Using Directional Changes for Searching Head and Shoulders Bottom Pattern

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Abstract—Head and Shoulder pattern is a well-known technical trading strategy. Exactly what clearly constitutes a Head and Shoulder pattern under time series is sometimes ambiguous. In this paper, we show how this pattern can be rigorously defined using Directional Change. Directional change is a new way to summarize price changes in the market. It records a transaction price only when the market has moved to an opposition direction by a significant degree, where the significance is defined by the observer. Unlike time series, it records data at irregular points. We will also show how Head and Shoulder patterns can be recognized using Directional Change. Presenting a precise definition of the Head and Shoulder pattern allows us to rigorously examine the effectiveness of a Head and Shoulder based trading strategy. We demonstrate that the Head and Shoulder patterns found under our definition supports a profitable trading strategy.

1 Introduction

Technical analysis is based on observed patterns in historical data. Such patterns are then used to develop trading rules. Robert and John [1] concluded that technical analysis is the science of recording, usually in graphic form. Brian [2] presented the view that a technical pattern behaved in a certain way in the past and therefore should continue to do so in the future. Hence he suggested that chart patterns could therefore help technical analysts to predict movement of share prices. In this paper, we propose a rigorous definition and assessment of the trading strategy, the Head and Shoulder Bottom Pattern (HSBP), which is a popular pattern used in the financial market for technical trading. Osler and Chang [3] gave a definition of the Head and Shoulder pattern, where they used the concept of a zigzag [4] to determine a head and shoulder top pattern, which is formed by five extreme points. Osler [5], specified the zigzag conditions which defined the Head and Shoulder pattern. However, applying the pattern to trading in U.S. equities markets, Osler concluded that it was not profitable. But, our work has led us to conclude differently, in that HSBP, can be regarded as a useful trading tool.

Thus, in our view, HSBP has been previously defined ambiguously, which has made it difficult for it to be clearly recognized by computers. So, to avoid such ambiguity, we have based our research on the framework of Directional Change (DC), which will be fully introduced in Section 2. In Section 3, we explain how HSBP can be recognized using conditions defined under DC. In Section 4, we go on to develop the use of a possible trading strategy based on the use of HSBP. In Section 5, we explain how we assess the performance of our HSBP-based trading strategy. And the findings of our research will be discussed in Section 6.

1.1 Introduction to HSBP

The concept of Head and Shoulders pattern has been introduced repeatedly by technical analysts, for example, in Robert and John and Arthur [1, 4]. It is one of the patterns that had attracted extensive attention from technical analysts, which has been much used. To further explain the significance of the pattern, the basic shape of the HSBP pattern has to be considered, which is shown in Figure 1. There are four main parts in this pattern: left shoulder (in which price moves from A to B to C), head (price movements from C to D to E) and right shoulder (price movements from E to F to G). The

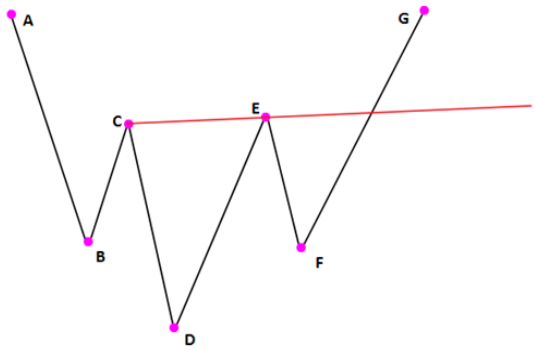

FIGURE 1. The HSBP pattern.

line that connects C and E is called the neckline. It is believed that whenever the price rises from F beyond the neckline (see Figure 1), the price will move to a relatively high point marked as G.

Although the HSBP has been used widely, researchers have varying views about the clear technical definition of HSBP. For example, how big is the rise from B to C, when it, would be called a shoulder? How low must D be, compared to B and F, before it would be

considered as a head? If E is much higher than C, or F above/below B, would the pattern still be considered to be a HSBP? If we want to scientifically assess the explanatory power and utility of this pattern, we must start with an unambiguous definition. In this paper, we attempt to provide such a rigorous and clear cut definition of HSBP; we propose to use seven points to define the HSBP. We identify the points of a HSBP, under the use of the Directional Change (DC) framework, and go on to define and develop the mathematical and logical relationship between these points.

1.1.1 Left shoulder

A left shoulder forms the first part of the HSBP pattern. Usually, there should be a period of decline in prices at the beginning of the left shoulder, as shown in the Figure 1, the line between points AB. The downward trend would stop, and then be followed by an upward movement (Here the first valley point B), and form the trough of the left shoulder. The upturn trajectory ends at point C, we call C a resistant point, as it indicates the end of the left shoulder.

1.1.2 Head

In order to form a HSBP pattern, a head shape should appear, following the left shoulder. The trajectory would turn downward after meeting the resistance point of C. The price would decline, and even exceed the first low point B, until it reaches a support point, i.e. point D in Figure 1, which is the lowest point the price would reach. Then the price trend would change direction again, upwards until it meets the second resistance point, which is E in Figure 1. If it cannot exceed the resistance point, the trajectory would be reversed (end of the Head).

1.1.3 Right Shoulder

A right shoulder would follow a Head event when the trend meets resistance, and turns down again. The downward trajectory would keep decreasing until it meets the support point F. The trend would start to turn up to point G.

1.1.4 Neckline

The two resistant points can be joined by a line, which is close to horizontal. This line is called the neckline. In Figure 1, the line that joints point C and E is therefore called the neckline.

2 Directional Change (DC) Definition

The research that is reported in this paper is based on the idea of Directional Change (DC) [6], details of which will be explained in this Section. Under the DC framework, peaks and troughs in the market are determined procedurally (this procedure will be explained in the next Section). They are equivalent to peaks and troughs in the zigzag [4].

When HSBC is defined verbally, what counts as an extreme point and what does not is subject to interpretation. Under DC, the extreme points are determined procedurally. A HSBC pattern can be defined by mathematical and logical relationships between the extreme points, as we shall explain below.

2.1 Directional Change (DC) events

Directional Change (DC) is an alternative to using time series for summarizing price changes in a financial market [7]. In time series, one samples a data point at each fixed time period. For example, one may summarize the market with the end of day transaction prices. By contrast, in DC, one does not sample the financial market at fixed time periods. Instead, one samples when price has changed by a 'significant' margin, where 'significance' is defined by the observer. The observer would define the minimum percentage price change that he/she would consider significant. When the price has dropped from the last peak by a predetermined threshold, the market is said to be in a downtrend. On the other hand, when the price has risen from the last trough by the same predetermined threshold, the market is said to be in an uptrend. Thus, under the DC framework, the market is seen to be alternating between uptrends and downtrends.

A downward (upward) DC event is confirmed when price has dropped (risen) from the last peak (trough) by the predetermined threshold. That means we only learn that a measurable state of DC has started in hindsight. The peaks, troughs and DC confirmation points are all signals, which together could tell us a lot about the market. DC provides a different perspective than time series to summarize price movements. This allows the researcher to see features that have not been observed under the use of time series. For example, James et al [8] observed scaling laws in foreign exchange markets through the use of DC. Edward et al [9] explained how stock markets could be studied with DC-based summaries. It introduced two new metrics for measuring volatility. Used in conjunction with time series, DC enhances our analysis of price movements in markets.

It is worth reiterating that different observers may choose different DC thresholds, which they consider to be significant. For instance, one who considers a change of 0.1% to be significant, would observe more directional changes than one who considers the threshold of 5% to be significant.

2.2 A more formal definition of DC

A formal definition of DC Events can be found in Edward [6]. This is briefly recapitulated here.

In an uptrend, one keeps record of the highest price Ph. When the current price P_c drops below P_h by a threshold θ or more, we say that it confirms a downtrend:

$$Pc \geq P_h * (1 + \theta) \quad (1)$$

The event of price change from P_h to P_c is called a

downtrend directional change event. It is important to note that before Pc is observed, one does not know that the downtrend has started.

