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Abstract

We analyze the impact of the real-estate/mortgage crisis on the dependence between the market for common stocks and returns on Real Estate Investment Trusts (REIT's), using a flexible mixed-copula approach. We find that the impact of the crisis on the levels of the tail dependence is very different from the impact on the values of the linear correlations. For this asset class all correlations are lower in the post-crisis period, whereas all other correlations have increased. In contrast, only the tail dependence values between the different REIT's indices seem to be impacted by the crisis, with the level of the tail dependence between each of the different REIT's indices and the stock market being less affected. That is, looking at the correlations the effect of the crisis appears to be a weakening of the connection between residential mortgage REIT's and the rest of the financial market, whereas the effect on the tail dependence suggest that the crisis mainly has an intra-REIT's effect.

Key Words: Real-estate, asymmetric tails, extreme-value dependence, copulas, semiparametric estimation.

JEL Classifications: $C13, C22, G22$

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1 Introduction

The connection between securitised real estate and stock markets has been analyzed by several authors, see for instance Knight *et al.* (2005) and Westerheide (2006, and references therein. In particular, the characteristics of Real Estate Investment Trusts (REIT's) have been the subject of several studies. These instruments allow investors to invest in real estate without suffering from its main disadvantage - its illiquidity. REIT's can invest in either actual real estate or montages and mortgage products, giving rise to equity REIT's and mortgage REIT's respectively. There also exists a smaller class of hybrid REIT's, investing in both asset classes. Unsecuritised real estate is usually seen as protection against stock market downturns. This has sparked a series of studies on the type of linkage there exists between securitised real estate and the stock market. In an early study, Ling and Naranjo (1999) find that the securitised commercial real estate market is integrated with the stock market. More recently, Westerheide (2006) tests for cointegration between REIT's on the one hand and stock markets and bond markets on the other for different countries. The results indicate that REIT's form an asset class on their own, distinct from both stocks and bonds.

Real estate and real estate securities are often seen as a protection during market downturns. Knight *et al.* (2005) test this hypothesis by estimating the tail dependence between a REIT's index and a stock market index, both for a UK data set and for a global one. They find strong tail dependence between common-stock and REIT's returns. These results are in line with those of Goldstein and Nelling (1999) who, using US data, find that equity betas for REIT's are higher in bear stock markets than in up markets. In both cases negative as well as positive tail dependence is found.

This paper focuses on the tail dependence between returns on different REIT's indices on the one hand and the common stock market on the other in the US. In particular, we compare the level of tail dependence before the bursting of the real estate/mortgage bubble, with the levels of tail dependence in the market since the bubble started to burst in early 2007. We estimate the upper and lower tail coefficients of the dependencies between the S&P 500 market index and three different REIT's indices: equity REIT's, retail mortgage REIT's and non-retail mortgage REIT's, both before and after the outbreak of the crisis. We compare whether the bursting of the bubble has had a different impact on the tail dependence with the S&P 500 index for the different REIT's indices. As the recent credit crisis resulted from the bursting of a real estate/mortgage bubble in the residential real estate segment, one might expect that the impact would be the strongest for the retail mortgage REIT's. In this letter we show that the main impact of the crisis on the tail dependence between the different indices is a lowering of the lower tail dependence coefficients between the different REIT's indices, a result very different from the impact of the crisis on the linear correlations.

The paper is structured as follows. Section 2 briefly introduces the concept of copulas as well as model estimation and diagnostics. Section 3 presents the data and empirical results. Section 4 concludes.

2 The Copula Concept

Consider two random variables X and Y with continuous univariate distribution functions $F_X(x) = P(X \leq x)$ and $F_Y(y) = P(Y \leq y)$ and their joint distribution function $F_{X,Y}(x,y) = P(X \leq x, Y \leq y)$. Sklar (1959) states that there exists a function called copula C that connects the univariate distributions F_X and F_Y to a bivariate distribution function

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)) \quad . \tag{1}$$

The copula C is the bivariate joint distribution function of the transformed random variables $U = F_X(X)$ and $V = F_Y(Y)$, i.e.

