Centre for Computational Finance and Economic Agents

WP069-13

Working Paper Series

Han Ao and Edward Tsang

Capturing Market Movements with Directional Changes

2013

University of Essex

CCFEA

www.ccfea.net

Capturing Market Movements with Directional Changes

Han Ao, Edward Tsang
Centre for Computational Finance and Economics Agents
Working Paper

June 21, 2013

Abstract

Every trade in the market is recorded. To study market movements, information of trading (such as price) is summarised. Most analysts summarise market movements with periodic sampling of data, such as daily closing prices.

1 Introduction

Financial data is in high quality, as it is recorded at trading. However, as a result, the quantity of the data tends to be large. Therefore, although information of every trading is recorded, it is not practically accessible until recent years with the developments of computer science. For example, in foreign exchange markets, not until the beginning of the 1990s, intra-daily data have been broadly studied, while daily data is very much used in 80s. The later represents only a very small subset of information available intra-daily, and the size of the former is 100 to 1000 times larger than daily data [1].

Even though, the daily data still does not use every piece of information that recorded. As a result, only tick-by-tick data contains every information, which we call raw data in the rest of this paper. And a fact is that except tick-by-tick data, all other data used in our daily research is somehow summarised from tick-by-tick data. Therefore, in this sense, except using tick-by-tick data, the ways of summarising become crucial. A good summary should reflect the information of raw data as much as possible.

A common way to summarise raw data is to first choose a time interval, and then sample raw data at fixed time points with the chosen interval; for example, hourly, daily or monthly. We call data summarised this way "interval-based summary". Naturally, an interval-based summary becomes a time series. In such a summary, the time interval is the arbitrarily chosen parameter, and the amplitude of the change of price is variable [1]. And based on the summary, analyses can be performed, and our established knowledge is very much built

on it. For example, one might describe the trend or volatility in the last n days [2][1]

A possible explanation of why interval-based summarising becomes the prevailing way would be this: Before tick-by-tick data became available, the daily or hourly even second-by-second quotations were the most accurate data available. Knowing them is seen as knowing all information available. Later, as a convention, when trading became more frequent, data are still summarised in such ways.

Although raw data is not necessary to be summarised; in finance, each trade is recorded, this includes the price, volume and the time that a trade occurs. As a result, the amount of data collected is potentially large. To prevent combinatorial explosion [3], these recorded data, or raw data, are usually not directly used, but are normally sampled into summaries. So that a comparatively smaller number of data are used for analyses.

Guillaume *et al.* [1] introduced an alternative way to summarise raw data. In this approach, one summarises raw data by "directional change event". In this algorithm, compare to interval-based summarising, the change of price is fixed and time is the varying parameter. Briefly, a directional change is an event during which price momentum changes the direction - from upward to downward or vice versa. In this paper, we recapitulate the formal definition of DC, and evaluate its appropriateness in capturing market dynamics.

The remainder of this paper is organised as follows: section 2 gives definitions of interval-based and Directional Change event based summary. Section 3 examines potential use of Directional Change algorithm. Section 4 introduces existing researches based on Directional Changes. Section 5 discusses our main views. Section 6 concludes this paper.

2 Summarising Raw Data

Raw data can be summarised in many ways. However, in this paper, we focus on a traditional ways – the interval-based summary and an alternative event based summary – what we called the Directional Change event-based summary or DC-based summaries in short.

2.1 Interval-based Summary of Transactions

As mentioned above, financial data (See figure 1, graph *a* for raw data) is often summarised using fixed time intervals. In other word, it is sampled with regular observation frequencies (intervals are as shown in figure 1 graph *d*). Samples collected this way are called interval-based summaries (see figure 1, graph *b* and *c*), i.e. time series. For example, using 400-business-day's daily[2] (figure 1 graph *a*) closing price data (from 06/07/2011 to 01/02/2013) of HSBC as raw

[1] A time series is a collection of observations indexed by the date of each observation, pp.25

[2] Although a daily data is already an interval-based summary, for simplicity, we use daily data as raw data to illustrate the concept.

data. With a monthly sampling, we can have an interval-based summary of 22 observations (21 intervals, shown in figure 1 graph *d*).

2.2 Using Directional Change Events to Summarise Transactions

Instead of summarising transactions with a chosen time interval, we can summarise them by events. A DC-based summary is a summary of raw data sampled at each Directional Change event. Although, there are many ways of defining events; in this paper, we focus on using one specific type of events defined by Guillaume *et al* [1], namely Directional Changes (or DC for short).

In order to understand what a DC-based summary is, it is necessary to first introduce the concept of "Directional Change".

Tsang [4] formally defined Directional Changes (See appendix 1 for a more detailed definition). Briefly, a Directional Change is an event, at which the current momentum (or direction) of price changes[3]. Obviously, there are two types of Directional Change events, "Upturn Event" and "Downturn Event" [4]. However, not every change in directions is called a Directional Change event. Instead, only when the price changes a certain rate in the opposite direction is called a Directional Change event. This certain rate is pre-determined, called a threshold.

A Directional Change Event is usually followed by an overshoot event. When the change of price reaches the threshold, then a Directional Change Event is confirmed. However, usually, the price would not start another Directional Change Event immediately but continuously goes in the same trend until another Directional Change Event is confirmed.

After understanding what a Directional Change event is, a DC-based summary is possible to be explained. A DC-based summary of raw data is a summary resulted by sampling raw data at each Directional Change event with a certain threshold.

A DC-based summary is depicted in figure 2, graph *a* shows the raw data, which is the same as in figure 1. Graph *b* and *c* show the DC-based summary of the raw data, and graph *d* gives the intervals defined by Directional Changes. It is also shown in graph *d* (vertical lines) that the interval widths are not fixed.

In figure 2, from graph *a* to *d*, it shows how Directional Changes summarise raw data (graph *a*) into a DC-based summary (graph *c* to *d*). And in graph *d*, blue vertical lines represent sample points. For comparison, in figure 3, graph *a* and *b* show the Interval-based summary and DC-based summary respectively. And in graph *c*, the dramatic difference between them is shown. A more detailed analysis is shown in next section.

[3] In terms of price, there are only two directions; which are up and down.

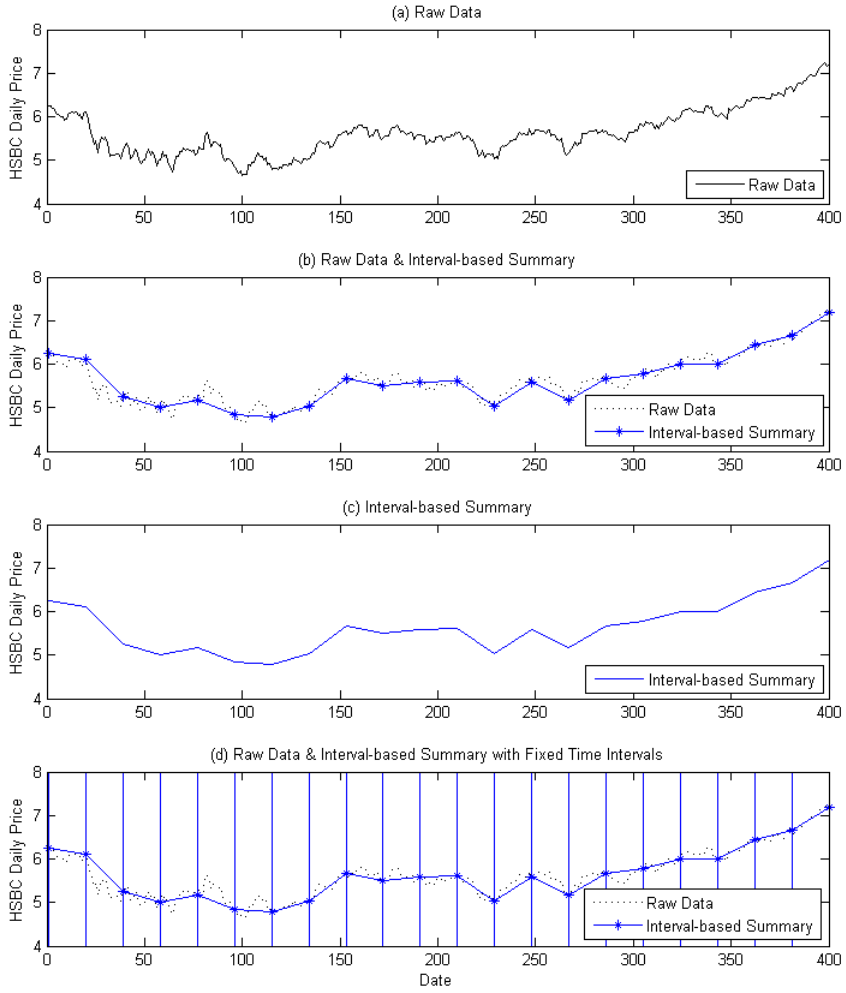

Figure 1. Summarising Raw Data
Interval-based summary of HSBC stock daily price (400 business days from 06/07/2011 to 01/02/2013). Blue curves is the interval-based summary of the original price curve (the black curve, chart a*). For simplicity, we use daily closing prices as raw data, and contrast them with monthly (20 days) interval-based summary*