Similarly, in a downtrend, one keeps record of the lowest price Pl. When the current price Pc rises above Pl by a threshold or more, it signals, in hindsight, an uptrend:

$$Pc \geq Pl * (1 + \theta) \quad (2)$$

Here Pc is called a DC confirmation point. P_h and P_l are called extreme points. We call the price changes from the DC confirmation point (Pc) to the next extreme point (P_h or P_l) an Overshoot (OS) Event.

To summarize: price movements are partitioned into alternating uptrend and downtrends. An uptrend comprises an upturn DC event and an OS event. A downtrend comprises a downturn DC event and an OS event. DC events are defined by the threshold θ. OS events are determined by price movements. The main concepts of DC summaries are summarised in Figure 2.

Figure 2 shows three complete trends: two uptrends and one downtrend. It shows the DC events, each of which records a price change by the threshold (the threshold is indicated by the vertical double arrow lines). For reference, time series, which samples data points at fixed intervals, is reported in the figure. It is

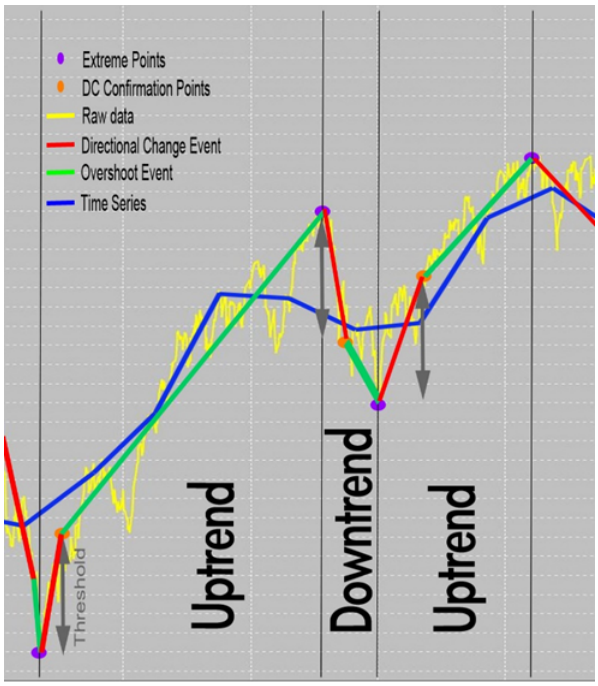

FIGURE 2. Key concepts in DC Summaries.

worth noting that the extreme points in DC are not captured by the time series.

The following contrast between time series and DC summaries is useful to consider: in time series, one determines the time intervals (normally presented in the x-axis in a graph) and observes price changes (y-axis) in each time interval. In the contrary, in DC summaries, one determines what a significant price change is (the y-axis), and let the data determine when to pick a sample (i.e. sampling at irregular intervals in the x-axis).

3 Basic Head and Shoulder Bottom Pattern under DC (HSBP+DC)[1]

The principle of HSBP+DC is utilizing the method of DC to search for HSBPs in the financial market. According to the definition of HSBP in Section 1, a HSBP comprises seven points, which we name A, B, C, D, E, F and G as shown in Figure 1. However, these seven points are the extreme points according to the theory of DC in Section 2. In addition, we would define the HSBP conditions that would enable us to search for the pattern. It is similar to the concept of the criteria mentioned by Osler and Chang [3]. But in this following section, we mainly focus on the mathematical and logical conditions to demonstrate what an HSBP is.

3.1 HSBP Conditions

The concepts of HSBP conditions are based on the definition of HSBP in section 1. To determine whether an HSBP is present, we look at a window of seven

C1: EP1 >= EP2, EP3, ..., EP6 (3)

C2: EP3, EP5 >= EP2, EP4, EP6 (4)

C3: $\left|\frac{EP2-EP6}{EP6}\right| \leq \theta \times T_{AL}$ (5)

C4: $\left|\frac{EP3-EP5}{EP5}\right| \leq \theta \times T_{AL}$ (6)

C5: $\left|\frac{AveEP35-AveEP26}{AveEP35-EP4}\right| \leq T_{max}$ (7)

C6: $\left|\frac{AveEP35-AveEP26}{AveEP35-EP4}\right| \geq T_{min}$ (8)

C7: EP 7 >= EP2, ..., EP6 (9)

consecutive extreme points. For convenience, we name these points EP1 to EP7, which corresponds to the points A to G in Figure 1. We define a HSBP-0 as a sequence of points which satisfy the following conditions:

Where:

- θ = threshold used by the analyst to generate DC summaries
- T_{AL}, T_{max} and T_{min} are tolerances given by the analyst
- AveEP35 in Condition 4 and Condition 5 is the mean of EP3 and EP5
- AveEP26 in Condition 4 and Condition 5 is the mean of EP2 and EP6

[1] We call our basic Head and Shoulder Bottom Pattern HSBP-0, to distinguish it from more complex patterns under development.

These conditions are explained below.

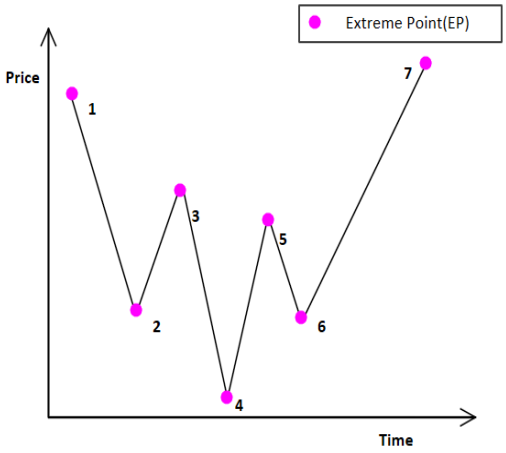

FIGURE 3. An example of a valid HSBP.

Condition 1:

The purpose of Condition 1 is to ensure that EP1 is higher than EP2 to EP6. It specifies that HSBP starts with a declining trend. Therefore, this condition corresponds to the definition of Left Shoulder as explained in Section 1 (See Figure 3).

Condition 2:

The purpose of Condition 2 is to confirm that there exists two resist points (EP3 and EP5), which are both higher than the support points (EP2, EP4 and EP6).

Condition 3:

The aim of Condition 3 is to limit the difference between EP2 and EP6 to an acceptable level. That level is specified by the analyst, in the form of a multiplier (T_{AL}) of the DC threshold (θ). Otherwise, if there is an excessive difference between the two points, it is hard to define an HSBP. Different threshold of magnitude would affect the final size of the HSBP. We define condition this way to ensure that the acceptable level is in relation to the DC threshold.

Condition 4:

The function of Condition 4 is similar with Condition 3, while the purpose is to limit the difference between EP3 and EP5 to an acceptable level.

Condition 5:

Figure 4 shows a non-HSBP. There EP2 and EP6 are too close to EP4, compared to their differences from EP3 and EP5. Condition 5 is defined to prevent that. There we introduce a ratio to compare the gap α (AveEP35 subtracting AveEP26) and the gap β (AveEP26 subtracting EP4). When the ratio α/β is 1, EP2, EP4 and EP6 are at the same horizontal level, which is invalid for a HSBP. Condition 5 specifies the maximum limit (parameterized by T_{max}) on the ratio α/β acceptable by the observer.

Condition 6:

Figure 5 shows another non-HSBP. There EP2 and EP6 are too close to EP3 and EP5, compared to their distance from EP4. Condition 6 is defined to prevent that. In Condition 5, we specified the maximum limit of

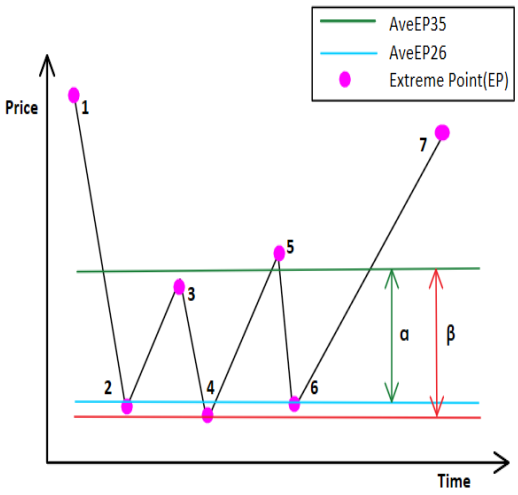

FIGURE 4. Example of a non-HSBP; Condition 5 is not met because the ratio between α and β is too big (α is the gap between AveEP35 and AveEP26; β is the gap between AveEP35 and EP4).