$$C(u,v) = P(U \leq u, V \leq v) \quad . \tag{2}$$

In this paper, we will not consider single parametric copulas, but a mixed copula to obtain a better fit of the dependence patterns within the data as the mixture will allow for asymmetric tail dependence (which is not possible when applying the common t-Copula). From Nelsen (1999) it is well known, that any convex linear combination of copulas is also a copula. In particular, we are looking at a mixture of a Gumbel-Copula

$$C_G(u,v;\theta_G) = \exp\left\{-\left[(-\ln u)^{\theta_G} + (-\ln v)^{\theta_G}\right]^{\frac{1}{\theta_G}}\right\}$$

(to account for upper tail dependence), a Clayton-Copula

$$C_C(u,v;\theta_C) = \left(u^{-\theta_C} + v^{-\theta_C} - 1\right)^{-1/\theta_C}$$

(to account for lower tail dependence) and a Frank-Copula

$$C_F(u,v;\theta_F) = -\frac{1}{\theta_F}\ln\left(1 + \frac{\left(e^{-u\theta_F} - 1\right)\left(e^{-v\theta_F} - 1\right)}{\left(e^{-\theta_F} - 1\right)}\right)$$

(to consider the case of no tail dependence), yielding

$$C(u,v;\ \theta) = w_G C_G + w_C C_C + (1 - w_G - w_C) C_F$$

with $w_G, w_C > 0$, $w_G + w_C \leq 1$ and $\theta = (\theta_G, \theta_C, \theta_F, w_G, w_C)$. The copula density and the tail dependence coefficients can be easily obtained by deriving the Copula function respectively. The parameters $\theta_{(.)}$ and $w_{(.)}$ play different roles in the mixed copula. Typically, $\theta_{(.)}$ as an association parameter controls the degree of dependence, whereas $w_{(.)}$ as the weighting parameter of the copula controls the structure of the dependence function. (For more discussions on the theory of copulas and specific examples, we refer the reader to the textbooks of Joe (1997) and Nelsen (1999).)

For the estimation of the mixed copula model we adopt the canonical maximum likelihood method (see Cherubini *et al.* (2004)). Our primary interest lies in the dependence function itself, so we do not specify a particular parametric form for the marginals, avoiding misspecification and overfitting of the model, and estimate only the copula. Now let X and Y denote two different return series. The semiparametric estimation is performed in two stages. In a first step we estimate the marginal distributions nonparametrically (n) using the empirical distribution

$$\hat{F}_n(x) = \frac{1}{N+1} \sum_{k=1}^{N} \mathbf{1} \quad ,$$

respectively for Y. The copula can now be written in the form

$$F(x_i, y_i; \theta) = C\left(\hat{F}_n(x_i), \hat{F}_n(y_i); \theta\right)$$

and the density of an observation (x_i, y_i) is

$$f(x_i, y_i; \theta) = c\left(\hat{F}_n(x_i), \hat{F}_n(y_i); \theta\right) \cdot \hat{f}_n(x_i) \cdot \hat{f}_n(y_i) \quad .$$

In a second step we then estimate the copula parameter vector θ by maximizing a log-likelihood function

$$\hat{\theta} = \arg\max_{\theta} \sum_{j=1}^{N} \ln c\left(\hat{F}_n(x_i), \hat{F}_n(y_i); \theta\right) \quad ,$$

yielding the maximum likelihood estimator $\hat{\theta}$, which is consistent and asymptotically normally distributed (see Genest *et al.* (1995) and Chen and Fan (2006)).

In order to check the goodness-of-fit of the estimated mixed copulas, we apply the probability integral transform approach similar to the density forecasts evaluation introduced by Diebold et al. (1998), and, later suggested by Embrechts et al. (2003) for application on copulas. Considering the conditional distribution $Z_i = C(F_X(X) \mid F_Y(Y))$, if $(F_X(X), F_Y(Y))$ has the joint distribution C, then $\Phi^{-1}(Z_i)$ are i.i.d. normally distributed. Applying their approach, the transform

$$S(X, Y) = (\Phi^{-1}(F_Y(Y))) + (\Phi^{-1}(C(F_X(X) \mid F_Y(Y))))$$

will follow a χ^2-distribution with two degrees of freedom under the correct model specification . Hence, we check the goodness-of-fit of the distribution of $S(X, Y)$ using nonparametric tests, like those of Kolmogorov-Smirnov (KS), Cramér-von-Mises (CM) and Anderson-Darling (AD), and calculating their respective p-values.

