cash handling and could differ across payment methods. These are identical for all merchants for a given card. The domain $\mathbb{D}_{\beta_p}$ is a subset of real numbers. Note that the benefits β_p could have a negative value. In other words, the variable fees paid by the merchant to the card issuer is bigger than the benefits he received from that particular electronic payment method. Cash payments do not produce any benefits.
Consumers also receive net benefits from paying by card, b_p, but no benefits from cash payments. Here, the benefits may arise from the delayed payment, insurance cover or cash-back options. The benefits are the same for all consumers, but could differ across card purveyors. The $\mathbb{D}_{b_p}$ is a subset of real number and also could include negative values as in the case of the merchants.
In addition, the issuer of the payment method has to decide how much it should spend in publicity $l_p \in \mathbb{D}_{l_p}$, in order to increase the number of consumers and merchants using the electronic card that he is providing. The publicity domain, $\mathbb{D}_{l_p}$, is a subset of real numbers. Finally the variables controlled by the card purveyors: F_p, Γ_p, β_p, b_p and l_p form its strategy.

2.3.2 Decision

The payment card providers' decision is to define what strategy they are going to use. For that reason we define the solution space of the payment card's strategy as

$$\mathbb{S} = \mathbb{D}_{F_p} \times \mathbb{D}_{\Gamma_p} \times \mathbb{D}_{b_p} \times \mathbb{D}_{\beta_p} \times \mathbb{D}_{l_p}$$

rewritten as

$$\begin{array}{l} \mathbb{S} = \mathbb{D}_1 \times \cdots \times \mathbb{D}_5 \text{ with} \\ \mathbb{D}_1 = \mathbb{D}_{F_p}, \ldots, \mathbb{D}_5 = \mathbb{D}_{l_p} \end{array} \tag{6}$$

In addition we assume that the cost of publicity, l_p, spend by the card issuer in each interaction, has a direct impact in the consumers' and the merchants' decisions to subscribe/cancel a card. The probabilities, π_c^+, π_c^- , π_m^+ , π_m^-, given from equation 1 to equation 5 are then adjusted according to the rule presented in the following equation

$$\boldsymbol{\Delta}\pi = \tau\pi\,(1-\pi) \tag{7}$$

Here π substitutes any of the above probabilities, Δ represents the differences between the original value of π and the adjusted π, and finally $\tau = \alpha\,(\varphi - exp\,(-l_p))$. The constants α and φ satisfy the constrains $\pi - \boldsymbol{\Delta}\pi \geq 0$ and $\pi + \Delta\pi \leq 1$.
Let $\vec{\mathbf{s}} = (\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ be the vector of sample strategies for all payment methods. The payment card providers' decisions consist of creating such a vector. The basic mechanism of sampling $\mathbf{s}_p$ from $\mathbb{S}$ is following a random process. Additionally, the vector $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ could be the result of an extensive search over the strategy space and its intersection with the complex shape of the modeled demand. This search is guided by particular criteria of interest. In the next section we propose the use of Generalised Population Based Incremental Learning algorithm [12] in order to find a joint probability distribution over this space.

3 Applying GPBIL in finding profit-maximizing strategy

The three elements of our model are in constant interactions. In order for the commercial transactions among consumers and merchants to take place, first each payment card purveyor has to determine his strategy[7]. In other words, the $\mathcal{APCM}$ simulates the interactions among consumers and merchants at the point of sale, given a vector of sample strategies and a specific number of interactions I. In this section we explain how the card issuers learn to select profit-maximizing strategies that guarantee an average participation in the market. This is performed by exploring the intersections between the modeled demand on both sides and the price structure of the competitors.
We have said earlier that the strategy's domain $\mathbb{D}_i$ are intervals of real numbers. Assume a probability distribution functions $\mathbb{F}_{\mathbb{D}_i} : \mathbb{R} \rightarrow [0, 1]$ for unconditional random variables over the ranges $\mathbb{D}_i$, we define the joint probability distribution $\mathbb{F}_{\mathbb{S}}$ over $\mathbb{S}$ by

$$\mathbb{F}_{\mathbb{S}} = \mathbb{F}_{\mathbb{D}_\mathbf{1}} \cdot \ldots \cdot \mathbb{F}_{\mathbb{D}_\mathbf{5}}. \tag{8}$$

[7]The set of strategies of all competitors is denoted $\vec{\mathbf{s}}$

All electronic cards issuers have the same joint probability distribution and we are using it firstly to sample individual strategies from the space, and secondly to modify $\mathbb{F}_\mathbb{S}$ through learning.

We have defined $\vec{\mathbf{s}} = (\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})$ as the vector of strategies of all payment methods in one execution of the $\mathcal{APCM}$. Additionally, we define $\phi_p = \left(\Phi_p, N_{T_p^*}, p\right)$ as the measurement of the performance achieved in one execution of the model for one payment method. The three elements that compose it are the profit of the card issuer Φ_p, its market share measured in terms of the total number of transactions $N_{T_p^*}$ and the corresponding index of the card p. The vector $(\phi_1, \ldots, \phi_{N_\mathcal{P}})$ represents the performance of all payment cards in one execution of the $\mathcal{APCM}$.