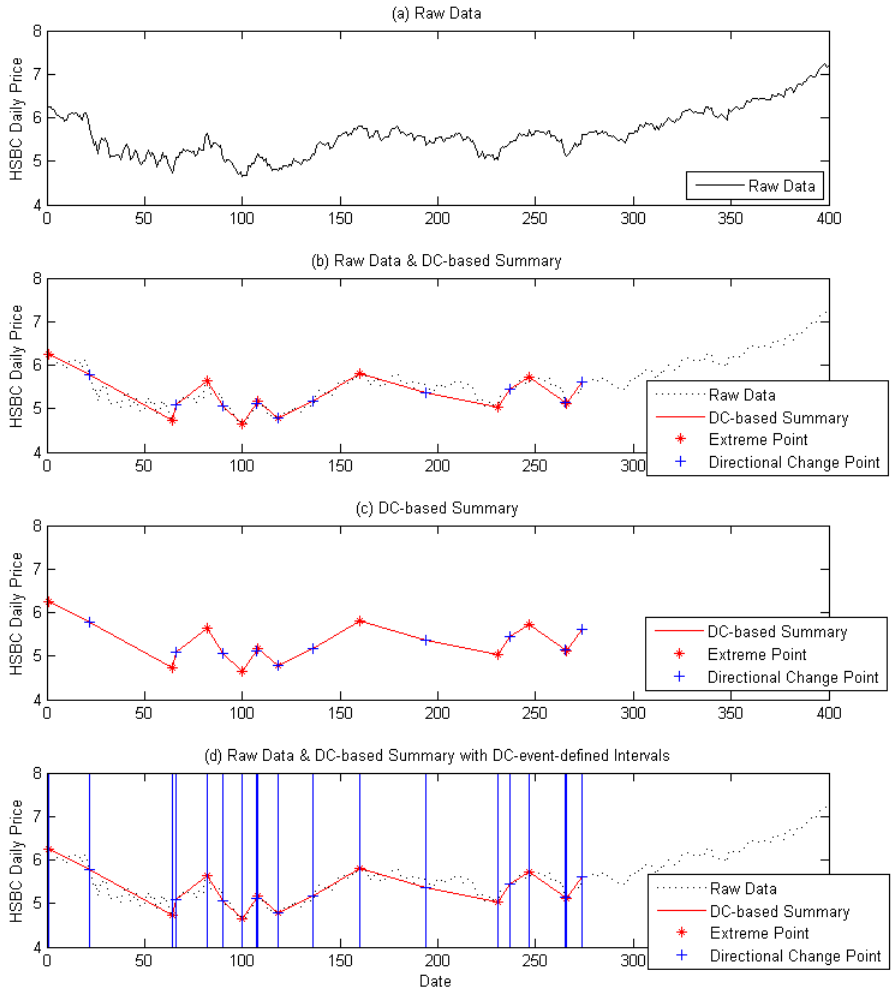

Figure 2. DC-based summary of HSBC stock daily price (400 business days from 06/07/2011 to 01/02/2013, red curve), under threshold of 0.075 (20 observations). Like figure 1, for simplicity, we use daily closing prices as raw data

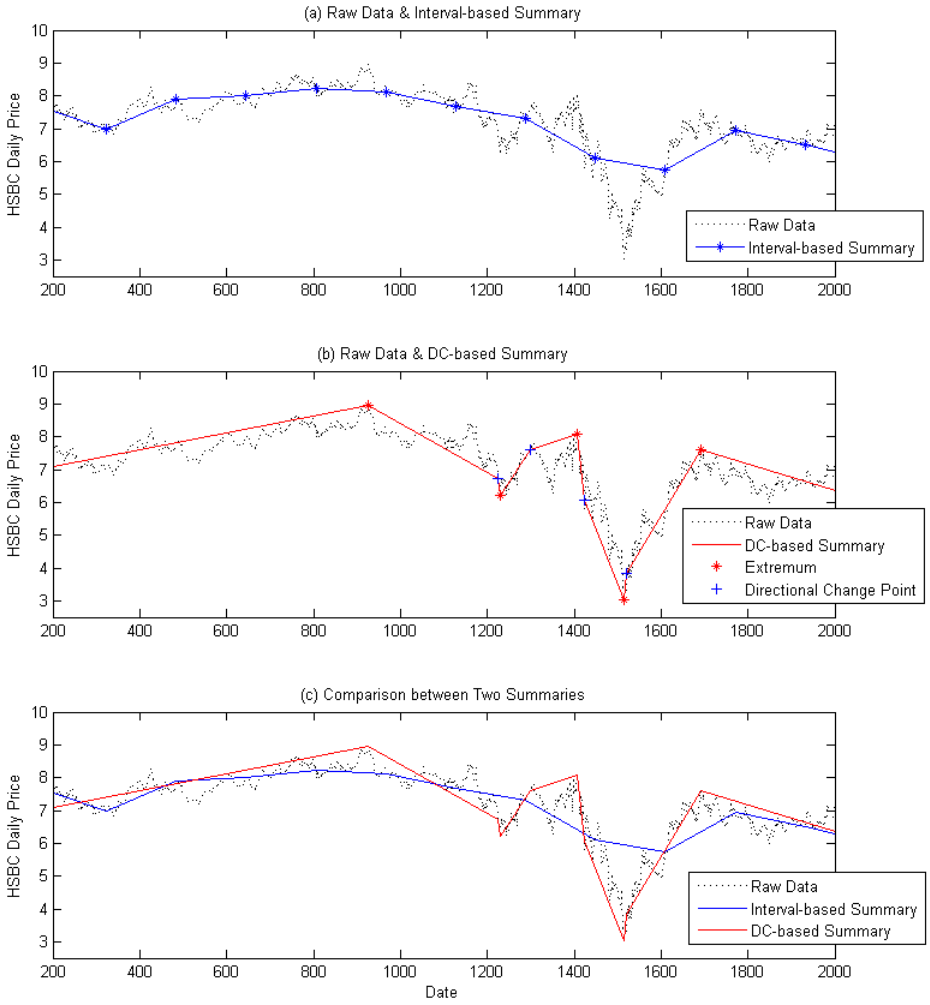

Figure 3. Comparison between interval-based and DC-based summaries (HSBC stock daily price from 24/12/2003 to 16/02/2009, under threshold 0.22) In this figure, red curves are DC-based summaries and blue curves are interval-based summaries. As can be seen in the figure, extreme are missed by the Interval-based Summary but captured by the DC-based Summary.

3 The Value of DC-based Summaries

3.1 DC-based Summaries Focus on Periods That Matter More

As an alternative of summarising raw data, DC-based summarising deals data in a different way. Whenever a change of price reaches the threshold, Directional Changes capture it. In contrast, interval-based summary records only at pre-determined (such as hourly, daily or second-by-second etc.) time points. Therefore, using Directional Changes means more data in periods with violently changing prices, fewer data in calm periods. For example, price P_1 starts to increase at t_1. Then at t_2, P_2 becomes $p + \Delta p$, and it starts to decrease. After, when at t_3, price P_3 becomes p again (becomes the same as at t_1, i.e. $p_1 = p_3$). Assume that t_1 and t_3 are sampling time points of an interval-based summary; the observation result would be no-change. But if the threshold T determined is smaller than p (this means that one actually cares changes that are bigger than T), then this move of price is definitely captured by Directional Changes.

For example, in figure 4, there are six extreme points marked as significant move — A, B, C, D, E and F. It appears that only A' is partly captured by the Interval-based Summary (not exact captures the peak point, but a sub-peak). As interval-based summaries sample at fixed points, it has a certain chance to capture peaks as A. In other word, no matter how big the change is, Interval-based Summary has a certain chance to miss it. As shown in figure 4, points B, C, D, E and F are completely missed by the Interval-based Summary, in which, a dramatic move (from D to E, and E to F) is inappropriately sampled to a much gentle decreasing trend (D' to F'). In a contrast, DC-based Summary captures all those significant moves except A'.

3.2 DC-based summaries Offer Longer Coastlines

Directional Changes offer a longer price coastline than Interval-based summaries. With consideration of profitability, Directional Changes capture all events that reach the threshold; of which is determined to find out changes in prise that concerning ones interests. Aloud et al (2012) show that price-curve coastlines measured by intrinsic time are longer than those measured by physical time. A longer coastline indicates higher potential of profitability. This is because longer coastline measures a bigger accumulative change (i.e. bigger $\sum_{i=1}^{n} |\frac{p_{i+1}-p_i}{p_i}|$).

A possible way of comparing coastlines summarized by intervals or DC (Directional Changes) is to calculate the cumulative changes. This is because both summaries are samples of original data, that is, the horizontal length of both summaries should be the same (see figure 1, 2)[4]. Therefore, the only matter of

[4]Usually, DC-based coastlines are, horizontally, shorter than interval based ones. This is because that at the end of raw data, the remainder data do not confirm another Directional Change event (see figure 2). But this is not a problem as: First, when calculating the coastline, the interval based summary coastline is counted up to where the DC-based one ends.