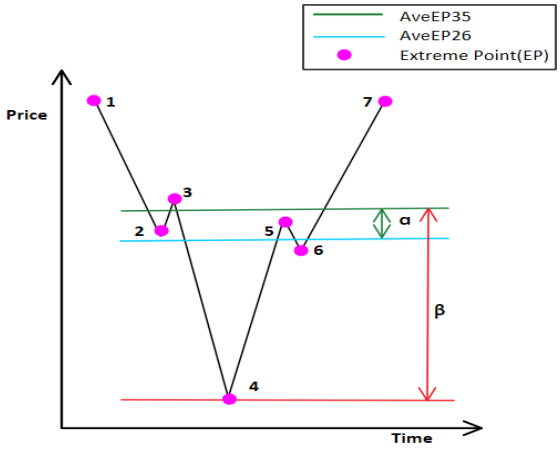

FIGURE 5. Example of a non-HSBP, Condition 6 is not met because the ratio between α and β is too small (α is the gap between AveEP35 and AveEP26; β is the gap between AveEP35 and EP4).

α/β. Here we specify the minimum limit (parameterized by T_{min}) of the ratio α/β.

Condition 7:

The function of this condition is similar with Condition 1. EP7 should be higher than EP2 to EP6.

4. HSBP TRADING STRATEGY

This section introduces a basic trading strategy, which we call Li-1. Li-1 attempts to exploit HSBP-0. The idea is to recognize buying opportunities when the price crosses the neckline [1]. Whenever sixth consecutive

extreme points satisfy the first six conditions of HSBP-0 (defined in Section 3.1), we have a potential HSBP, from which we look for a trading opportunity.

Li-1 comprises three trading rules, which are Open Position Rule, Selling for Profit Rule and Stop Loss Rule. The Open Position Rule in Li-1 only focuses on opening long positions. The Selling for Profit Rule and Stop Loss Rule are defined for closing positions.

Open (Long) Position Rule (OP_Rule)

In a potential HSBP, when the price rises to a point which crosses the neckline, Li-1 would consider it a trading opportunity to buy. Following is the buying rule for Li-1 (OP_Rule):

$$\frac{(BP.P - EP3.P)}{(BP.D - EP3.D)} \geq \frac{(EP5.P - EP3.P)}{(EP5.D - EP3.D)} \quad (10)$$

Where:

- BP.P is the buying price which is sought by Li-1;
- BP.D is the number of days counted from the first day of the series in the study to the buying point, BP.
- EP5.P and EP3.P are the prices at these two points.
- EP5.D and EP3.D are the number of days from the first day of the series to EP5 and EP3, respectively.

Note, A point in the market is defined by its date and price. We use X.D and X.P to denote the date and price of point X.

(EP5.P – EP3.P)/(EP5.D – EP3.D) measures the angle of the slope from EP3 to EP5, which is the neckline. We call the current price BP (for Buying Price) if it is above the extension of the line from EP3 to EP5, as shown in Figure 6. The condition above supports a mathematical condition to recognize BP. In mathematical terms, Δy is confirmed by BP.P subtracting EP3.P, meanwhile Δx can be found by BP.D subtracting EP3.D. Therefore, the slope would be obtained by Δy dividing Δx.

Selling for Profit Rule (SP_Rule) is a rule in Li-1 which defines when to sell for profit. Traders need to set a target return rate (T_p) in Li-1. Here T_p should reflect the scale of the threshold used in DC; e.g. T_p could be $0.5*\theta$ or $0.75*\theta$, where θ is the threshold used in the DC summary. Once a long position is opened (by OP_Rule), Li-1 would track the price movement and compute the potential profit. Should the potential profit reach T_p, Li-1 would fire the SP_Rule.

$$Tp \leq \frac{CP.P - BP.P}{BP.P} \quad (11)$$

Where:

- BP.P is buying price;
- CP.P is the current price.

Stop Loss Rule (SL_Rule) is a rule in Li-1 which is designed to limit losses should the price drop below a certain condition. As EP2 and EP6 are the support points, we use the mean of these two points to define a **SL point (SLP)**. In other words, the long position will be closed when price reaches or fall below the SL point (SL_Rule).

$$SLP.P = \frac{EP2.P + EP6.P}{2} \geq CP.P \quad (12)$$

Where:

- EP2.P is the price at extreme point 2;
- EP6.P is the price at extreme point 6.
- CP.P is the current price.

To summarize, Li-1 is a trading strategy based on the HSBP-0 conditions. It opens a long position whenever the current price crosses (from below) the neckline. It takes the profit when the price rises above a target return rate (T_p) defined by the investor. It closes its position to stop loss when the price drops to the stop loss price level (SLP).

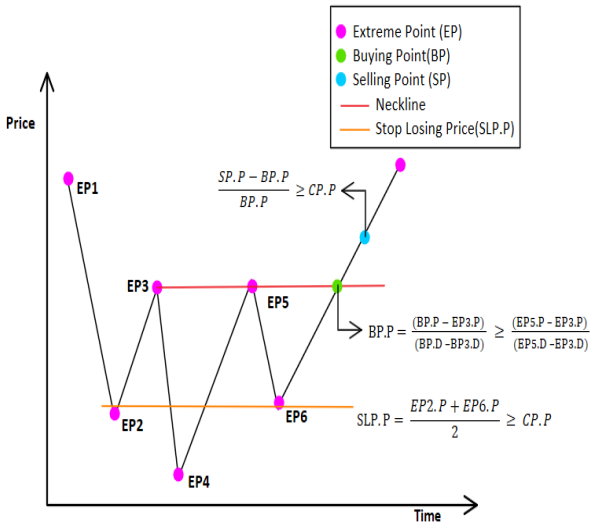

FIGURE 6. HSBP with trading strategy -- here CP is current price; SP_Rule and SL_Rule would be as candidate rules after opening a long position, and once current the price satisfies one of a rules SP_Rule or SL_Rule, Li-1 would close the position.

5. EXPERIMENT: TRADING UNDER HSBP

In this section, we will explain how to evaluate the potential of HSBP for use in trading. We start with Directional Changes summarized from the raw data, as described in Edward et al [10]. Here we will explain how to detect potential HSBPs. Then, we will explain how potential HSBPs could be turned into buy and sell actions. With all the above implemented in a computer programme, which we call SH-1, we will evaluate the use and performance of an HSBP trading strategy.

5.1: DCC: From raw data to Directional Changes

In this approach, we summarize raw data in the form of the use of DC. As explained in Section 2, DC is one way to summarize price changes. It is different from time series, which records data points at fixed time intervals. DC records (in hindsight) extreme points at which the market has changed direction. Thus, DC is data-driven, which means it samples data at irregular time points (hence completely different from time series). Those extreme points recorded by DC form the EPs in HSBP that we introduced in Section 3 (as shown in Figure 6).

Directional Change Computing (DCC) is a module in SH-1 that implemented the concept reported by Edward et al [10]. DCC takes time series data as input. It generates a sequence of extreme points – each extreme point indicates the date and price in the input time series. DCC outputs a sequence of extreme points.

5.2: HSBP_Detector: From DCs to Potential HSBP

This section will explain how to search for HSBPs. HSBP_Detector is the second module in SH-1. It receives as input EPs from DCC. Then we detect potential HSBPs in the following way: we scan EPs in a moving window to detect consecutive EPs that satisfy the first six HSBP conditions (described in Section 3). This will be elaborated in the following subsections.

5.2.1 Scanning for potential HSBPs

In Section 4, we have mentioned that when searching for buying opportunities in HSBP, we ignored EP 7, and focused instead on the current price after EP6 is found. According to the definition in Section 1, six EPs do not confirm a HSBP. However, we declare that six EPs determine a **Potential HSBP**.

In addition, a number of EPs will be summarized after the establishment of DC computing primary data. Therefore, the method below gives an idea for searching for where potential HSBPs are likely to be found.