3 Data and Empirical results

We use daily data from December 12, 2004 until June 30 2008, leading to a total of 852 days on which we observe the returns on the different REIT's indices and the S&P 500 index. We use three Dow Jones REIT's indices: the DJ Equity REIT Index, the DJ

Table 1: Descriptive statistics of daily returns

Series	Total Period		Until 31.01.2007		From 01.02.2007	
	Mean	Std	Mean	Std	Mean	Std
S&P 500	0.01%	0.89%	0.04%	0.61%	-0.04%	1.17%
Commercial Mortgages	-0.13%	1.87%	0.00%	1.01%	-0.31%	2.64%
Residential Mortgages	-0.15%	2.05%	-0.09%	1.24%	-0.23%	2.84%
Equity Reits	0.01%	1.41%	0.07%	0.96%	-0.09%	1.88%

Residential Mortgage REIT Index and the DJ Commercial Mortgage Index. The first index covers all the different types of equity REIT's, i.e. REIT's that exclusively invest in actual real estate. The other two indices cover the two segments of the mortgage REIT's. The first one invests exclusively in residential mortgages, and related mortgage products. The latter invests in non-household mortgages, and related products.

Table 1 gives the mean and standard deviations for the daily returns for two different periods. The first period starts on December 12, 2004 and finishes on January 31, 2007. The second periods starts on February 1, 2007, and ends on June 30, 2008. We see that the returns in the second period are much lower, and negative, than in the first period, whereas the standard deviations have all increased dramatically. Both features illustrate that the second period is one of turmoil for the financial markets.

Table 2 gives the correlations between the different indices. All correlation coefficients are positive. Comparing the period before the bursting of the real-estate bubble with the second one, we find that, as expected, correlation levels are distinctly higher in the second period than in the first, except for those involving residential mortgage REIT's. The latter showing much lower levels of correlation with the other asset classes since the real estate bubble started to burst in early 2007. These results suggest that there is a weakening in the dependence between residential montage REIT's and the rest of the financial market. However, the common correlation coefficient only measures the general linear (symmetric) dependence.

The scatter plots of Figure 1 and Figure 2 show again the impact of the crisis. We see that in the post-crisis period the scatter plots involving the residential mortgages REIT's are very far from the first diagonal. In fact, these plots suggest values for the corresponding correlations in the second period still lower than those in Table 2. However, taking a closer look at Figure 2, we see that there are still observations in the noth-west and south-east corners of the plots involving the residential mortgages REIT's, leading to higher levels for the corresponding correlations than one would expect upon a first glance at the figure. In the next section we investigate whether a similar phenomenon can be observed for dependence far in the tails.