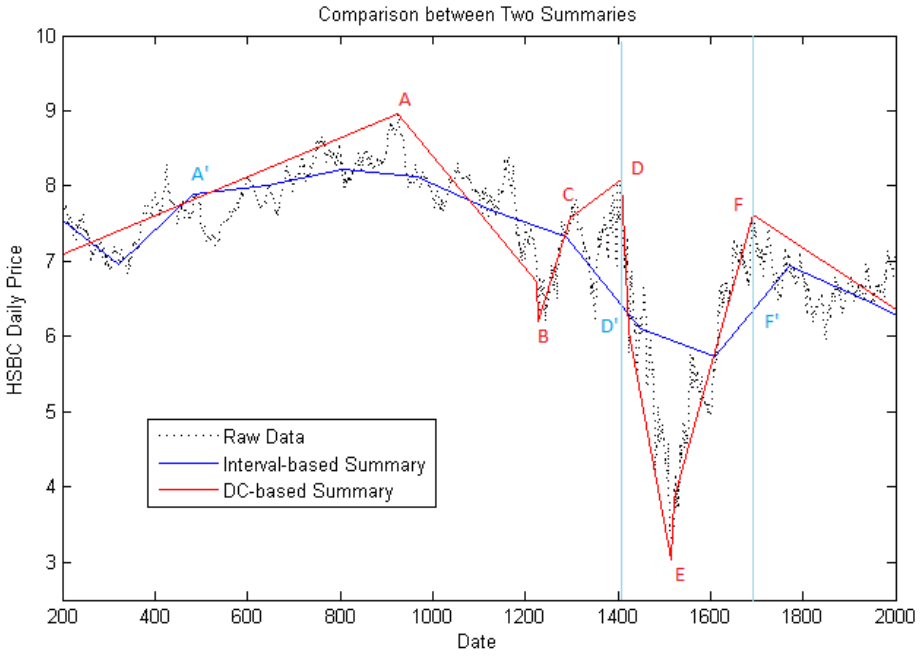

Figure 4. Comparison between interval-based and DC-based summaries (HSBC stock daily price from 24/12/2003 to 16/02/2009, under threshold 0.22)

Table 1: Vertical Coastline Calculation

Threshold	1%	2%	3%	5%
Observations	2403	1602	961	601
Interval based coastline	255.53	207.64	154.62	124.42
DC-based coastline	322.67	277.89	243.54	193.70

the length of the coastline is vertical movements, i.e. cumulative change of the price. In order to make comparison possible, we first define a threshold so that observation number (i.e. Directional Changes events) of a threshold is calculated. After, the interval of interval-based summary is defined by using raw data length divided by the observation number (DC event number). To compare the coastlines of both summaries, we test vertical coastline under thresholds of 1%, 2%, 3% and 5% on the stock daily price of HSBC. And the results show that the vertical DC-based summary coastlines are longer than interval-based ones (See table 1, a full experiments on FTSE 100 can be seen in appendix 2).

3.3 DC-based Summarising Skips Data Holes

A basic fact of tick-by-tick data (raw data) is that the prices is irregularly spaced in physical time, or the transactions take place irregularly in terms of physical time (τ_j). However, most statistical analyses rely upon the use of regularly spaced data (t_i). [1]

Consequently, interval-based summaries are often used, and the price at t_i can be defined as:

$$p(t_i, \Delta t)$$

Where t_i is a sequence of the regular spaced time data, and Δt is the time interval ($\Delta t = 1day$, $\Delta t = 1hour$, $\Delta t = 1second$, etc.)

However, when summarising raw data, there is a possibility that the sample point (t_i) is laid between ticks ($\tau_{j-1} < t_i < \tau_j$). In other word, there are data holes in interval-based summaries ($\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$ does not exist).

To fill the data holes (to obtain $\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$), linear interpolation can be adopted (Müller, 1990 [5]). In this case an estimate p^* of $\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$ can be calculated as

$$p^*(t_i) = wp(\tau_{j-1}) + (1-w)p(\tau_j)$$

Where

$$w = \frac{\tau_j - t_i}{\tau_j - \tau_{j-1}}$$

An alternative method is using $p(\tau_{j-1})$ as p^* (Wasserfallen and Zimmermann, 1985 [6]).

Second, even without adjustment, according to later experiments DC-based ones are longer than interval-based ones. If considering this issue, DC-based ones should be even longer than interval-based ones.

However, if raw data is summarised by Directional Change events, the above issue no longer needs to be considered. Because Directional Change events always take place at τ_j. By replacing t_i by τ_j, traditional statistical analyses can still be employed without data holes.

3.4 DC-based Summaries Offer a Potential New Risk Measure

The Directional Change frequency over period S can be defined [1]:

$$d(S) \equiv d(\Delta t, n, r_c) \equiv \frac{1}{S} N(\{k | m_k \neq m_{k-1}, 1 < k \leq n\})$$

where

$$S = n\Delta t$$

and $N(\{k\})$is the counting function, $n\Delta t$ is the sampling period on which the counting is performed. m_k indicates the event type – upturn event or downturn event – of current trend. r_c is a constant threshold. $d(S)$ calculates the frequency of Directional Change events in the period.

DC-based summaries can be used as a new risk measure in two senses. First, like volatility, measuring DC frequency gives an idea that how volatile the price is in a certain period. Second, unlike volatility, because the threshold is chosen by the traders, it gives the knowledge that the price is likely to move beyond the threshold. [1]

4 Research using DC-based summaries

4.1 Regularities Based on Directional Changes Have Been Discovered

Since Guillaume *et al.* [1] introduced Directional Changes, as regularities in a complex system, scaling laws are discovered in foreign exchange markets by [1]. A scaling law or power law is a simple polynomial function relationship: $f(x) \propto x^{-a}$. In the study of Directional Changes, [1] presented the Directional-Change count scaling law:

$$N(\Delta x_{dc}) = \left(\frac{\Delta x_{dc}}{C} \right)^E$$

Where, $N(\Delta x_{dc})$ is the number of directional changes measured for the threshold Δx_{dc}. What is more,[7] introduced a scaling law relates the length of the average overshoot segment to the directional change threshold:

$$\langle |\Delta x^{os}| \rangle = \left(\frac{\Delta x_{dc}}{C} \right)^E$$

And it turns out that the average length of overshoot Δx^{os} is about the same size as the threshold: $\langle|\Delta x^{os}|\rangle \approx \Delta x_{dc}$ [7]. In addition, another 12 empirical scaling laws are found in high-frequency foreign exchange data [8].

These scaling laws can be seen as the law of the nature [7], or regularities of science, or patterns in financial data. Because they are regularities, they happen under certain conditions. Therefore, trading strategies can be made upon these laws.

For example, according to the scaling laws and what Directional Changes reveal. After a confirmation of Directional Change event (a t% Directional Change), a t% overshoot will be expected (on average). A natural decision would be taking a long position when the price is expected to rise, and taking a short position when the price is expected to fall. Therefore, what the strategy suggest would be buying with all wealth at an upturn event confirmation point, as at which the lowest price is available to an investor when price is expected to rise. And selling (or short selling if possible) all assets at a down turn event confirmation point, as at which the price would be the highest to an investor when the price is expected to decline.

However, it is obvious that the strategy will not work when encounters zero overshoot[5] or when overshoot is smaller than expected. The scaling laws only apply on average, as a result the strategy may face a possibility of losing money. A possible solution will be selling a proportion of total assets, say (a_i%, where a_i decreases exponentially), whenever a small rise, say Δt% (Δt% is smaller than t%), happens after purchasing. The position will not be closed till next Directional Change event. And vice versa in short selling. Doing this, when meet zero overshoots and overshoot is smaller than expected, this rule will act as cut-loss strategy.

The trading strategy presented here is rather simple and more works need to be done to make it more sophisticated. However, though it is only tested in foreign exchange markets, these studies show that DC-based summaries hold the ability of revealing regularities of underlying financial data. Based on which trading rules are possible to be made.

4.2 Useful Market Indicators Have Been Proposed

A pioneer work for measuring the impact of major events to the markets is introduced by Zumbach *et al.* and applied in foreign exchange markets [9], it is so called the scale of market shocks. It quantifies market movements on a tick-by-tick basis. Later on, Maillet and Michel [10] applied the scale of market shocks to the stock market, and it is designed to detect and to compare the severity of various crises. Inspirited by [9], another unpublished work by Subbotin [11] is also mentioned in [12]. This later study proposed a probabilistic indicator for volatilities, of which seems usable for detecting crises and regime shifts rather than quantifying impact of individual events.