We use a moving window to scan each sequence of six EPs. Figure 7 illustrates the mechanism; for example, the programme will continuously move from window

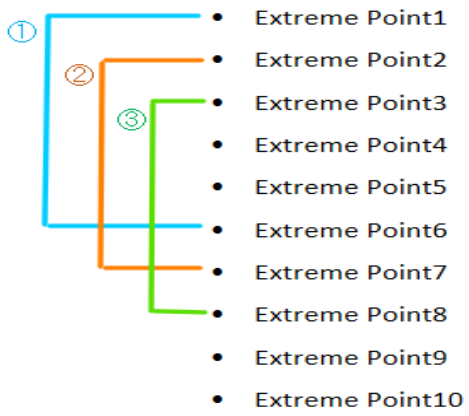

FIGURE 7. The Mechanism of scanning potential HSBPs.

① to window ②, ③ … and so on. Each of these windows will be checked to see if it exhibits a potential HSBP condition, as explained in the next section.

5.2.2 Determining Potential HSBP

Given a window of six EPs, we check whether they exhibit a potential HSBP. In Section 3.1, we explained that seven mathematical conditions determine a HSBP. But in this experiment, we only need the first six conditions to establish a potential HSBP.

Figure 8 summarizes the conditions for establishing a potential HSBP. In this example, HSBP_Detector initially scans window ① which comprises extreme points 1 to 6. It checks whether the first 6 HSBP

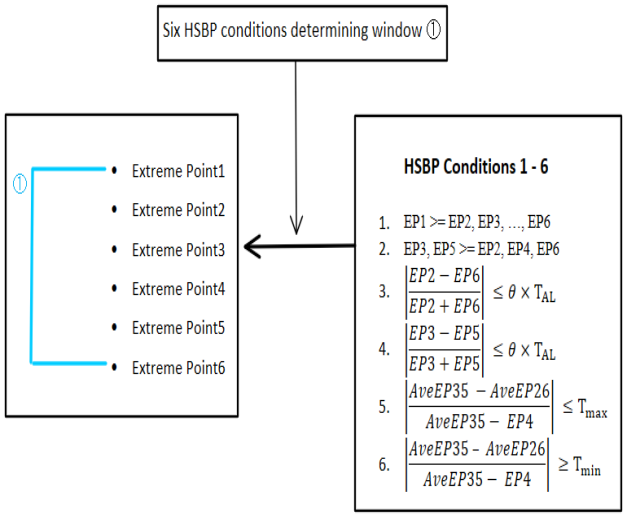

Figure 8. The mechanism of determining a potential HSBP by 6 conditions, an example.

conditions are satisfied.

5.3: Trading with potential HSBP

In Section 4, we introduced Li-1, a strategy that trades with a potential HSBP. Li-1 defines the conditions under which it opens, and conditions under which it closes positions for profit and for stopping loss. In this experiment, LI-1 is a module that implements Li-1. As a module in SH-1, LI-1 follows the second module HSBP_Detector. Once potential HSBPs are found, LI-1 will be executed. For each potential HSBP, LI-1 will look for conditions.

In this experiment, there is a slight difference with the introduction in Section 4 that Li-1 will read the raw data in which the data was used in DCC (These raw data will be as parameters to three the trading rules of Li-1). In detail, Li-1 initially requires the data of the first potential HSBP, which comprises six EP.Ps (EP.Ps are the prices at these six extreme points) with relative EP.Ds, and then checks the next daily closing price of the EP.P six from raw data. According to **Open Position Rule** (OP_Rule), we suppose that the next closing price is the BP.P. A position will be opened when the parameters of BP.P and related BP.D satisfy the OP_Rule (When the BP.P rises to a point which crosses the neckline, Li-1 would consider it a trading opportunity to buy, See figure 6). LI-1 would discard a potential HSBP without trading once the price drops below the stop

loss price (as defined in Section 4).

Once a position is opened, **Selling for Profit Rule** (SP_Rule) and **Stop Loss Rule** (SL_Rule) will be as candidate rules to fire. If the current price, CP (defined in Section 4), finally rises and satisfies the condition of the SP_Rule, the position will be closed for profit. However, when the price drops below the buying price (BP.P), and eventually drops to, or below the **Stop Loss Price** (SLP.P), the position will also be closed to stop loss. LI-1 terminates after every potential HSBP is examined.

5.4: Evaluating SH-1 through back-testing

SH-1 is a program created by the first author. The programme comprises three modules, DC Computing (DCC), HSBP_Detector and LI-1. In order to execute SH-1, an analyst has to set four parameters, which are Thresholds (θ, for computing DCs), T_{AL} (for conditions C3 and C4), Tmax and Tmin (for conditions C5 and C6). In addition, an input file is required to provide raw data.

The DCC module takes in raw data and produces extreme points(EP). HSBP_Detector receives these EPs, and then determines potential HSBPs. The module comprises two components, which is the method of "Moving window" and HSBP conditions. Thus, it would scan six sequential EPs, and meanwhile invokes conditions in order to determine potential HSBPs. Trading Actions are executed by Li-1, which was introduced in Section 4 and Section 5.3, it obtains all the data of Potential HSBPs and raw data (Daily closing prices with dates). This data is used for three trading rules, eventually all trading actions will be recorded

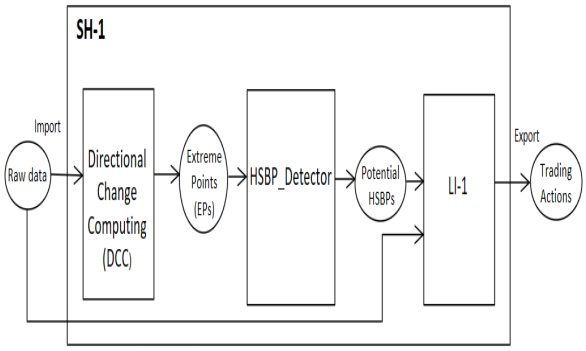

Figure 9. SH-1 flow diagram.

(See figure 9).

In this back testing, SH-1 firstly randomly selected 100 stocks from NASDAQ as original data (all original data are imported to SH-1 in the form of excel files). Then, DCC calculates this data, and works out EPs, moreover all EPs would save into the database. After that, HSBP_Detector invokes all EPs from the database, and scans each six sequential EPs through the method of a moving window. In the meantime, the six conditions would determine whether the six EPs satisfy the criteria to form a potential HSBP. Finally, LI-1 would acquire these potential HSBPs, and tracks the daily closing prices. Once a potential HSBP satisfies buying condition (**Open Position Rule),** Li-1 executes the buying decision. When the profits reach to T_p, **SP_Rule** would be performed. However, if the price drops down to the **Stop Loss Price**, Li-1 would execute the rule of SL.

Note, this back-test used a set of thresholds from 5% to 30% for getting various EPs of magnitude. Therefore, SH-1 first computes all original data by the threshold of 5%, and then increases the threshold each time by 1% until it reaches 30% (See Table 1).

Table 1: All data and parameters used in this experiment

Original data	Historical daily closing prices from 100 stocks in NASDAQ
Thresholds	5% - 30% (increase 1% each time)
T_{AL}	0.5
T_{max}	0.65
T_{min}	0.3
T_p	5%

5.5 Results

Li-1 executed 67 trading decisions. Among these, 49 decisions made a profit and 18 decisions executed Stop Loss (All the trading results are shown in Appendix). Table 2 summarized the performance of SH-1 from the above experiment. Besides this, we have shown one of the trading records to present the shape of an HSBP, with its trading record, on a historical graph in figure 10.

We summarized the necessary data for this test, in Table 2. To get the median return and average return, we collect the rate of returns from the 67 trading records. For this test, all the 49 profitable trades were taken at the same target return rate T_p 5%, while the other 18 trades were automatically closed by Li-1 at the stop loss level. In addition, we simulated a 1 million dollars account to run through the data of the 67 records. In the end, the financial capital was $2,050,085, and the average holding days for each trade was 32.12 days.