In figure 1 we present the process $\mathcal{MARKET} - \mathcal{GPBIL}$ used to find a

$\mathcal{MARKET} - \mathcal{GPBIL}()$
1 $I = \mathfrak{I}; \mathbb{S} = \mathfrak{S}; N_\mathcal{P} = \mathfrak{N}_\mathcal{P}; R = \mathfrak{R}$
2 $\mathbb{F}_\mathbb{S} =$ *initialisation* $(\mathbb{S})$
3 FOR $r = 1, \ldots, R$ DO
4 FOR $p = 1, \ldots, N_\mathcal{P}$ DO
5 $\mathbf{s}_p =$ *sampling* $(\mathbb{F}_\mathbb{S})$
6 $(\phi_1, \ldots, \phi_{N_\mathcal{P}}) = \mathcal{APCM}\left((\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}}), I\right)$
7 $\vec{\phi}' =$ *profitDescendingSort* $(\phi_1, \ldots, \phi_{N_\mathcal{P}})$
8 $\mathbb{F}_\mathbb{S} =$ *learning* $\left(\mathbb{F}_\mathbb{S}, \vec{\phi}', (\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})\right)$
9 RETURN $\mathbb{F}_\mathbb{S}$

Figure 1: The process $\mathcal{MARKET} - \mathcal{GPBIL}$ for profit-maximizing strategies

profit-maximizing strategies. In our application the strategy should fulfill two main objectives: obtain the highest possible profit Φ_p and achieve above average market share measured in terms of the total number of transactions $N_{T_p^*}$. The first step is to initialize the joint probability function. This is performed by the function *initialisation*, which receives as a parameter the solution space $\mathbb{S}$ and returns the initialized joint probability function, $\mathbb{F}_\mathbb{S}$.

The main part of the algorithm consists of a loop over R runs. At the beginning of each run every payment card provider selects a strategy $\mathbf{s}_p$. This process is carried out by the function *sampling* (line 4 of the GPBIL algorithm fig. 1), which returns a strategy $\mathbf{s}_p$ for each one of the payment cards based on the probability distribution function $\mathbb{F}_\mathbb{S}$.

Thereafter, in line 5, we instantiate the process $\mathcal{APCM}$ with the strategy vector $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})$ and number of interactions I. This process returns a

Table 1: Constants used in the end-users' decisions

Symbol	Description of the Constants	Value
ε	common constant for the inertia to changes	1
x_c^-	accounting for the consumers' inertia to drop cards	0.05
x_c^+	account for the consumers' inertia to add new cards	2
x_m^-	account for the merchant' inertia to drop cards	0.05
x_m^+	account for the merchant' inertia to add new cards	9
α	account for the impact of the publicity cost	0.1
φ	account for the impact of the publicity cost	5

vector of all payment cards performance measures $(\phi_1, \ldots, \phi_{N_\mathcal{P}})$.
Before the learning function is carried out, the performance of the payment cards providers $(\phi_1, \ldots, \phi_{N_\mathcal{P}})$ are sorted (line 6) according to the profit Φ_p achieved at the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). The new vector is denoted $\vec{\phi}$. Following this step, the joint probability function $\mathbb{F}_\mathbb{S}$ is modified by a learning process (line 7). This task is accomplished considering the market share $N_{T_p^*}$ obtained in line 5. The function receives as parameters the current values of the joint probability distribution $\mathbb{F}_\mathbb{S}$, the profit based order of the performance $\vec{\phi}$ and the vector of strategies $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})$.
Finally, in line 7, the GPBIL algorithm returns the resulting joint probability distribution. This function is used as a probabilistic model to generate strategies that fulfill the two main objectives: to achieve above average market share with the highest possible profit.

4 The Setting of the Experiment

We have tested the application of the GPBIL for the $\mathcal{APCM}$ through rigorous experimentation. For that reason we have executed the process $\mathcal{MARKET} - \mathcal{GPBIL}$ in a loop over considerable number of runs R. In order to test the efficiency of the algorithm, we have specified three different cases of nine, five and two[8] competitors in a market with local connections among consumers and merchants. The fact that the competitors are trying to achieve higher than the average market share, effectively means that the aim of the payment card providers in the cases of 2, 5 and 9 cards are different. We have executed ten examples for each one of the three cases. In this paper we include the results of the case with nine competitors.