Although, there is no right or wrong when choosing metric of measuring

[5] When there is not an overshoot period between two Directional Change events.

market evolution [12], and it shoulds like a natural choice to use volatility; [12] claimed that using volatility fails to maximise the criteria of simplicity and the ability of incorporating all details of the prise evolution, as aggregating activities into a volatility measurement mingles different price scales. For example, Bouchaud *et al.* [13] showed the dynamics of the market slowly 'digesting' the changes in supply and demand[6] involving market order book dynamic and market maker profits, of which certainly interesting. However, because of using volatility as the measurement of market dynamics, it is still not clear that what impact a event brings to the market, as the volatility is a measure calculated from all past prices of which from various scales.

Therefore, to quantify the trajectory of market prise evolution, Bisig *et al.* [12] proposed a frame work so called the scale of market quakes, in which the physic no longer exist, instead, time ticks at every confirmation of price Directional Changes. By calculating the average Overshoot and comparing the overshoot-at-event, a quake at a certain magnitude/scale can be calculated. Testing at major news announcement and analysing the evolution of those scales, [12] claim that the SMQ response to news announcements or a mismatch of demand and supply. And the SMQ is believed to be the first step to build a global information system [12].

5 Discussion

5.1 DC-based Summaries Reflect Properties of Original Raw Data

Although financial raw data are in high quality, it seems that there is not an efficient way to deal with the raw data but summarise them into either Interval-based or event-based summaries. When summarising data, it loses some information of the original data for sure. Therefore, it is important that the summary reflects the original data's properties/features.

When raw data are summarised as Interval-based summaries, without considering the properties/features, data are sampled at certain time points with fix interval lengths. This mechanism makes the summary actually regardless to the properties/features of original data. Although, one may claim that with smaller intervals, it has bigger probability to capture market 'significant' movements; still there is no guarantee (see figure 4). What is more, with smaller intervals it faces the problem of handling large size of data, which is one of the reasons that we summarise raw data. In other aspects, when dealing with smaller-interval data such as high frequency data, we may focus on more micro movements. But, the data are still time series with fixed interval lengths. This means that even with small intervals, Interval-based summaries still have the chance to miss significant movement at a much more micro scale (as in figure 4).

[6] Mainly, how transactions impact the market

However, with DC-based summaries, once the threshold is decided, the summary captures these movements reaching or exceeding the threshold. Unlike Interval-based summaries, all DC based summaries' sampling are at extreme points, (as shown in figure 3 and figure 4). As a result, when markets have significant move, they are represented as a big Overshoots. And the size of the Overshoot actually reflects how significant a movement is. DC-based summaries work when facing high frequency data, and all need to be done is, depending on the frequency, to use a smaller threshold.

5.2 A Longer Coastline Potentially Offers More Profits

As mentioned in section 4.2, comparing to Interval-based summaries, DC-based summaries have longer coastlines. Because Directional Changes are always confirmed on extreme points (see figure 2). And Interval-based summaries can sample at potentially anywhere on the original price curve (see figure 1). Horizontally, both DC-based and Interval-based summaries pass the same path; vertically, because DC-based summaries are always at extreme which Interval-based summaries are not, DC-based summaries are considered more volatile than Interval-based summaries (see figure 4). i.e. DC-based summaries' curves are longer than Interval-based summaries'.

Considering the measurement of return $\frac{p_t - p_{t-1}}{p_{t-1}}$, in a market allowing short selling with proper trading strategies (as mentioned in section 4.2), higher volatility means potentially higher profitability. Compare with Interval-based summaries, DC-based summaries are the ones giving longer coastlines. Yet, the problem becomes how to find a proper trading rule.

5.3 DC-based Summaries Prevent Distorting Raw Data

Although there are methods to fill data holes, doing it makes the data distorted. Because, when a data hole is filed, artificial data has been added. One may claim that a big enough observation number could eliminate the artificial data's effects. However, there is no guarantee that the number of artificial data is not proportional to the observation number. While using DC-based summaries does not need to worry about this issue at all. By skipping data holes, DC-based summaries make sure that all data used to run statistical analyses exist.

5.4 A Potential New Risk Measure

Volatility gives investors an idea that the amplitude of change of price. According to a unrealistic assumption that the change or price is subject to normal distribution, predictions of future volatilities can be made. However, a high volatility does not only mean that a investor is like to have a higher chance to loose, it also indicates that there is a chance to gain more.

While interval-based summaries fix time intervals and change of price's amplitude is changing; DC-based summaries choose a constant threshold while time is varying. This means that the change of price is fixed and it gives the

idea of how likely the price is to move a certain rate in a certain direction. This is helpful to traders to decide whether to open or close a position.

Although volatility tells us the general environment of the market, we are actually more interested in the timing of our trades. ([14])

5.5 DC-based Summaries Reveal Regularities

Actually, there are works on regularities in DC-based summaries. As mentioned in section 5.1, although trading strategies are not necessary to be built on regularities. But if there are regularities found, then trading strategies can be made upon.

According to Guillaume *et al.* [1], scaling laws, as regularities in complex systems, are found in foreign exchange markets. And empirical works are done in foreign exchange markets showing positive results [1, 7, 8]. Based on the scaling laws found in DC-based summaries (see section 5.1), trading strategies are possible to be made.

5.6 DC-based Summaries as the Building Brick of Global Economical/Financial Information System

The Scale of Market Quakes (SMQ) is introduced by Bisig *et al.* [12]. This system is built on DC-based summaries and it is for detecting the market dynamics. This system detects the quake scales of market by comparing the overshoot at event to the average overshoot to give a description of market status. Inspirited by the work, a further development can perhaps be using the overshoot distribution to make a value-at-risk-like risk measurement. Because this new measurement is based on DC-based summary, it may not have the drawback that volatility has (price activities at different scale are mingled). However, further works need to be done.

6 Conclusions

This paper introduces two ways of summarising raw data. Interval-based summarising and DC-based summarising. As a well known method, interval-based summarising is not redundantly explained. Focusing on DC-based Directional Change events are firstly introduced, as the summarising based on Directional Change events, its uses are stated mainly included: DC-based summaries focus on periods matters more, they offer longer price coast lines, skip data holes and potentially can be a new risk measure telling investors the timing of closing or opening a position.

In addition, researches based on DC-based summaries are introduced – the discovery of scaling laws and the Scale of Market Quakes (SMQ). Scaling laws can be seen as regularities in the market, with which, trading strategies can be built. SMQ is a market indicator, it shows the affects that major events bring to the market.

References

[1] D. M. Guillaume, M. M. Dacorogna, R. R. Davé, U. A. Müller, R. B. Olsen, and O. V. Pictet, "From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets," *Finance and Stochstics*, vol. 1, pp. 95–129, 1997.

[2] J. D. Hamilton, *Time Series Analysis*. Princeton, 1994.

[3] K. Krippendorff, "Combinatorial explosion," *Web Dictionary of Cybernetics and Systems*, November 2010.

[4] E. Tsang, "Directional changes, definitions." Working Paper WP050-10 Centre for Computational Finance and Economic Agents (CCFEA), University of Essex Revised 1, 2011.

[5] U. Müller, M. Dacorogna, R. Olsen, O. Pictet, M. Schwarz, and C. Morgenegg, "Statistical study of foreign exchange rates, empirical evidence of a price scaling law, and intraday analysis.," *J. Banking Finance*, vol. 14, pp. 1189–208, 1990.

[6] W. Wasserfallen and H. Zimmermann, "The behaviour of intra-intra exchange rates," *J. Banking Finance*, vol. 9, pp. 55–72, 1985.

[7] J. B. Glattfelder, T. Bisig, and R. B. Olsen, "R&D strategy document," vol. ZH.2010-12-16, 2010.

[8] J. Glattfelder, A. Dupuis, and R. Olsen, "Patterns in high-frequency fx data: Discovery of 12 empirical scaling laws," *Quant. Financ*, vol. 11(4), pp. 599 – 614, 2011.

[9] G. Zumbach, M. Dacorogna, J. Olsen, and R. Olsen, "Measuring shock in financial markets," *Int. J. Theoretical Appl.*, p. 3:347, 2000.

[10] B. Maillet and T. Michel, "An index of market shocks based on multiscale analysis," *Quant. Finan.*, p. 3:88, 2003.

[11] A. Subbotin, "A multi-horizon scale for volatility," *http://hal-paris1.archives-ouvertes.fr/docs/00/26/15/14/PDF/Bla08020.pdf*, 2008.

[12] T. Bisig, A. Impagliazzo, and R. Olsen, "The scale of market quakes," 2009.

[13] J.-P. Bouchaud, J. D. Farmer, and F. Lill, "How markets slowly digest changes in supply and demand," *Handbook of Financial Markets: Dynamics and Evolution*, 2008.

[14] M. Dacorogna, U. Müller, R. Nagler, R. Olsen, and O. Pictet, "A geographical model fro the daily and weekly seasonal volatility in the fx market," *J. Intern. Money Finance*, vol. 12(4), pp. 413–38, 1993.

Appendices

1 Directional Changes, Definitions

In this Appendix, we summarize the key definitions of directional changes, based on [4].

A **Directional Change Event** can be a **Downturn Event** or an **Upturn Event**.