Table 2: Summarizes for trading results of Li-1

Median return	5.00%
Average return	1.29%
Standard deviation (risk)	0.064176
Maximum drawdown	-16.62%
Number of taking profit trades	49
Number of stop loss trades	18
Ave Holding days	32.12
Initial Capital	USD 1,000,000
Final Capital	USD 2,050,085

Figure 10 is an example of a HSBP pattern found by Li-1

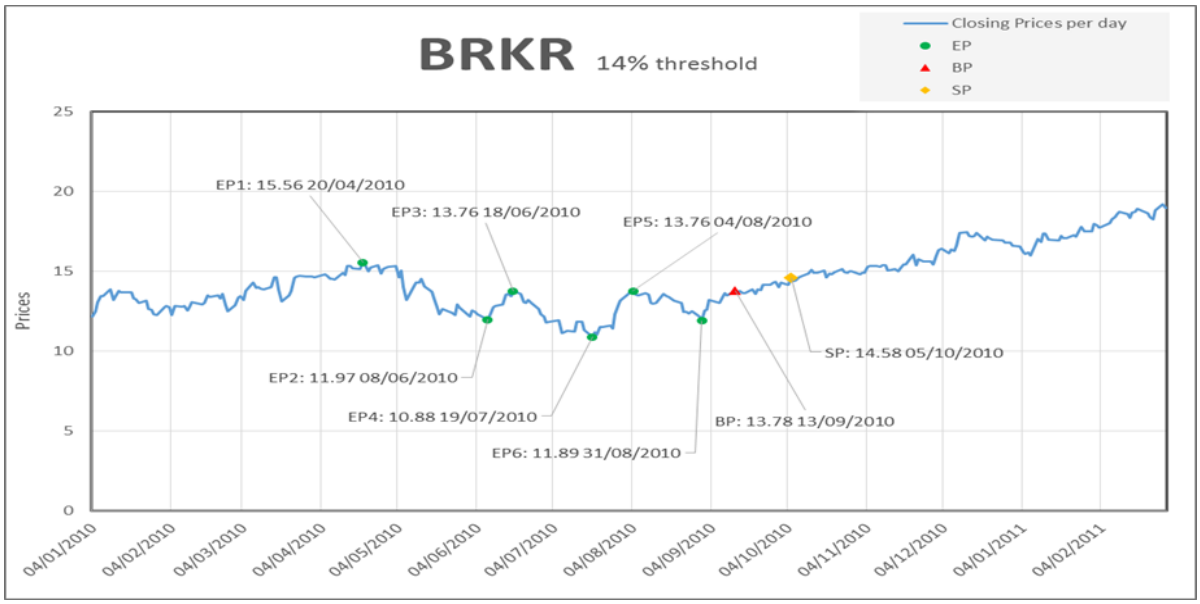

Figure 10. A HSBP found by SH-1, an example.

with the threshold 14%. In this graph, Li-1 found a potential HSBP after identifying six EPs. At the price of 13.78 on 13/09/2010, Li-1 executed its Open Position Rule (explained in Section 4) and opened its long position. When the price went up to 14.58, Li-1 executed its Selling for Profit Rule and closed its position on 05/10/2010, taking a profit of 0.80.

6 Discussion

6.1. DC is useful for defining HSBP

In the experiment conducted above, we can observe that Directional Change is a constructive method for working with HSBP trading strategy, because it focuses on the price change, rather than the specific time series. It realizes the summary of the price changes without being affected by the time series, which is beneficial for analysis of the price trend in the financial market. Because of the unstable volatility of prices in the stock market, it is difficult to confirm a significant trend change at a specific time point. HSBP is a noticeable example of the use of trend trading using technical analysis, it comprises six up and down trends, and seven extreme points. The HSBP conditions are also an important element in this experiment, which rigorously defines six mathematical conditions as discussed in Section 3, in order to determine a potential HSBP. In addition, Li-1 is a valuable trading strategy for HSBP, as discussed in Section 4. The main strategy of Li-1 is to open long positions, which was the idea brought forward by Robert and John [1].

6.2. Different thresholds produce different number of potential HSBPs.

Some 67 trading actions were formed in this back test, 49 of which are have made a profit and 18 trades were closed at stop loss points. Within these trades, we observe that 48 trading actions were produced within the thresholds, from 5% to 10%, and 11 trading actions were from the thresholds between 11% and 15%. Three trading actions were performed with the thresholds over 15%. In addition, entire 18 Stop Loss decisions are from the threshold from 5% to 15%. According to the trading records, most of the trading actions were executed within the thresholds between 5% and 10%, which is an important point for helping us in setting a reasonable range of thresholds in the further experiments. Based on these results, we conclude that some thresholds produce more potential HSBPs than others. We believe that we should concentrate on thresholds that produce more potential HSBPs in future studies, and we assume that the different markets exhibit different volatilities, which would need adjustments in the threshold, relative to the specific stock market.

6.3. Uncommon HSBPs

Although we have a clear definition of HSBP, some of the HSBPs that we found exhibit a variable pattern within the parameters of HSBP. Without a formal definition, these patterns can be difficult to clearly interpret as being HSBPs. According to the results, 23 uncommon HSBPs exhibited asymmetrical features. These uncommon patterns are classified by two extreme situations. The first is the asymmetries of the two shoulders in a HSBP, so that one shoulder was formed through longer periods (days) than another. Another

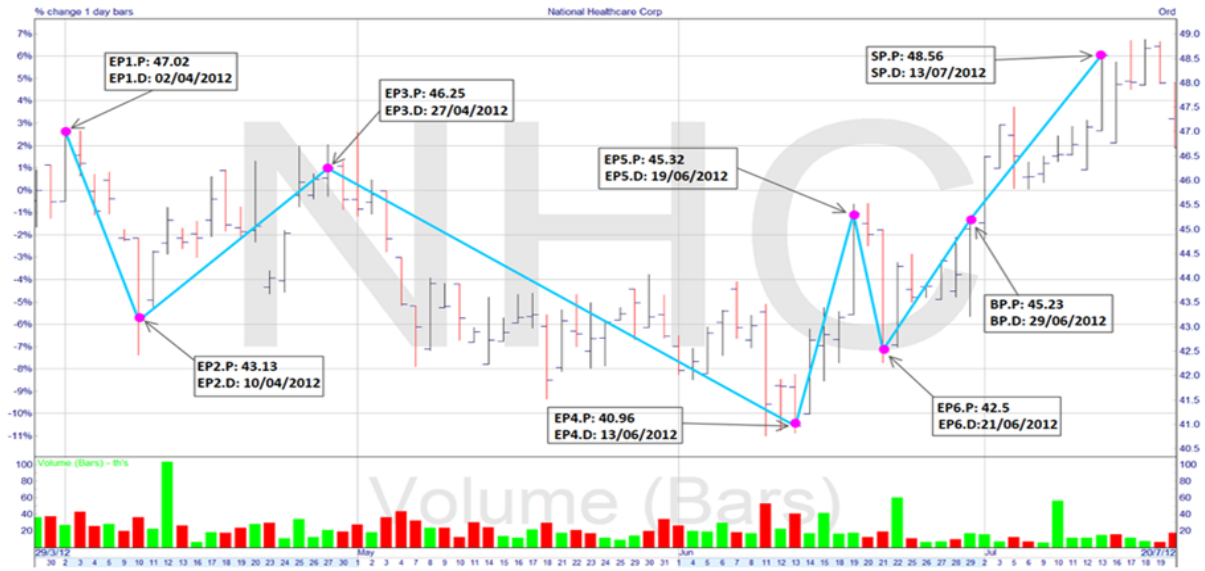

Figure 11. An extreme HSBP, here left shoulder and right shoulder are asymmetric – the left shoulder took 18 trading days and the right shoulder took 8 trading days. Besides, the periods from EP3 to EP4 is much longer then the periods from EP4 to EP5.

situation is the asymmetry between two trends around EP4 (See figure 11, where the period from EP3 to EP4 is much longer then the period from EP4 to EP5). Although, these uncommon HSBPs deviate from the classical HSBP, the test indicated that 17 out of these 23 uncommon HSBPs led to profitable trades. However, those 23 trading records of uncommon HSBPs alerted us to whether those uncommon HSBPs could be confirmed as HSBPs, and in our view, they conform to be considered as HSBPs. This is because they follow the seven EPs patterns, and hence they are confirmed by our definition.

Levy [11] suggested that the core of forming head and shoulders is caused by the support and resistance conditions which have implications for future price behaviour. Furthermore, the price movements are simply noise in the gap between the support and resistance level. Therefore, it is likely that some HSBPs will exhibit asymmetric, or extremely asymmetric, patterns.