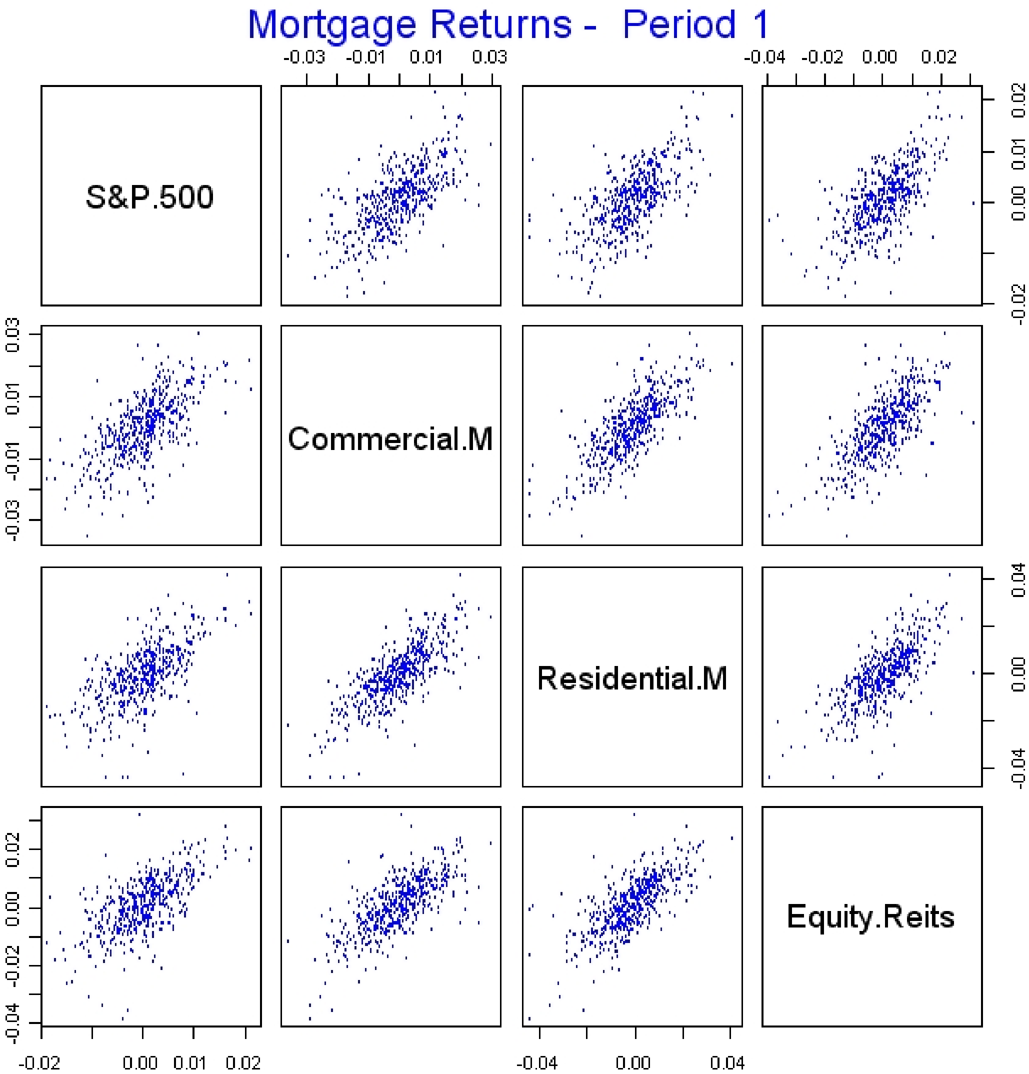

Figure 1: Scatterplot of Returns for Period 1

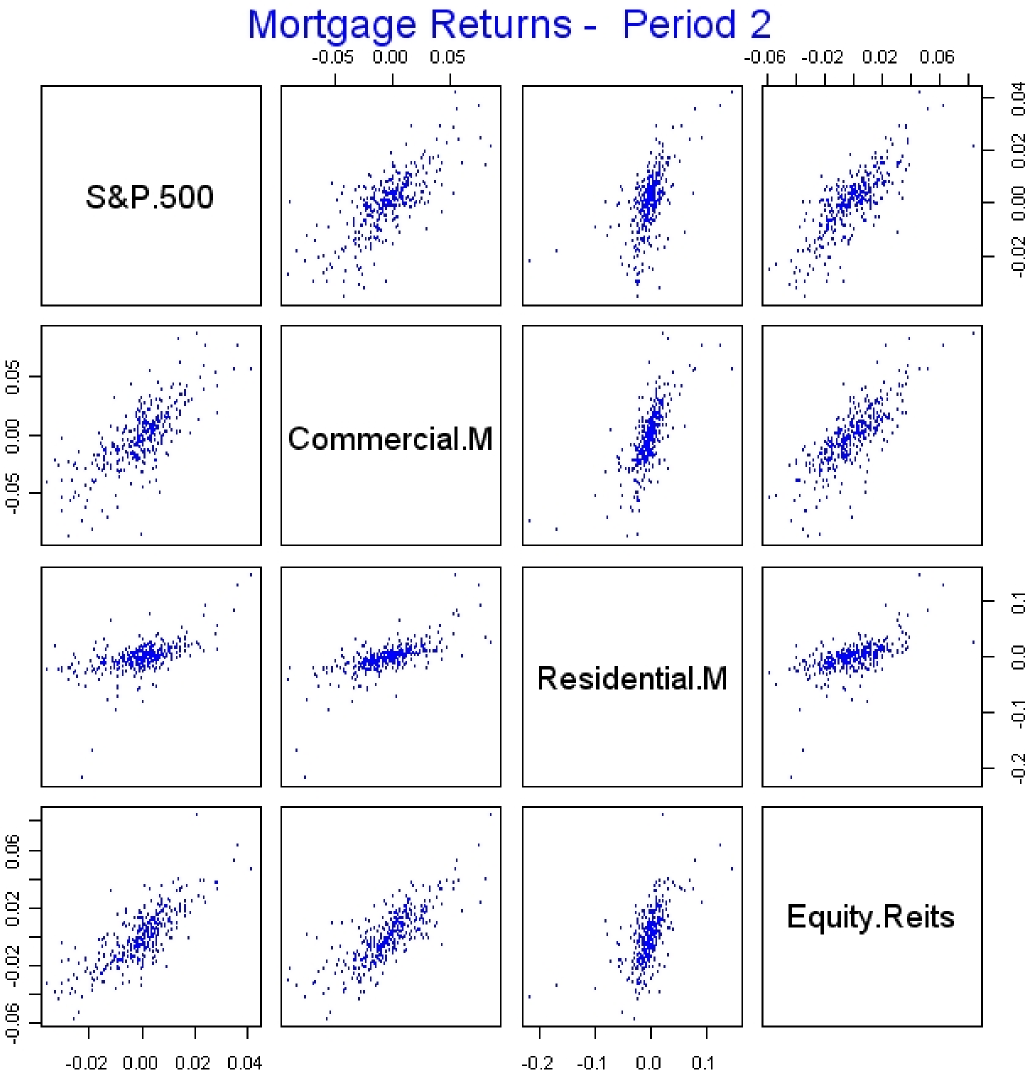

Figure 2: Scatterplot of Returns for Period 2

Table 2: Correlations between the daily returns

Total Period				
	S&P 500	Comm. Mortg.	Resid. Mortg.	Equity REITS
S&P 500	100.00%	70.32%	57.63%	74.50%
Comm. Mortg.	70.32%	100.00%	70.24%	75.97%
Resid. Mortg.	57.63%	70.24%	100.00%	61.99%
Equity	74.50%	75.97%	61.99%	100.00%
Until January 31, 2007				
	S&P 500	Comm. Mortg.	Resid. Mortg.	Equity REITS
S&P 500	100.00%	66.09%	63.89%	64.20%
Comm. Mortg.	66.09%	100.00%	76.42%	70.92%
Resid. Mortg.	63.89%	76.42%	100.00%	71.60%
Equity	64.20%	70.92%	71.60%	100.00%
From February 1, 2007				
	S&P 500	Comm. Mortg.	Resid. Mortg.	Equity REITS
S&P 500	100.00%	72.22%	55.74%	78.41%
Comm. Mortg.	72.22%	100.00%	68.87%	77.96%
Resid. Mortg.	55.74%	68.87%	100.00%	59.05%
Equity	78.41%	77.96%	59.05%	100.00%

Table 3: Estimated parameters of the mixed copula

	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2	