A **Downward Run** is a period between a Downturn Event and the next Upturn Event. An **Upward Run** is a period between an Upturn Event and the next Downturn Event.

In a Downward Run, a **Last Low** is constantly updated to the minimum of (a) the current price and (b) the Last Low. In an Upward Run, a **Last High** is constantly updated to the maximum of (a) the current price and (b) the Last High. Last Low and last High are called **Extremum (EXT)**.

In a Downward Run, given a **Threshold** (a percentage), an **Upturn Event** is an event when the price is higher than the Last Low by the Threshold. An Upturn Event terminates a Downward Run, and starts an Upward Run.

In an Upward Run, given a **Threshold** (a percentage), a **Downturn Directional Change Event** is an event when the price is lower than the Last High by the Threshold. A Downturn Event terminates an Upward Run, and starts a Downward Run.

The point when the price reaches the Threshold is called the point of **Directional Change (DC)**.

The above definitions are mutual recursive. Operationally, we set both the Last High and Last Low to the price at the beginning of the sequence, where neither downward run nor upward run is defined.

A Downturn Event is followed by a **Downward Overshoot Event**, which is ended by the next Upturn Event, which is itself followed by an **Upward Overshoot Event**, which is ended by the next Downturn Event. So time is defined by sequences of event cycles of four events, as shown below:

... → Downturn Event →
Downward Overshoot Event →
Upturn Event →
Upward Overshoot Event →
Downturn Event → ...

2 Tables of Vertical Coastline Calculation of FTSE 100

Table 2: Vertical Coastline Calculation 1

EPIC	Threshold	0.01	0.02	0.03	0.04	0.05
ADN	Interval-based Summaries	978.076	828.128	662.133	573.338	536.621
ADN	DC-based Summaries	1159.747	1058.833	935.646	869.549	804.243
ADM	Interval-based Summaries	220.984	165.612	134.372	101.15	95.036
ADM	DC-based Summaries	294.064	243.124	208.892	173.554	158.18
AGK	Interval-based Summaries	215.292	172.714	139.032	115.688	105.118
AGK	DC-based Summaries	271.892	239.848	207.932	186.588	170.468
AMEC	Interval-based Summaries	74.214	64.326	56.825	51.945	45.823
AMEC	DC-based Summaries	98.188	90.678	84.564	78.03	70.766
AAL	Interval-based Summaries	272.054	227.574	197.882	173.778	143.594
AAL	DC-based Summaries	340	302.64	281.712	252.968	228.67
ANTO	Interval-based Summaries	217.31	172.594	155.188	120.904	115.5
ANTO	DC-based Summaries	284.102	252.67	227.454	197.994	187.132
ARM	Interval-based Summaries	259.626	210.58	172.302	151.412	134.096
ARM	DC-based Summaries	360.1	329.76	300.308	273.432	251.288
ABF	Interval-based Summaries	202.625	175.899	152.496	137.13	124.832
ABF	DC-based Summaries	242.485	227.846	215.159	201.208	186.994
AZN	Interval-based Summaries	383.285	318.286	261.103	221.533	191.618
AZN	DC-based Summaries	466.095	419.511	376.241	341.176	303.898
AV.	Interval-based Summaries	1003.325	799.57	657.545	545.01	479.2
AV.	DC-based Summaries	1283.58	1115.27	1002.12	863	755.24
BAB	Interval-based Summaries	113.21	84.246	72.535	63.901	59.303
BAB	DC-based Summaries	156.402	135.387	120.957	106.481	94.019
BA.	Interval-based Summaries	237.924	194.065	169.311	148.699	127.519
BA.	DC-based Summaries	288.756	260.809	236.694	214.074	197.213
BARC	Interval-based Summaries	390.861	291.578	230.328	187.306	156.188
BARC	DC-based Summaries	482.055	407.273	335.75	292.405	250.359
BG.	Interval-based Summaries	168.079	135.869	111.925	94.593	83.09
BG.	DC-based Summaries	210.676	185.832	166.083	148.895	132.817
BLT	Interval-based Summaries	307.407	246.91	203.236	167.555	139.84
BLT	DC-based Summaries	382.374	338.882	303.14	267.432	232.889
BP.	Interval-based Summaries	243.6	188.386	154.652	127.549	111.383
BP.	DC-based Summaries	307.941	271.251	238.964	212.878	188.904
BATS	Interval-based Summaries	547.303	457.708	396.39	338.146	306.364
BATS	DC-based Summaries	654.794	589.782	536.061	486.025	447.788
BLND	Interval-based Summaries	169.266	125.184	104.336	87.548	75.97
BLND	DC-based Summaries	218.352	181.876	155.84	133.546	120.492
BSY	Interval-based Summaries	211.671	156.897	127.543	103.721	90.594
BSY	DC-based Summaries	263.228	220.986	192.241	167.799	146.966
BT.A	Interval-based Summaries	209.166	174.698	144.062	122.28	104.204
BT.A	DC-based Summaries	256.49	231.486	210.3	189.626	170.326
BNZL	Interval-based Summaries	337.821	281.741	222.957	191.865	176.603
BNZL	DC-based Summaries	422.531	380.08	343.925	315.671	291.687
BRBY	Interval-based Summaries	181.658	150.56	125.222	98.784	93.32
BRBY	DC-based Summaries	226.096	205.988	184.371	169.698	154.066
CPI	Interval-based Summaries	693.142	556.486	449.206	407.98	358.01
CPI	DC-based Summaries	852.332	740.736	658.69	598.696	545.852
CSCG	Interval-based Summaries	82.32	62.606	48.427	41.45	33.018
CSCG	DC-based Summaries	102.771	86.392	73.395	64.297	52.445

Table 3: Vertical Coastline Calculation 2

EPIC	Threshold	0.06	0.07	0.08	0.09	0.1
ADN	Interval-based Summaries	446.981	403.691	397.326	358.18	320.159
ADN	DC-based Summaries	744.861	702.497	651.807	590.712	547.713
ADM	Interval-based Summaries	81.386	57.892	58.33	55.73	55.6
ADM	DC-based Summaries	133.372	109.234	102.38	94.87	89.48
AGK	Interval-based Summaries	95.694	85.804	78.61	74.852	70.398
AGK	DC-based Summaries	159.728	145.28	132.51	128.34	118.25
AMEC	Interval-based Summaries	43.994	39.762	36.545	34.228	30.138
AMEC	DC-based Summaries	66.87	62.274	57.774	55.11	52.249
AAL	Interval-based Summaries	122.676	109.778	106.316	92.57	81.914
AAL	DC-based Summaries	206.95	186.646	176.176	158.142	146.478
ANTO	Interval-based Summaries	102.714	96.736	79.99	77.854	70.796
ANTO	DC-based Summaries	169.436	159.56	141.116	132.29	125.514
ARM	Interval-based Summaries	120.764	117.346	106.792	99.856	85.084
ARM	DC-based Summaries	230.602	222.506	203.212	192.352	175.144
ABF	Interval-based Summaries	110.918	106.37	89.826	81.924	72.225
ABF	DC-based Summaries	173.6	165.934	154.552	146.378	137.774
AZN	Interval-based Summaries	170.531	153.693	140.936	119.8	114.693
AZN	DC-based Summaries	277.981	251.437	232.182	216.05	208.038
AV.	Interval-based Summaries	369.275	333.4	299.825	255.535	225.465
AV.	DC-based Summaries	663.945	593.56	548.195	472.045	435.64
BAB	Interval-based Summaries	51.731	42.548	37.231	38.443	33.805
BAB	DC-based Summaries	84.077	68.062	62.891	55.247	52.119
BA.	Interval-based Summaries	107.673	94.175	92.677	80.45	80.085
BA.	DC-based Summaries	175.826	160.027	150.067	137.481	131.097
BARC	Interval-based Summaries	132.854	121.2	106.944	91.79	86.44
BARC	DC-based Summaries	214.773	191.66	173.406	160.701	133.381
BG.	Interval-based Summaries	72.418	67.956	61.508	48.438	46.382
BG.	DC-based Summaries	119.305	112.054	99.968	88.039	82.562
BLT	Interval-based Summaries	120.2	106.634	97.851	88.244	76.766
BLT	DC-based Summaries	208.463	188.288	173.823	162.347	142.732
BP.	Interval-based Summaries	96.943	87.252	74.051	67.828	58.828
BP.	DC-based Summaries	165.73	149.9	130.038	114.562	102.376
BATS	Interval-based Summaries	268.396	245.494	213.301	185.524	183.076
BATS	DC-based Summaries	423.735	385.645	355.873	323.994	305.47
BLND	Interval-based Summaries	70.768	57.114	52.2	45.53	41.278
BLND	DC-based Summaries	111.4	95.474	83.114	75.828	72.068
BSY	Interval-based Summaries	76.795	72.794	57.829	57.878	45.323
BSY	DC-based Summaries	132.198	122.1	104.878	95.974	85.882
BT.A	Interval-based Summaries	102.226	91.236	80.946	74.574	61.018
BT.A	DC-based Summaries	158.612	145.474	130.618	128.086	106.232
BNZL	Interval-based Summaries	144.853	137.81	127.317	107.569	101.815
BNZL	DC-based Summaries	268.389	246.993	236.953	211.878	201.778
BRBY	Interval-based Summaries	76.058	68.486	65.614	57.113	48.402
BRBY	DC-based Summaries	138.266	126.837	115.878	107.453	90.5
CPI	Interval-based Summaries	320.602	315.724	263.968	236.786	236.182
CPI	DC-based Summaries	500.832	468.274	417.26	374.752	353.518
CSCG	Interval-based Summaries	29.331	25.497	25.173	19.333	18.146
CSCG	DC-based Summaries	48.559	43.569	38.698	33.772	31.153