Therefore, we have focused on establishing the foundations of the conditions for HSBP, as it is important to establish the basic structure of common HSBPs, and then on this basis, any further improvements in HSBP conditions could be adapted to recognise any specific situations of HSBPs.

6.4 Compared to previous work

The closest work to ours is the work of Osler and Chang [3]. They used zigzag summaries in technical analysis to evaluate the Head and Shoulder trading strategy [4]. Like DC, zigzag indicator summarizes up trends and down trends. It retrospectively records each peak and trough according to a given threshold, given the same threshold, it finds the same extreme points as DC. A peak is defined by a decline of a constant percentage threshold after a local maximum price. A trough is defined by an ascent of a constant percentage threshold after a local minimum price [4].

Osler and Chang [3] tested Head and Shoulder top pattern (HSTP) in FX market, and the core technical method of searching HSTP is utilizing zigzag plus five (HSTP) conditions rules. They used five conditions to define HSTP. Those conditions are introduced in words, without the use of mathematical equations or algorithms [3]. In addition, they concluded that compared with other technical indicators HSTP is not a significant pattern that could support a considerably profitable trading strategy.

By contrast this paper focuses on the rigorous definition of conditions for the purpose of analysing the effective search for HSBPs. Our definition is unambiguous given a threshold and parameters in the conditions. The purpose of introducing the trading strategy (Section 4) is to test the usefulness of the HSBP pattern defined under our conditions in the stock markets.

7. Conclusion

Head and Shoulders is a widely used pattern in technical trading. As the pattern was normally described in words in the literature, what constitutes a Head and Shoulder pattern was sometimes ambiguous. In this paper we have defined the Head and Shoulder Bottom Pattern (HSBP) under the Directional Change (DC) framework. In a DC summary, extreme points are determined procedurally for given thresholds (which is chosen by the observer) – in other words, there is no ambiguity in what is an extreme point. We have defined a HSBP pattern in terms of mathematical and logical relationship on those extreme points. In other words, as soon as a threshold is given and the conditions met,

HSBPs can be recognized without ambiguity.

We have defined a trading strategy, Li-1, in which trading decisions are based on the HSBP defined. We have examined the performance of Li-1 with data from 100 stocks from NASDAQ. Results suggest that Li-1 can be a profitable trading strategy based on SH-1.

This paper has made two contributions to the field. First, it proposed a rigorous definition of HSBP. Our approach supported alternative Head and Shoulder patterns to be defined (by changing the conditions in Section 3.1). Secondly, it demonstrated that the HSBP that we have defined supports a profitable trading strategy.

HSBP-0 only defined the basic conditions of a Head and Shoulder Bottom Pattern. The conditions in HSBP can be refined to increase the reliability of trading. The approach that we have taken, namely to use DC to identify extreme points and to specify the relationship between them, is a general, scientific approach. It can be applied to other patterns and trading strategies. These will be left to future work.

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APPENDIX—Table 3[2]

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.05	ELTK	1	3.19	11/27/2013	0.14	BRKR	1	15.56	4/20/2010
		2	2.3	12/6/2013			2	11.97	6/8/2010
		3	2.44	12/9/2013			3	13.76	6/18/2010
		4	2.2	12/17/2013			4	10.88	7/19/2010
		5	2.4	12/19/2013			5	13.76	8/4/2010
		6	2.27	12/20/2013			6	11.89	8/31/2010
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	2.428	12/23/2013	2.66	1/7/2014		13.78	9/13/2010	14.58	10/5/2010
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.06	ELTK	1	3.19	11/27/2013	0.08	AMZN	1	96.3	1/2/2008
		2	2.3	12/6/2013			2	68.5	2/6/2008
		3	2.44	12/9/2013			3	77.7	2/13/2008
		4	2.2	12/17/2013			4	62.4	3/3/2008
		5	2.5	12/24/2013			5	75.9	3/24/2008
		6	2.29	12/30/2013			6	69.8	3/28/2008
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SP.P	SP.D
	2.66	1/7/2014	2.29	1/24/2014		76.7	4/1/2008	81	4/23/2008
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.27	ELTK	1	5.35	3/28/2005	0.13	AMZN	1	96.3	1/2/2008
		2	2.15	5/2/2005			2	68.5	2/6/2008
		3	3.12	5/17/2005			3	77.7	2/13/2008
		4	1.69	6/7/2005			4	62.4	3/3/2008
		5	2.82	7/14/2005			5	82	4/28/2008
		6	2.21	7/19/2005			6	72.4	5/9/2008
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SLP.P	SLP.D
	3.06	7/26/2005	3.28	8/8/2005		84.5	6/5/2008	68.5	7/11/2008
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.1	PLKI	1	17.03	7/5/2011	0.18	AMZN	1	88.1	8/11/2008
		2	13.51	8/8/2011			2	48.7	10/15/2008
		3	14.94	8/9/2011			3	58.5	11/4/2008
		4	11.62	10/3/2011			4	35	11/20/2008
		5	15.69	11/18/2011			5	57.4	1/6/2009
		6	13.93	11/25/2011			6	48.4	1/27/2009
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	15.95	11/28/2011	16.85	1/31/2012		58.8	1/30/2009	63.6	2/3/2009
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.07	NSSC	1	10.707	5/5/2006	0.06	FCBC	1	32.83	3/1/2005
		2	9.167	5/22/2006			2	27.5	3/24/2005
		3	10.147	6/2/2006			3	29.32	4/7/2005
		4	8.32	6/13/2006			4	26.31	4/15/2005
		5	10.15	7/7/2006			5	29.01	5/9/2005
		6	9.45	7/14/2006			6	27.32	5/13/2005
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SP.P	SP.D
	10.45	7/26/2006	8.17	9/11/2006		29.33	5/26/2005	31.18	6/15/2005
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.07	NSSC	1	3.13	3/30/2012	0.09	FCBC	1	40.05	3/21/2007
		2	2.89	4/11/2012			2	29.16	6/12/2007
		3	3.11	5/11/2012			3	31.91	6/19/2007
		4	2.67	5/18/2012			4	25.52	7/24/2007
		5	3.06	6/7/2012			5	32.17	8/8/2007
		6	2.82	6/22/2012			6	29.1	8/14/2007
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	2.98	7/19/2012	3.15	8/24/2012		33.12	8/16/2007	35.21	8/22/2007
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.14	NSSC	1	3	6/24/2011	0.07	EEI	1	14.65	4/17/2009
		2	2.4	8/11/2011			2	13.43	5/1/2009
		3	2.78	9/13/2011			3	14.43	5/6/2009
		4	1.97	11/17/2011			4	11.85	5/13/2009
		5	2.9	1/31/2012			5	14.22	5/29/2009
		6	2.44	2/15/2012			6	13.24	6/3/2009
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SLP.P	SLP.D
	2.95	3/15/2012	3.13	3/26/2012		14.22	6/4/2009	13.01	6/22/2009

[2] In Table 3, the first left column indicates the threshold which Li-1 found a potential pattern at that threshold. On the left second column is the name of the share. The columns, from third to fifth show 6 EPs with their relative prices and dates. The last two rows record the results of trading decisions. Among these, BP.P is Buying Price; BP.D is Buying Point Date; SP.P is the Selling Price; SP.D is Selling Point Date; SL.P means Stop Loss Price; SL.D means Stop Loss Date.