S&P 500	1.8027 (0.1775)	1.8638 (0.1524)	1.7921 (0.1363)	1.8528 (0.1196)	1.8165 (0.1044)	2.2267 (0.1560)	θ_G
	1.6113 (0.7073)	2.9632 (2.2891)	1.3888 (2.6359)	2.9065 (1.4973)	2.0737 (0.7881)	6.2735 (2.0597)	θ_C
	6.5228 (1.7573)	20.0492 (9.2848)	6.5953 (2.7716)	-7.7504 (9.3903)	12.9314 (38.4830)	-16.8708 (14.3655)	θ_F
	0.4845 (0.1574)	0.6867 (0.13886)	0.7533 (0.1496)	0.8714 (0.0867)	0.8383 (0.1065)	0.7503 (0.0776)	w_G
	0.1707 (0.1046)	0.1650 (0.0986)	0.0337 (0.1082)	0.0882 (0.0787)	0.1582 (0.0823)	0.2351 (0.0743)	w_C
		Commercial M.	2.2328 (0.1775)	2.3741 (0.2823)	2.2257 (0.1750)	2.3920 (0.2260)	θ_G
			2.3226 (0.4380)	0.8461 (0.3461)	1.9831 (0.3578)	2.0673 (0.9421)	θ_C
			10.0402 (5.3503)	42.2448 (24.5144)	-3.3938 (10.4353)	12.0105 (5.1093)	θ_F
			0.5438 (0.1279)	0.6450 (0.1030)	0.5399 (0.0763)	0.6836 (0.1391)	w_G
			0.3365 (0.0878)	0.2511 (0.0958)	0.4521 (0.0786)	0.1308 (0.0898)	w_C
				Residential M.	2.4230 (0.3396)	1.6743 (0.1189)	θ_G
					2.2595 (1.0477)	0.0001 (0.0000)	θ_C
					4.6352 (1.7625)	10.9918 (3.6367)	θ_F
					0.5585 (0.1483)	0.7891 (0.1302)	w_G
					0.2131 (0.1006)	0.0001 (0.0000)	w_C
						Equity REITS	