Table 4: Vertical Coastline Calculation 3

EPIC	Threshold	0.01	0.02	0.03	0.04	0.05
CCL	Interval-based Summaries	120.808	92.941	74.186	57.274	49.387
CCL	DC-based Summaries	147.842	126.993	108.568	92.454	81.342
CNA	Interval-based Summaries	181.178	148.652	120.846	102.272	93.292
CNA	DC-based Summaries	240.828	213.51	186.935	167.048	151.646
CPG	Interval-based Summaries	246.766	194.05	162.622	148.106	130.92
CPG	DC-based Summaries	330.916	289.874	254.544	232.538	206.744
CRH	Interval-based Summaries	604.638	494.348	415.402	369.614	316.698
CRH	DC-based Summaries	768.958	695.812	636.17	574.478	531.192
CRDA	Interval-based Summaries	154.002	122.447	100.311	85.614	79.87
CRDA	DC-based Summaries	211.775	184.226	161.613	146.754	129.042
DGE	Interval-based Summaries	290.151	209.509	162.407	133.631	107.9
DGE	DC-based Summaries	367.139	299.25	258.697	219.397	185.873
ENRC	Interval-based Summaries	227.091	197.996	177.86	163.257	143.75
ENRC	DC-based Summaries	256.879	242.923	225.397	213.263	196.018
EVR	Interval-based Summaries	19.736	17.865	14.958	12.609	10.868
EVR	DC-based Summaries	22.694	21.48	19.357	17.615	16.654
EXPN	Interval-based Summaries	208.192	166.492	141.888	116.584	97.9
EXPN	DC-based Summaries	258.45	225.198	199.006	171.716	149.288
FRES	Interval-based Summaries	244.428	215.382	177.124	148.516	143.226
FRES	DC-based Summaries	279.42	253.364	226.584	207.436	190.306
GFS	Interval-based Summaries	38.302	28.36	21.921	17.25	15.557
GFS	DC-based Summaries	46.955	38.997	32.703	27.757	24.809
GKN	Interval-based Summaries	105.455	85.814	75.158	65.662	57.59
GKN	DC-based Summaries	130.88	118.478	106.606	96.639	89.089
GSK	Interval-based Summaries	25.539	18.38	17.18	15.548	11.533
GSK	DC-based Summaries	30.148	27.599	25.762	22.547	20.307
GLEN	Interval-based Summaries	558.29	437.1	352.13	284.11	227.53
GLEN	DC-based Summaries	713.81	620.005	537.705	451.71	398.34
HMSO	Interval-based Summaries	64.079	53.225	45.868	39.113	34.093
HMSO	DC-based Summaries	76.598	68.471	63.057	55.641	50.336
HL.	Interval-based Summaries	169.218	135.124	112.402	99.512	87.626
HL.	DC-based Summaries	225.243	198.022	174.995	158.485	143.413
HSBA	Interval-based Summaries	257.105	198.713	162.72	138.351	122.25
HSBA	DC-based Summaries	324.285	277.893	243.459	216.792	193.623
IMI	Interval-based Summaries	211.062	175.369	147.953	132.05	115.773
IMI	DC-based Summaries	251.336	227.23	203.485	184.147	167.49
IMT	Interval-based Summaries	234.055	190.873	148.219	127.611	113.139
IMT	DC-based Summaries	285.739	252.385	216.795	194.517	169.831
IHG	Interval-based Summaries	229.956	185.708	156.26	145.519	119.002
IHG	DC-based Summaries	286.198	254.078	230.556	209.18	189.596
IAG	Interval-based Summaries	385.038	293.112	241.888	200.486	162.512
IAG	DC-based Summaries	489.766	417.722	356.918	301.216	261.362
ITRK	Interval-based Summaries	258.319	203.507	153.583	134.149	108.765
ITRK	DC-based Summaries	322.959	276.159	238.543	217.597	186.377
ITV	Interval-based Summaries	22.761	17.773	15.068	13.122	11.022
ITV	DC-based Summaries	26.668	23.236	20.102	17.798	16.112
WG.	Interval-based Summaries	549.036	425.908	359.262	328.25	260.882
WG.	DC-based Summaries	714.21	632.188	552.16	490.336	446.358

Table 5: Vertical Coastline Calculation 4

EPIC	Threshold	0.06	0.07	0.08	0.09	0.1
CCL	Interval-based Summaries	42.355	37.911	32.594	26.947	21.745
CCL	DC-based Summaries	73.928	67.205	58.346	46.525	42.795
CNA	Interval-based Summaries	79.966	71.258	66.059	61.892	52.107
CNA	DC-based Summaries	133.734	123.52	116.086	105.385	99.288
CPG	Interval-based Summaries	107.942	103.142	97.442	76.25	70.296
CPG	DC-based Summaries	189.668	167.818	159.228	133.148	121.968
CRH	Interval-based Summaries	287.954	238.35	209.596	197.096	180.674
CRH	DC-based Summaries	485.112	426.038	389.742	358.828	334.012
CRDA	Interval-based Summaries	68.063	60.599	54.456	50.569	43.419
CRDA	DC-based Summaries	118.235	110.057	96.391	88.787	83.274
DGE	Interval-based Summaries	93.428	82.538	71.263	60.297	55.663
DGE	DC-based Summaries	162.125	148.575	135.592	115.801	105.391
ENRC	Interval-based Summaries	131.468	119.254	107.926	111.078	94.91
ENRC	DC-based Summaries	184.232	173.247	164.367	161.783	150.94
EVR	Interval-based Summaries	11.278	8.49	9.154	5.832	6.088
EVR	DC-based Summaries	15.293	13.484	12.958	11.053	11.091
EXPN	Interval-based Summaries	83.152	74.95	67.222	62.702	47.737
EXPN	DC-based Summaries	128.384	115.916	104.412	95.884	85.587
FRES	Interval-based Summaries	119.594	108.396	97.27	102.672	87.842
FRES	DC-based Summaries	170.736	159.228	145.356	141.012	138.662
GFS	Interval-based Summaries	13.58	12.129	10.064	8.669	8.605
GFS	DC-based Summaries	23.414	20.158	17.783	16.505	15.173
GKN	Interval-based Summaries	48.956	46.152	41.769	39.416	37.562
GKN	DC-based Summaries	80.863	73.929	70.565	65.572	60.423
GSK	Interval-based Summaries	12.06	11.291	9.075	8.301	7.525
GSK	DC-based Summaries	18.987	17.497	15.01	15.045	12.792
GLEN	Interval-based Summaries	194.635	174.075	146.36	130.375	129.77
GLEN	DC-based Summaries	344.14	312.81	276.185	254.19	240.25
HMSO	Interval-based Summaries	30.415	26.976	24.362	22.336	20.248
HMSO	DC-based Summaries	45.503	42.326	39.627	35.615	32.347
HL.	Interval-based Summaries	72.956	66.962	60.147	58.066	51.608
HL.	DC-based Summaries	125.21	114.475	105.913	98.216	88.43
HSBA	Interval-based Summaries	107.942	92.354	80.773	75.156	73.507
HSBA	DC-based Summaries	173.941	155.689	142.991	126.767	122.779
IMI	Interval-based Summaries	104.481	94.385	81.942	76.106	70.449
IMI	DC-based Summaries	151.315	133.95	122.881	115.304	107.014
IMT	Interval-based Summaries	98.347	79.567	71.377	72.115	58.967
IMT	DC-based Summaries	153.945	134.307	124.287	117.617	109.367
IHG	Interval-based Summaries	112.858	91.669	82.596	74.406	67.327
IHG	DC-based Summaries	167.006	150.79	138.949	131.154	115.92
IAG	Interval-based Summaries	142.264	138.806	122.062	107.784	82.538
IAG	DC-based Summaries	223.514	209.312	197.714	169.282	153.476
ITRK	Interval-based Summaries	94.973	73.545	65.795	58.265	56.555
ITRK	DC-based Summaries	162.389	138.829	118.279	106.245	101.725
ITV	Interval-based Summaries	9.831	9.083	8.157	8.083	6.859
ITV	DC-based Summaries	14.33	13.778	12.292	11.83	10.645
WG.	Interval-based Summaries	233.164	220.494	191.372	180.98	149.694
WG.	DC-based Summaries	398.4	358.264	332.504	307.682	266.258