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.07	EEI	1	11.088	1/28/2015	0.07	CIA	1	7.049	4/16/2004
		2	9.75	2/5/2015			2	6.49	4/20/2004
		3	10.47	2/19/2015			3	7.014	4/22/2004
		4	8.58	3/18/2015			4	5.267	5/13/2004
		5	10.71	6/26/2015			5	6.813	6/16/2004
		6	9.91	7/13/2015			6	6.332	6/22/2004
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	11.18	7/24/2015	11.75	9/2/2015		6.935	6/25/2004	7.284	6/29/2004
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.12	EEI	1	12.35	2/10/2014	0.13	CHEF	1	18.5	8/3/2011
		2	9.328	4/17/2014			2	12.98	9/6/2011
		3	11.14	5/27/2014			3	15.37	9/16/2011
		4	8.35	11/25/2014			4	11.7	10/3/2011
		5	10.75	1/14/2015			5	14.82	10/28/2011
		6	9.19	1/26/2015			6	13.09	11/29/2011
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SP.P	SP.D
	11.088	1/28/2015	9.18	3/16/2015		14.47	12/2/2011	15.84	12/6/2011
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.3	DMND	1	40.56	12/9/2011	0.07	CETV	1	26.36	10/13/2010
		2	26.92	12/20/2011			2	18.75	11/29/2010
		3	38.04	1/27/2012			3	21.2	1/3/2011
		4	12.98	11/21/2012			4	17.96	2/22/2011
		5	34.93	3/31/2014			5	20.68	3/3/2011
		6	24.58	1/30/2015			6	19.31	3/16/2011
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	33.33	6/25/2015	35.65	10/26/2015		20.7	3/21/2011	21.79	4/7/2011
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.15	DGI	1	31.61	9/11/2014	0.09	CE	1	47.01	8/31/2011
		2	26.79	10/13/2014			2	35.6	9/22/2011
		3	31.02	10/29/2014			3	39.41	9/27/2011
		4	24.07	12/11/2014			4	31.49	10/4/2011
		5	31.34	12/26/2014			5	40.85	10/14/2011
		6	26.89	1/30/2015			6	37.27	10/19/2011
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	33.25	2/27/2015	35.18	3/3/2015		42.17	10/24/2011	44.76	10/27/2011
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.08	DDD	1	5.306	5/12/2010	0.06	HCP	1	28.85	10/26/2004
		2	4.232	5/24/2010			2	26.18	11/19/2004
		3	4.756	5/27/2010			3	28.31	12/22/2004
		4	3.919	6/8/2010			4	23.45	4/6/2005
		5	4.636	6/18/2010			5	28.43	6/1/2005
		6	4.282	6/24/2010			6	26.6	6/27/2005
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SLP.P	SLP.D
	4.639	6/28/2010	4.185	6/30/2010		28.58	8/2/2005	25.39	8/8/2005
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.09	CLC	1	34.19	12/8/2008	0.06	HCP	1	40.6	4/28/2011
		2	30.03	12/24/2008			2	35.03	6/10/2011
		3	33.68	1/5/2009			3	38.2	7/8/2011
		4	27.49	1/14/2009			4	28.77	8/8/2011
		5	32.43	1/15/2009			5	37.28	8/31/2011
		6	29.73	1/26/2009			6	34.94	9/9/2011
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SLP.P	SLP.D
	31.84	1/28/2009	29.79	2/19/2009		37.42	9/16/2011	34.86	9/28/2011
Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.11	CLC	1	33.68	1/5/2009	0.09	HCP	1	40.6	4/28/2011
		2	27.49	1/14/2009			2	35.03	6/10/2011
		3	32.82	2/6/2009			3	38.2	7/8/2011
		4	23.41	3/20/2009			4	28.77	8/8/2011
		5	32.51	5/8/2009			5	37.55	9/20/2011
		6	27.89	6/22/2009			6	33.83	10/3/2011
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	32.48	7/23/2009	35.25	1/14/2010		37.97	21/10/2011 0	40.19	10/28/2011

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.1	HCP	1	43.95	9/18/2013		0.07	FNGN	1	22.84	4/30/2012
		2	39.05	10/4/2013				2	19.85	5/17/2012
		3	43.05	10/25/2013				3	21.58	6/6/2012
		4	35.66	12/12/2013				4	18.16	8/1/2012
		5	43.73	9/5/2014				5	22.03	8/7/2012
		6	39.47	9/24/2014				6	20.53	8/23/2012
	BP.P	BP.D	SP.P	SP.D			BP.P	BP.D	SP.P	SP.D
	43.97	10/31/2014	47.14	1/7/2015			22.4	9/13/2012	23.72	9/21/2012

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.06	HBHC	1	33.31	10/24/2011		0.05	FMC	1	12.5	12/1/2004
		2	29.08	11/1/2011				2	11.67	12/9/2004
		3	31.59	11/8/2011				3	12.255	12/28/2004
		4	27.58	11/25/2011				4	10.855	1/24/2005
		5	31.43	12/9/2011				5	12.07	2/2/2005
		6	29.5	12/14/2011				6	11.463	2/10/2005
	BP.P	BP.D	SP.P	SP.D			BP.P	BP.D	SP.P	SP.D
	31.43	12/16/2011	33.8	1/9/2012			12.063	2/15/2005	12.893	3/1/2005

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.06	HBHC	1	35.63	11/12/2014		0.06	FMC	1	15.65	7/22/2005
		2	29.11	12/15/2014				2	12.68	10/12/2005
		3	31.09	12/29/2014				3	13.655	11/1/2005
		4	25.19	1/21/2015				4	12.3	11/18/2005
		5	31.07	3/12/2015				5	13.852	12/2/2005
		6	28.7	3/19/2015				6	12.963	12/19/2005
	BP.P	BP.D	SLP.P	SLP.D			BP.P	BP.D	SP.P	SP.D
	31.07	4/15/2015	28.3	4/27/2015			14.403	2/1/2006	15.305	2/15/2006

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.05	GLDC	1	4.75	8/22/2005		0.07	FMC	1	46.43	7/21/2011
		2	4.4	8/23/2005				2	35.145	8/8/2011
		3	4.713	8/30/2005				3	37.78	8/15/2011
		4	4.2	9/6/2005				4	33.375	8/22/2011
		5	4.678	9/9/2005				5	37.965	8/31/2011
		6	4.43	9/12/2005				6	35.21	9/12/2011
	BP.P	BP.D	SLP.P	SLP.D			BP.P	BP.D	SLP.P	SLP.D
	4.67	9/13/2005	4.4	9/19/2005			38.22	9/15/2011	34.36	9/22/2011

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.07	GILD	1	24.185	4/6/2009		0.08	INT	1	40.91	3/30/2011
		2	21.865	4/21/2009				2	36.44	4/19/2011
		3	23.8	4/28/2009				3	39.58	4/29/2011
		4	20.72	5/27/2009				4	33.11	6/13/2011
		5	23.94	6/25/2009				5	39.02	7/21/2011
		6	22.22	7/10/2009				6	35.54	8/2/2011
	BP.P	BP.D	SLP.P	SLP.D			BP.P	BP.D	SLP.P	SLP.D
	24.275	7/21/2009	21.915	10/23/2009			39.49	8/3/2011	35.6	8/5/2011

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.05	GABC	1	24.89	9/24/2012		0.1	INT	1	19.23	12/30/2008
		2	21.75	10/19/2012				2	15.41	1/12/2009
		3	22.86	11/1/2012				3	17.57	1/28/2009
		4	19.98	11/15/2012				4	13.075	3/9/2009
		5	22.73	12/20/2012				5	17.045	3/26/2009
		6	21.24	12/28/2012				6	15.43	4/1/2009
	BP.P	BP.D	SLP.P	SLP.D			BP.P	BP.D	SP.P	SP.D
	23.02	1/2/2013	21.29	4/9/2013			17.005	4/9/2009	18.005	4/16/2009

Threshold	Name of Share	EP	Price	Date		Threshold	Name of Share	EP	Price	Date
0.09	GABC	1	13.6	9/12/2008		0.14	IILG	1	17.63	2/14/2011
		2	11.66	9/30/2008				2	12.55	6/13/2011
		3	12.9	10/1/2008				3	14.42	7/7/2011
		4	10.4	3/18/2009				4	10.4	8/8/2011
		5	12.98	4/17/2009				5	14.75	11/7/2011
		6	11.85	4/24/2009				6	12.35	11/25/2011
	BP.P	BP.D	SP.P	SP.D			BP.P	BP.D	SP.P	SP.D
	13.9	5/20/2009	14.99	6/24/2009			15.51	3/9/2012	16.42	3/14/2012

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.06	HOT	1	62.82	6/1/2006	0.09	MBLX	1	159.48	8/9/2007
		2	55.55	6/14/2006			2	130.8	8/15/2007
		3	61.51	7/3/2006			3	142.62	8/17/2007
		4	50.19	8/11/2006			4	123.9	9/13/2007
		5	60.52	9/20/2006			5	148.38	9/27/2007
		6	56.79	9/28/2006			6	134.52	10/11/2007
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	60.64	10/5/2006	65.61	11/21/2006		162.78	10/12/2007	173.34	10/31/2007