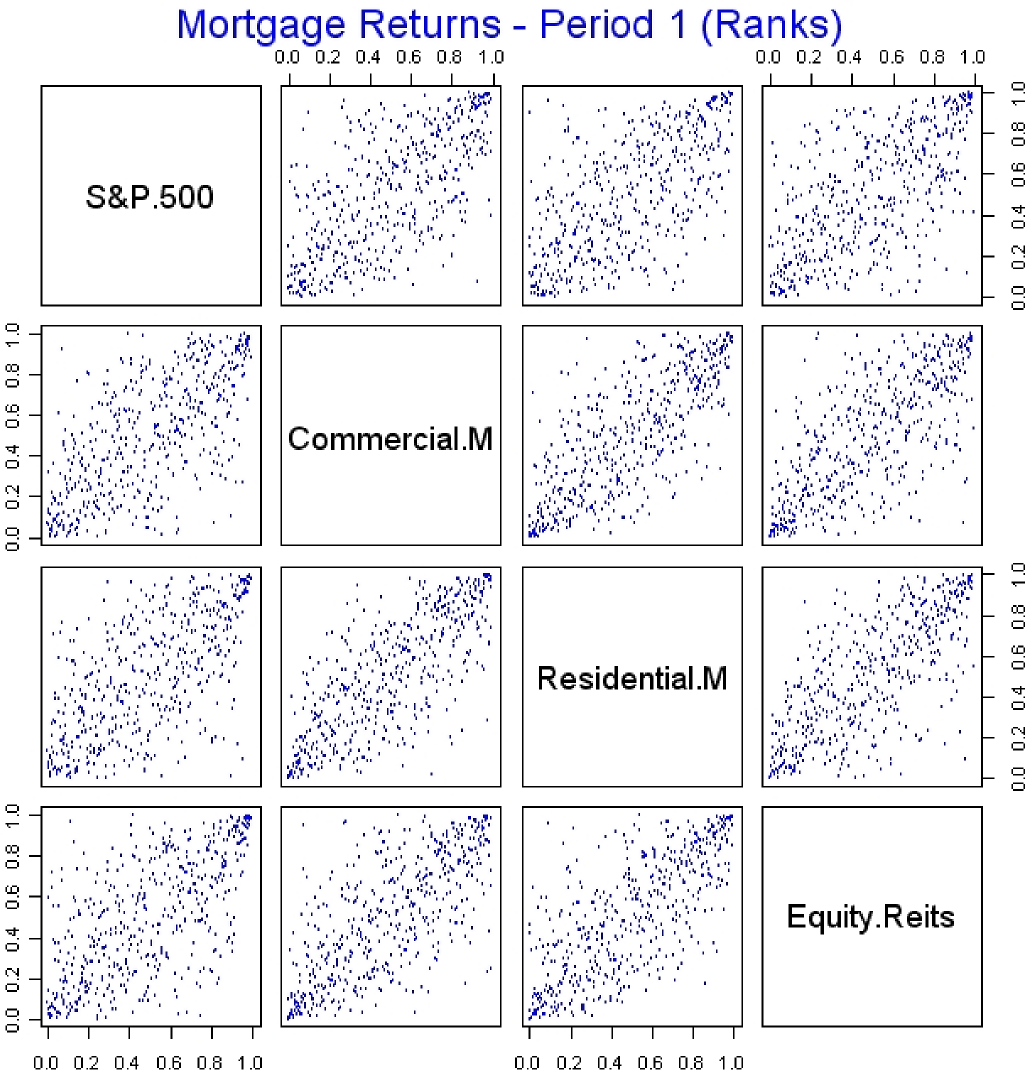

Figure 3: Scatterplot of Ranks of Returns for Period 1

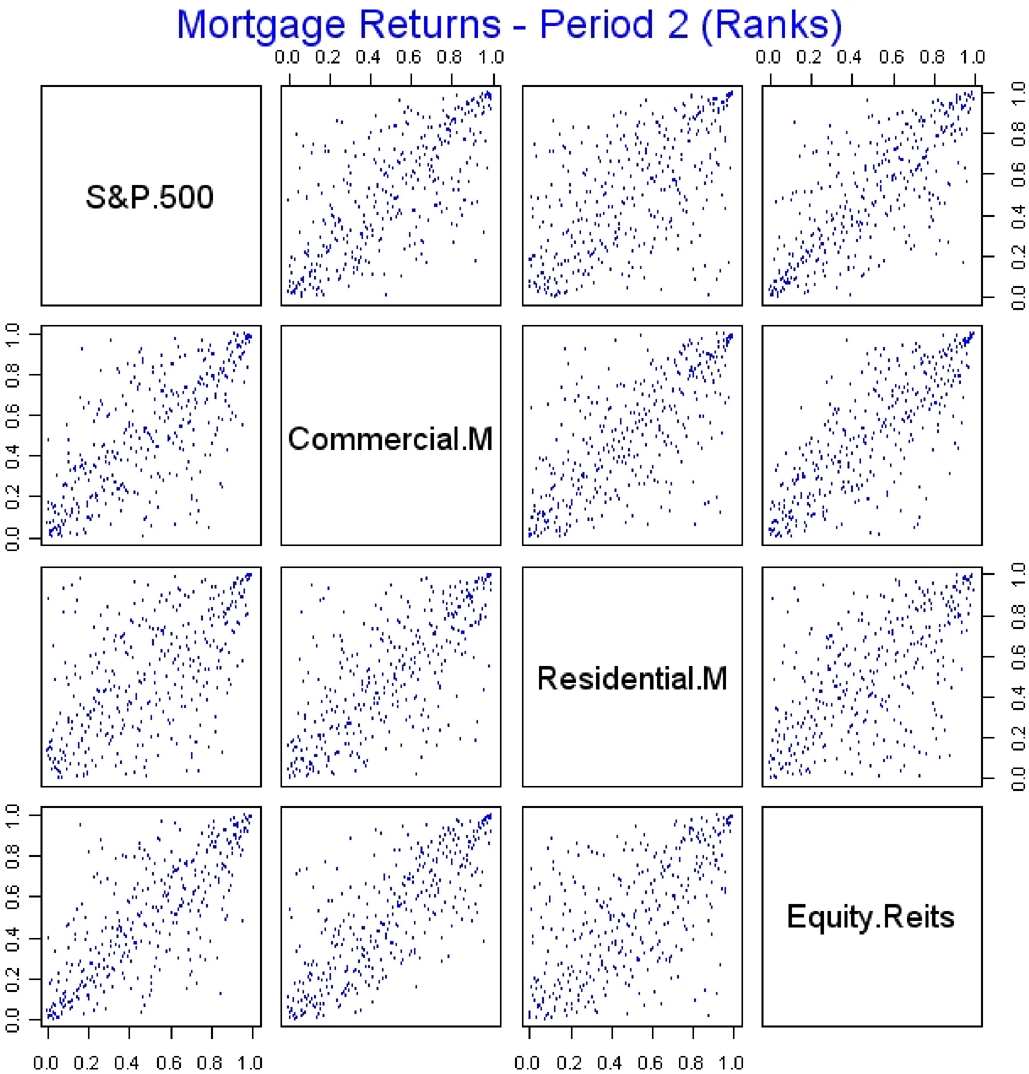

Figure 4: Scatterplot of Ranks of Returns for Period 2

Table 4: Upper and lower tail dependence coefficient implied by the estimated copula.