Table 6: Vertical Coastline Calculation 5

EPIC	Threshold	0.01	0.02	0.03	0.04	0.05
JMAT	Interval-based Summaries	422.064	358.756	322.142	272.934	248.556
JMAT	DC-based Summaries	474.682	445	412.326	373.377	350.232
KAZ	Interval-based Summaries	146.599	119.736	95.468	81.387	70.569
KAZ	DC-based Summaries	180.342	160.133	141.654	124.165	110.655
KGF	Interval-based Summaries	333.81	255.879	210.911	176.41	152.108
KGF	DC-based Summaries	428.045	364.26	322.956	280.421	249.283
LAND	Interval-based Summaries	69.45	54.901	45.595	38.053	32.393
LAND	DC-based Summaries	84.399	75.827	66.541	59.252	53.358
LGEN	Interval-based Summaries	127.858	107.045	94.789	79.18	68.979
LGEN	DC-based Summaries	153.317	140.439	127.426	115.968	106.43
LLOY	Interval-based Summaries	88.889	73.013	62.153	55.684	50.506
LLOY	DC-based Summaries	114.811	101.964	92.516	84.994	75.714
MKS	Interval-based Summaries	184.035	143.755	118.813	103.232	83.502
MKS	DC-based Summaries	226.005	194.129	169.963	150.777	128.777
MGGT	Interval-based Summaries	27.741	23.452	19.893	18.046	15.468
MGGT	DC-based Summaries	36.21	32.802	29.564	27.146	24.233
MRO	Interval-based Summaries	76.531	58.335	47.759	39.172	34.977
MRO	DC-based Summaries	99.065	84.736	73.118	62.234	55.521
MRW	Interval-based Summaries	150.462	109.524	86.667	68.387	57.977
MRW	DC-based Summaries	191.062	159.988	134.169	111.767	97.96
NG.	Interval-based Summaries	554.222	445.222	357.63	302.07	264.518
NG.	DC-based Summaries	680.48	595.756	512.394	455.592	405.202
NXT	Interval-based Summaries	54.163	43.128	37.521	31.942	28.128
NXT	DC-based Summaries	66.133	58.641	53.102	47.797	42.774
OML	Interval-based Summaries	230.166	192.3	158.028	142.73	114.368
OML	DC-based Summaries	272.078	248.646	216.216	195.592	170.122
PSON	Interval-based Summaries	40.81	35.82	28.41	23.26	18.41
PSON	DC-based Summaries	48.01	43.63	37.83	32.66	29.01
PFC	Interval-based Summaries	379.228	318.811	267.64	230.832	193.542
PFC	DC-based Summaries	466.6	425.993	382.731	350.099	313.274
POLY	Interval-based Summaries	457.999	365.671	289.067	252.643	225.795
POLY	DC-based Summaries	584.761	526.487	467.129	421.621	386.145
PRU	Interval-based Summaries	1550.874	1331.318	1122.167	966.3	869.052
PRU	DC-based Summaries	1918.538	1735.816	1558.418	1375.748	1271.872
RRS	Interval-based Summaries	606.894	451.26	352.262	280.076	235.606
RRS	DC-based Summaries	788.314	658.52	539.806	443.546	382.12
RB.	Interval-based Summaries	693.78	496.16	416.173	360.529	298.471
RB.	DC-based Summaries	863.08	735.38	643.466	567.814	508.604
REL	Interval-based Summaries	233.418	178.697	147.652	126.607	108.19
REL	DC-based Summaries	296.603	257.61	222.728	198.506	175.588
RSL	Interval-based Summaries	172.703	133.992	107.538	92.754	82.01
RSL	DC-based Summaries	210.685	183.456	159.29	140.554	127.242
REX	Interval-based Summaries	1238.743	1044.714	852.171	763.531	679.865
REX	DC-based Summaries	1524.942	1388.531	1256.313	1154.005	1051.227
RIO	Interval-based Summaries	1024.652	894.622	776.478	669.738	615.738
RIO	DC-based Summaries	1358.246	1230.986	1103.85	993.802	907.79
RR.	Interval-based Summaries	174.013	142.304	114.468	98.504	90.476
RR.	DC-based Summaries	211.576	187.98	164.27	142.253	129.019

Table 7: Vertical Coastline Calculation 6

EPIC	Threshold	0.06	0.07	0.08	0.09	0.1
JMAT	Interval-based Summaries	227.187	199.99	196.471	184.477	172.244
JMAT	DC-based Summaries	328.115	303.971	291.551	278.886	261.39
KAZ	Interval-based Summaries	63.207	51.301	47.34	42.695	35.569
KAZ	DC-based Summaries	99.923	90.153	80.566	75.903	70.351
KGF	Interval-based Summaries	121.09	113.413	105.907	89.63	74.805
KGF	DC-based Summaries	213.552	190.998	171.36	158.282	140.31
LAND	Interval-based Summaries	27.715	25.417	23.475	22.496	19.433
LAND	DC-based Summaries	47.297	44.213	40.663	38.135	34.883
LGEN	Interval-based Summaries	64.966	56.49	50.236	48.62	42.182
LGEN	DC-based Summaries	99.604	88.926	79.339	74.754	70.714
LLOY	Interval-based Summaries	41.968	39.074	37.054	34.226	31.992
LLOY	DC-based Summaries	68.578	62.013	58.302	56.046	54.462
MKS	Interval-based Summaries	76.668	63.883	56.664	51.195	45.87
MKS	DC-based Summaries	115.678	101.512	92.544	83.948	80.601
MGGT	Interval-based Summaries	13.211	11.432	10.302	9.813	8.437
MGGT	DC-based Summaries	22.37	19.762	17.75	16.674	15.228
MRO	Interval-based Summaries	30.397	27.509	24.067	18.823	17.799
MRO	DC-based Summaries	48.94	43.34	38.817	33.081	31.354
MRW	Interval-based Summaries	50.675	43.386	35.308	31.18	28.901
MRW	DC-based Summaries	85.277	75.694	65.871	58.027	56.37
NG.	Interval-based Summaries	223.064	185.26	191.83	154.964	134.584
NG.	DC-based Summaries	352.652	321.892	299.738	262.194	237.434
NXT	Interval-based Summaries	25.38	23.264	20.324	18.523	17.719
NXT	DC-based Summaries	39.864	36.251	33.315	30.923	29.221
OML	Interval-based Summaries	107.51	88.136	78.4	72.288	62.452
OML	DC-based Summaries	158.476	141.746	127.032	111.236	101.8
PSON	Interval-based Summaries	14.97	15.93	15.74	15.22	15
PSON	DC-based Summaries	26.64	24.66	22.96	19.81	19.93
PFC	Interval-based Summaries	174.614	145.866	134.771	123.958	109.125
PFC	DC-based Summaries	282.342	254.072	233.412	214.708	199.79
POLY	Interval-based Summaries	190.387	157.789	137.277	127.105	116.233
POLY	DC-based Summaries	343.801	310.579	286.465	256.731	238.591
PRU	Interval-based Summaries	747.33	668.643	632.515	617.696	541.147
PRU	DC-based Summaries	1113.641	997.477	952.18	907.587	861.016
RRS	Interval-based Summaries	196.356	166.356	157.714	143.87	124.996
RRS	DC-based Summaries	325.584	283.696	271.006	242.906	217.556
RB.	Interval-based Summaries	275.541	234.234	214.136	181.322	170.202
RB.	DC-based Summaries	457.978	409.032	385.078	340.555	327.117
REL	Interval-based Summaries	94.987	87.527	77.131	69.356	58.714
REL	DC-based Summaries	159.024	146.021	129.981	115.642	107.472
RSL	Interval-based Summaries	76.4	67.803	56.331	48.314	49.49
RSL	DC-based Summaries	114.44	104.286	92.212	82.754	77.279
REX	Interval-based Summaries	608.093	563.438	462.763	421.581	393.915
REX	DC-based Summaries	987.355	922.229	837.823	784.735	714.453
RIO	Interval-based Summaries	531.49	487.932	452.196	418.312	382.444
RIO	DC-based Summaries	843.166	775.97	740.81	661.46	632.06
RR.	Interval-based Summaries	74.948	62.351	58.674	53.786	46.468
RR.	DC-based Summaries	114.111	102.671	96.245	85.4	76.502