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.16	LBY	1	16.81	7/7/2011	0.17	LIOX	1	7.3	3/5/2014
		2	11.36	8/22/2011			2	5.11	5/8/2014
		3	13.31	8/29/2011			3	6.16	7/3/2014
		4	10.08	10/5/2011			4	4.13	10/10/2014
		5	13.27	10/27/2011			5	5.85	12/29/2014
		6	11.25	11/25/2011			6	4.98	1/30/2015
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	14.03	1/19/2012	14.89	1/25/2012		5.81	2/17/2015	6.2	6/8/2015

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.1	LBIX	1	15.7	3/12/2007	0.05	OII	1	16.89	1/6/2009
		2	13.25	3/21/2007			2	14.29	1/14/2009
		3	14.95	3/23/2007			3	15.025	1/15/2009
		4	11.788	4/11/2007			4	14.055	1/20/2009
		5	14.8	4/16/2007			5	15.36	1/21/2009
		6	13.45	4/23/2007			6	14.62	1/22/2009
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	15.65	5/7/2007	16.7	5/25/2007		15.88	1/23/2009	17.525	1/28/2009

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.18	KOP	1	45.73	4/29/2011	0.05	OBCI	1	4.49	10/17/2014
		2	28.39	8/22/2011			2	3.47	10/29/2014
		3	33.55	8/30/2011			3	3.73	10/31/2014
		4	24.13	9/23/2011			4	3.3	11/10/2014
		5	36.23	11/4/2011			5	3.68	11/14/2014
		6	28.45	11/25/2011			6	3.46	11/17/2014
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SLP.P	SLP.D
	39.79	2/1/2012	41.83	1/22/2013		3.64	11/28/2014	3.45	12/2/2014

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.05	KBAL	1	12.7	7/30/2015	0.3	OBCI	1	1.95	2/9/2004
		2	10.3	8/25/2015			2	1.21	7/23/2004
		3	10.93	8/31/2015			3	1.59	8/4/2004
		4	9.31	9/29/2015			4	0.93	12/1/2004
		5	10.7	10/12/2015			5	1.79	1/10/2005
		6	10.16	10/21/2015			6	1.23	1/11/2005
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SP.P	SP.D
	10.79	10/23/2015	11.64	11/3/2015		1.97	1/28/2005	2.37	1/31/2005

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.05	MGLN	1	38.46	10/18/2004	0.06	NHC	1	47.02	4/2/2012
		2	33.7	11/11/2004			2	43.13	4/10/2012
		3	35.66	11/29/2004			3	46.25	4/27/2012
		4	31.39	12/10/2004			4	40.96	6/13/2012
		5	35.565	12/21/2004			5	45.32	6/19/2012
		6	33.41	1/5/2005			6	42.5	6/21/2012
	BP.P	BP.D	SLP.P	SLP.D		BP.P	BP.D	SP.P	SP.D
	36.21	1/14/2005	33.39	2/15/2005		45.23	6/29/2012	48.56	7/13/2012

Threshold	Name of Share	EP	Price	Date	Threshold	Name of Share	EP	Price	Date
0.05	MGLN	1	47.79	6/13/2007	0.06	NCT	1	16.886	2/7/2007
		2	39.57	8/9/2007			2	13.642	3/5/2007
		3	42.22	8/16/2007			3	14.676	3/9/2007
		4	38.29	9/17/2007			4	12.885	3/14/2007
		5	42.62	10/12/2007			5	14.747	3/26/2007
		6	40.55	11/1/2007			6	13.854	4/2/2007
	BP.P	BP.D	SP.P	SP.D		BP.P	BP.D	SLP.P	SLP.D
	45.48	11/2/2007	47.95	12/20/2007		15.075	4/18/2007	13.622	6/25/2007

Threshold	Name of Share	EP	Price	Date
0.05	PII	1	78.62	5/24/2012
		2	71.97	6/1/2012
		3	75.59	6/6/2012
		4	68.63	6/13/2012
		5	75.03	7/19/2012
		6	70.98	7/24/2012
	BP.P	BP.D	SP.P	SP.D
	75.21	7/26/2012	80.76	9/10/2012

Threshold	Name of Share	EP	Price	Date
0.07	SBR	1	56.01	7/9/2012
		2	44	11/15/2012
		3	47.93	11/29/2012
		4	39.37	12/28/2012
		5	48.28	2/8/2013
		6	45	2/21/2013
	BP.P	BP.D	SP.P	SP.D
	49.2	4/5/2013	51.71	5/10/2013

Threshold	Name of Share	EP	Price	Date
0.07	PII	1	22.435	2/1/2008
		2	20.025	2/19/2008
		3	21.635	2/26/2008
		4	18.415	3/10/2008
		5	21.845	4/8/2008
		6	20.345	4/11/2008
	BP.P	BP.D	SP.P	SP.D
	22.215	4/16/2008	23.595	5/1/2008

Threshold	Name of Share	EP	Price	Date
0.15	SBR	1	44.76	11/26/2008
		2	37.6	12/22/2008
		3	44.475	1/6/2009
		4	28.83	3/9/2009
		5	42.92	5/14/2009
		6	37.27	5/21/2009
	BP.P	BP.D	SP.P	SP.D
	42.76	6/1/2009	44.92	6/11/2009

Threshold	Name of Share	EP	Price	Date
0.1	PPP	1	2.92	5/29/2012
		2	2.59	6/13/2012
		3	2.88	6/20/2012
		4	2.37	6/28/2012
		5	2.82	7/3/2012
		6	2.54	7/12/2012
	BP.P	BP.D	SP.P	SP.D
	2.79	7/18/2012	2.93	7/19/2012

Threshold	Name of Share	EP	Price	Date
0.19	RMBS	1	46.8	4/18/2006
		2	14.79	7/27/2006
		3	17.63	7/31/2006
		4	10.26	8/11/2006
		5	19.23	10/13/2006
		6	15.99	11/9/2006
	BP.P	BP.D	SP.P	SP.D
	21.72	11/16/2006	23.1	11/24/2006

Threshold	Name of Share	EP	Price	Date
0.11	PPP	1	4.835	1/20/2015
		2	3.18	2/20/2015
		3	3.55	2/27/2015
		4	2.9	3/10/2015
		5	3.72	3/23/2015
		6	3.35	3/30/2015
	BP.P	BP.D	SP.P	SP.D
	3.85	4/6/2015	4.05	5/12/2015

Threshold	Name of Share	EP	Price	Date
0.05	YUM	1	78.3	11/22/2013
		2	71.6	12/12/2013
		3	76.56	1/7/2014
		4	66.16	2/3/2014
		5	77.4	3/10/2014
		6	73.2	3/27/2014
	BP.P	BP.D	SP.P	SP.D
	78.65	6/3/2014	83.23	7/9/2014

Threshold	Name of Share	EP	Price	Date
0.14	RIC	1	9.85	4/28/2011
		2	7.15	5/5/2011
		3	8.55	5/27/2011
		4	6.46	6/16/2011
		5	8.65	8/3/2011
		6	7.53	8/8/2011
	BP.P	BP.D	SP.P	SP.D
	8.75	8/10/2011	9.61	8/15/2011

Threshold	Name of Share	EP	Price	Date
0.08	YUM	1	32.87	1/5/2009
		2	27.57	2/2/2009
		3	30.29	2/9/2009
		4	23.47	3/9/2009
		5	29.86	3/26/2009
		6	27.48	3/31/2009
	BP.P	BP.D	SP.P	SP.D
	30.03	4/2/2009	31.82	4/16/2009

Threshold	Name of Share	EP	Price	Date
0.06	UNM	1	23.82	6/15/2010
		2	21.26	7/2/2010
		3	23.24	8/2/2010
		4	19.45	8/26/2010
		5	22.92	11/4/2010
		6	21.48	11/23/2010
	BP.P	BP.D	SP.P	SP.D
	23.26	12/8/2010	24.58	12/22/2010