	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2	
	0.2573	0.3773	0.3975	0.4760	0.4488	0.4763	*up*
S&P 500	0.1110	0.1306	0.0204	0.0695	0.1133	0.2105	*low*
			0.3459	0.4263	0.3426	0.4538	*up*
		Commercial M.	0.2497	0.1106	0.3187	0.0935	*low*
					0.3735	0.3844	*up*
				Residential M.	0.1568	0.0000	*low*
						Equity REITS	

Table 3 shows the estimated parameters of the mixed copula for both periods (with the standard errors in brackets). At a first glance, it is discernible that the weights for the Gumbel Copula is always higher than for the Clayton, $w_G > w_F$. However, a general clear picture is not visible as the tail dependent coefficients in a mixed copula is not only controlled by the Copula parameters θ_G and θ_C but also by the respective Copula weights w_G and w_C. In contrast, Table 4 gives the values for the lower and upper tail dependence coefficients implied by the calibrated Copula (considering both weights and Copula parameter). For the upper tail dependence all values have increased from the first period to the second, whereas the lower tail dependence values show changes in two directions. All lower tail dependence coefficients related to S&P 500 have increased, while the remaining coefficients for tail dependencies between REITS mortgages have decreased. This finding is very different from the one found for the correlations in Table 2. In the latter case, all correlations are higher in the second period, except for those involving the residential mortgages REIT's, which are all lower. We do not find a similar effect here.

As visible in Table 4, in both periods all the lower tail dependence coefficients for REIT's-stock market combinations are significantly lower than the corresponding upper tail dependence coefficients. Such a level asymmetry is rather unusual for financial asset, see for instance Poon *et al.* (2004), and Hartmann *et al.* (2004). This result implies that REIT's might indeed provide some protection during stock market downturns. A different picture emerges for the lower tail dependence coefficients for the intra-REIT's combinations. In the first period, these values are quite close to those for the corresponding upper tail dependence coefficients, whereas in the second period they are significantly lower, more in line with the levels for the REIT's-stock market combinations. We find that, in contrast to the results for the correlations, the crisis mainly seems to affect the tail dependence between RET's indices.

Table 5: Goodness-of-fit of the mixed copula.

	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2	
	148.46	138.37	145.72	94.09	149.03	177.02	LL
	−286.92	−266.75	−281.44	−178.18	−288.06	−344.05	AIC
S&P 500	−265.85	−247.44	−260.36	−158.88	−266.98	−324.75	BIC
	0.86	0.87	0.69	0.98	0.90	0.38	KS
	0.71	0.92	0.66	0.96	0.87	0.39	AD
	0.58	0.29	0.96	0.31	0.70	0.71	CM
			223.81	144.22	189.42	178.10	LL
			−437.63	−278.44	−368.85	−346.21	AIC
Commercial M.			−416.56	−259.14	−347.78	−326.91	BIC
			0.71	0.96	0.93	0.83	KS
			0.50	0.98	0.94	0.87	AD
			0.78	0.97	0.79	0.33	CM
					200.14	101.06	LL
					−390.28	−192.12	AIC
					−369.21	−172.82	BIC
Residential M.					0.99	0.93	KS
					0.89	0.99	AD
					0.99	0.70	CM
					Equity REITS		

Finally, Table 5 presents the goodness-of-fit measures for all estimated copulas. For each estimated mixed copula, the table lists the log-likelihood value (LL), AIC, BIC, as well as the p-values of the Kolmogoroff-Smirnoff test (KS), the Anderson-Darling test (AD) and the Cramer-von-Mises test (CM). Since all p-values are higher than the significance level of 10%, the null hypothesis that the distribution of the probability transform $S(X, Y)$ is χ^2_2 can not be rejected, implying that the estimated copula models are correctly specified.

4 Conclusions

We analyzed the impact of the real-estate/mortgage crisis on the dependence between the market for common stocks and REIT's. We find that the impact of the crisis on the tail dependence is very different from the impact on the values of the linear correlations. In the first case the effect is mainly restricted to tail dependence values between the different REIT's indices, whereas in the latter the impact is restricted to residential mortgage REIT's. Put differently, the impact on the correlations suggests that the crisis leads to a weakening of the dependence between on the one hand the residential mortgage REIT's and on the other the rest of of the financial markets, whereas the impact on the levels of the tail dependence seems to indicate that the crisis mainly effects the dependence between the different REIT's indices.

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