We have used the same setting for all cases[9]. We have assigned a number of

[8]The case of two competitors is aimed to find a strategy designed to obtain better than the average market share, without considering maximization of the profit

Table 2: Strategy's Domains

Symbol	Domains	Value
$\mathbb{D}_{\mathcal{F}_\mathbf{p}}$	Consumer Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{\Gamma_p}$	Merchant Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits	$[-1, 1]$
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits	$[-1, 1]$
$\mathbb{D}_{l_p}$	Publicity Cost's Domain	$[0, \infty]$

runs $R = 6000$ and a number of interaction $I = 3000$; the poisson distribution, used to determine the decision period of consumers and merchant, has a mean $\lambda = 20$. The sets of consumers $\mathcal{C}$ and merchants $\mathcal{M}$ are instantiate with $N_{\mathcal{C}} = 1100$ and $N_{\mathcal{M}} = 125$. The rest of the user defined parameters are divided in two tables. In table 1 we have listed the values of the constants, which impact the decisions of consumers and merchants. Finally in table 2 the domain of each element of the strategy space is presented.

5 Conclusions

In this section we present the main results and the conclusions related to the application of the GPBIL algorithm on the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. The aim of the implementation is to explore the areas of intersection between the demand at the point of sale of payment card instruments and the price level determined by the card issuers in order to find a profit-maximizing strategy of the payment providers.

In table 3 the final strategies, obtained for each one of the ten executions, are presented. The columns correspond to each element of the strategy: consumers fixed fees F_p, merchants fixed fees Γ_p, consumers benefits b_p, merchants benefits β_p and publicity cost l_p. We can see from the results that the algorithm has reached the same price level on the merchants side (columns 2 and 4). In the case of the consumers (columns 1 and 3) in eight of ten execution the price level is similar. The mayor difference between the price structure of the card holders' and the sellers' consists on the fixed fees. We observe that the merchants do not pay any fixed fees ($\Gamma_p = 0$), whereas the consumers' side is charged with high fixed fees (on average $\mathcal{F}_p > 5.00$), considering the domain of the variable.

Furthermore in table 4 we make a comparison between the average profit

[9]For more details of the results of the other cases please consult

Table 3: Final Strategies

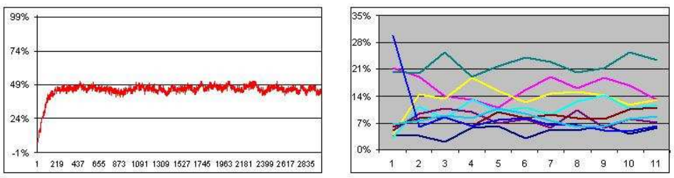
F_p	Γ_p	b_p	β_p	l_p
7.57	**0.00**	**-1.00**	**-1.00**	**11.11**
5.33	**0.00**	**-1.00**	**-1.00**	**7.66**
3.51	**0.00**	**1.00**	**-1.00**	**11.81**
6.03	**0.00**	**0.48**	**-1.00**	**11.82**
5.46	**0.00**	**-1.00**	**-1.00**	**10.49**
6.03	**0.00**	**-1.00**	**-1.00**	**13.85**
5.98	**0.00**	**-1.00**	**-1.00**	**8.39**
6.48	**0.00**	**-1.00**	**-1.00**	**9.97**
5.38	**0.00**	**-1.00**	**-1.00**	**10.24**
5.66	**0.00**	**-1.00**	**-1.00**	**10.82**

Table 4: Initial and final profit of the competitors

Average profit in the first 100 runs	**Average profit in the last 100 runs**
4.092.329,50	**5.179.486,23**

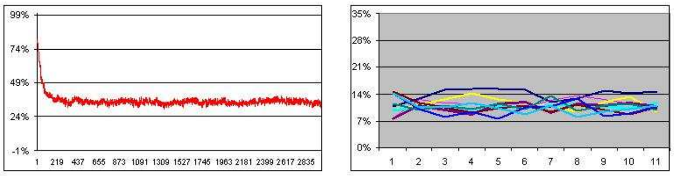
obtained by the competitors in the first 100 runs with the average profit obtained in the last 100 runs. We can see from the results that there is a significant improvement in the profit of the payment cards providers.

In addition, in figures 2 and 3 we present a comparison between the cash and cards transactions in the initial and the final runs. We can see in figure 2 that the cash transactions has slightly decreased after the learning process. Whereas, with respect of the cards transactions, we observe in figure 3 that the competitors are getting closer to the average transactions in the accomplishment of one of the GPBIL's requirements.
In general, given the complex shape of the aggregated end-users' demand



(a) Cash transactions (b) Cards transactions

Figure 2: First run



(a) Cash transactions (b) Cards transactions

Figure 3: Last run

for electronic payment methods, explicitly modeled in the artificial market, we can say that the GPBIL algorithm has found a price structure and price level that maximize the profit of the card purveyors and has successfully fulfill the main objectives of the search. Due to the relevance of the market and the complex relationships among the market participants, we consider that the use of agent-based models will allow us to gain better understanding of the payment card market. More the all so, we conclude that applying evolutionary techniques, as the GPBIL, in studying relevant aspect of this market opens a new research opportunities, untractable with the analytical models.

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