Table 8: Vertical Coastline Calculation 7

EPIC	Threshold	0.01	0.02	0.03	0.04	0.05
RBS	Interval-based Summaries	139.964	110.241	91.669	81.133	70.581
RBS	DC-based Summaries	169.888	153.143	135.842	124.232	111.065
RDSB	Interval-based Summaries	30.617	24.771	19.343	16.991	15.342
RDSB	DC-based Summaries	39.125	34.344	29.667	26.904	24.576
RSA	Interval-based Summaries	345.089	272.943	220.017	175.87	154.472
RSA	DC-based Summaries	429.367	371.378	313.71	264.93	245.992
SAB	Interval-based Summaries	152.756	120.786	95.726	84.373	73.459
SAB	DC-based Summaries	197.689	170.035	145.355	130.955	119.514
SGE	Interval-based Summaries	562.616	471.084	395.818	344.744	301.822
SGE	DC-based Summaries	721.85	655.356	592.932	546.398	489.358
SBRY	Interval-based Summaries	134.701	113.689	93.859	85.897	74.355
SBRY	DC-based Summaries	176.622	165.042	151.801	142.819	131.009
SDR	Interval-based Summaries	402.74	320.312	275.214	229.894	204.484
SDR	DC-based Summaries	506.608	451.486	406.136	358.288	323.424
SRP	Interval-based Summaries	49.931	38.384	30.88	27.834	23.299
SRP	DC-based Summaries	60.259	53.756	48.015	44.271	38.919
SVT	Interval-based Summaries	370.43	285.432	233.176	199.7	163.688
SVT	DC-based Summaries	470.754	398.692	353.886	313.196	272.09
SHP	Interval-based Summaries	170.101	126.437	106.707	90.378	74.285
SHP	DC-based Summaries	216.522	186.448	160.342	138.176	127.606
SN.	Interval-based Summaries	136.445	115.232	93.99	81.571	71.627
SN.	DC-based Summaries	184.273	166.587	149.902	134.964	123.632
SMIN	Interval-based Summaries	284.3	201.828	156.328	129.29	112.82
SMIN	DC-based Summaries	365.9	303.374	253.374	209.96	180.954
SSE	Interval-based Summaries	578.858	466.209	404.651	329.009	294.672
SSE	DC-based Summaries	699.145	626.298	567.305	506.052	465.19
STAN	Interval-based Summaries	358.826	274.666	220.462	182.138	159.072
STAN	DC-based Summaries	465.306	390.728	337.576	288.004	249.332
SL.	Interval-based Summaries	181.572	138.305	115.038	96.957	85.423
SL.	DC-based Summaries	230.492	196.715	173.944	147.433	129.401
TATE	Interval-based Summaries	245.818	207.598	180.45	158.824	136.412
TATE	DC-based Summaries	325.556	294.09	268.618	236.25	214.394
TSCO	Interval-based Summaries	103.953	81.558	63.375	50.794	43.419
TSCO	DC-based Summaries	132.816	113.202	96.136	81.553	70.682
TT.	Interval-based Summaries	46.309	37.807	33.643	27.014	23.631
TT.	DC-based Summaries	54.139	49.528	44.61	39.492	35.756
TLW	Interval-based Summaries	471.276	363.262	296.688	225.858	205.228
TLW	DC-based Summaries	618.602	526.896	434.662	364.76	329.128
ULVR	Interval-based Summaries	216.31	160.396	124.484	103.206	88.543
ULVR	DC-based Summaries	272.2	229.788	194.004	166.546	147.369
UU.	Interval-based Summaries	636.322	554.882	478.388	400.548	380.372
UU.	DC-based Summaries	732.562	684.104	633.148	585.1	541.33
VED	Interval-based Summaries	87.376	68.147	58.145	49.614	42.722
VED	DC-based Summaries	105.936	94.932	86.398	76.954	70.224
VOD	Interval-based Summaries	318.547	269.553	232.007	195.671	175.291
VOD	DC-based Summaries	417.587	382.705	351.501	319.243	295.597
WEIR	Interval-based Summaries	124.132	105.912	88.007	75.198	68.868
WEIR	DC-based Summaries	150.184	137.968	123.903	108.099	99.246

Table 9: Vertical Coastline Calculation 8

EPIC	Threshold	0.06	0.07	0.08	0.09	0.1
RBS	Interval-based Summaries	61.409	54.429	48.795	45.325	37.942
RBS	DC-based Summaries	99.261	90.068	82.94	77.374	68.954
RDSB	Interval-based Summaries	15.378	12.492	9.955	10.85	10.473
RDSB	DC-based Summaries	22.225	19.771	17.845	16.503	16.527
RSA	Interval-based Summaries	132.586	121.178	109.658	113.701	91.48
RSA	DC-based Summaries	219.894	193.374	187.104	177.164	155.489
SAB	Interval-based Summaries	65.992	55.735	54.332	48.805	45.895
SAB	DC-based Summaries	107.427	96.227	86.722	78.497	72.17
SGE	Interval-based Summaries	278.66	241.59	208.118	190.744	166.736
SGE	DC-based Summaries	445.056	392.082	355.006	326.438	304.098
SBRY	Interval-based Summaries	66.003	56.073	56.929	49.26	44.69
SBRY	DC-based Summaries	120.696	110.976	104.221	97.184	92.984
SDR	Interval-based Summaries	172.3	167.764	151.694	131.994	130.566
SDR	DC-based Summaries	297.15	270.292	254.304	223.664	214.032
SRP	Interval-based Summaries	18.982	20.542	18.182	16.581	15.156
SRP	DC-based Summaries	36.082	33.8	32.139	29.561	27.939
SVT	Interval-based Summaries	141.494	123.746	115.27	102.068	94.996
SVT	DC-based Summaries	239.394	217.07	197.776	182.74	165.736
SHP	Interval-based Summaries	67.04	54.769	50.576	45.492	40.705
SHP	DC-based Summaries	113.084	97.518	90.322	83.644	74.341
SN.	Interval-based Summaries	66.522	60.322	53.714	51.66	46.084
SN.	DC-based Summaries	116.48	108.81	99.632	92.228	84.806
SMIN	Interval-based Summaries	97.304	80.636	68.94	63.444	62.806
SMIN	DC-based Summaries	164.164	146.138	132.97	120.98	112.74
SSE	Interval-based Summaries	256.198	230.977	193.756	182.082	174.698
SSE	DC-based Summaries	417.464	391.034	351.572	332.763	314.163
STAN	Interval-based Summaries	134.572	119.37	113.63	93.06	80.608
STAN	DC-based Summaries	221.978	196.372	184.112	161.08	142.454
SL.	Interval-based Summaries	74.196	66.579	63.426	55.706	54.431
SL.	DC-based Summaries	116.889	107.88	100.896	89.126	83.158
TATE	Interval-based Summaries	120.022	111.574	101.532	86.742	86.648
TATE	DC-based Summaries	194.57	182.868	168.544	159.268	148.062
TSCO	Interval-based Summaries	35.986	29.708	24.544	22.338	21.003
TSCO	DC-based Summaries	60.888	49.286	41.78	38.766	35.046
TT.	Interval-based Summaries	19.629	17.095	16.522	15.713	13.229
TT.	DC-based Summaries	31.652	29.256	26.554	25.017	22.828
TLW	Interval-based Summaries	176.444	158.814	131.75	115.314	101.418
TLW	DC-based Summaries	296.67	266.794	229.174	204.204	181.416
ULVR	Interval-based Summaries	74.025	63.807	59.484	56.779	49.63
ULVR	DC-based Summaries	129.903	114.835	107.406	99.342	91.308
UU.	Interval-based Summaries	330.526	309.292	277.894	253.024	255.436
UU.	DC-based Summaries	497.59	467.73	443.866	417.172	387.062
VED	Interval-based Summaries	35.378	30.079	26.96	23.268	22.768
VED	DC-based Summaries	62.549	57.339	52.43	47.68	45.82
VOD	Interval-based Summaries	155.037	133.553	127.903	119.631	109.239
VOD	DC-based Summaries	271.851	241.423	231.993	210.859	196.755
WEIR	Interval-based Summaries	55.152	49.816	44.952	43.025	41.72
WEIR	DC-based Summaries	88.78	82.01	78.319	72.939	68.775

Table 10: Vertical Coastline Calculation 9

EPIC	Threshold	0.01	0.02	0.03	0.04	0.05
WTB	Interval-based Summaries	1257.506	1030.068	884.018	769.756	676.194
WTB	DC-based Summaries	1574.568	1389.82	1245.462	1100.746	1010.888
WOS	Interval-based Summaries	312.202	250.101	202.61	172.683	156.651
WOS	DC-based Summaries	390.183	348.691	308.105	278.45	259.16
WPP	Interval-based Summaries	425.138	322.498	259.92	220.054	180.714
WPP	DC-based Summaries	531.104	458.228	395.58	343.562	299.206
XTA	Interval-based Summaries	462.609	389.047	340.083	290.751	264.813
XTA	DC-based Summaries	533.009	495.16	454.097	413.125	381.603

Table 11: Vertical Coastline Calculation 10

EPIC	Threshold	0.06	0.07	0.08	0.09	0.1
WTB	Interval-based Summaries	620.824	541.948	477.24	454.94	419.838
WTB	DC-based Summaries	924.462	818.9	748.786	688.886	633.534
WOS	Interval-based Summaries	141.281	123.886	117.544	105.644	83.756
WOS	DC-based Summaries	236.108	218.024	203.394	185.028	171.294
WPP	Interval-based Summaries	155.026	141.874	129.548	111.75	112.49
WPP	DC-based Summaries	265.368	241.496	225.854	209.252	191.408
XTA	Interval-based Summaries	224.748	202.156	195.653	168.185	170.049
XTA	DC-based Summaries	351.268	322.927	311.238	296.217	276.644