A measure of relative intensity between two DC sequences

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Abstract

Directional Change (DC) is an alternative to time series [1]. A market over a period of time is recorded by a DC sequence: a sequence of extreme points that delimit alternating uptrends and downtrends in the market. In this paper, we introduce a measure for relating the volatility of two DC sequences. We call this measure "relative intensity".

1. Directional change

Price changes in a financial market is traditionally recorded as a time series. Guillaume et al. [1] introduced the concept of directional change (DC) for sampling data. Instead of sampling prices at fixed time intervals, DC samples a data point at significant price reversals, where significance is defined by a **threshold**, which is defined by the observer. In other words, sampling in DC is data-driven. Given a threshold, a DC sequence partitions a market in a given period into alternating uptrends and downtrends. These trends are delimited by **extreme points** (**EP**), at which directions change. A local peak is an extreme point from which price drops beyond the threshold. A local trough is an extreme point from which price rises beyond the threshold. Given a threshold, a **DC sequence** records the extreme points in a market over the specified period.

A formal definition of DC can be found in Tsang [2]. Tsang [3] and Tsang et al [4] explained how volatility can be measured under DC. One volatility measure is the number of directional changes (NDC) over a period of time, which we use in this paper. Given a period of time, more NDCs observed means higher volatility. This paper introduces a new measure called relative intensity. The scope of this paper is to add to the DC vocabulary. Application of this new measure is left to future papers.

2. A measure of relative intensity in the price changes under DC approach (DCRI)

In this paper, we introduce the concept of **DC relative intensity** (**DCRI**). It is a concept based on the DC approach. It relates the EPs between two markets. We shall explain this concept in two examples.

Figure 1 shows the extreme points of two markets, ordered chronologically. In Figure 1, the triangles and diamonds denote the EPs from market A and market B, respectively. Figure 1 presents the case that every confirmed EP in market A is followed by an EP in market B. In this case, we say that the relative intensity between these two DC sequences remained constant over time.

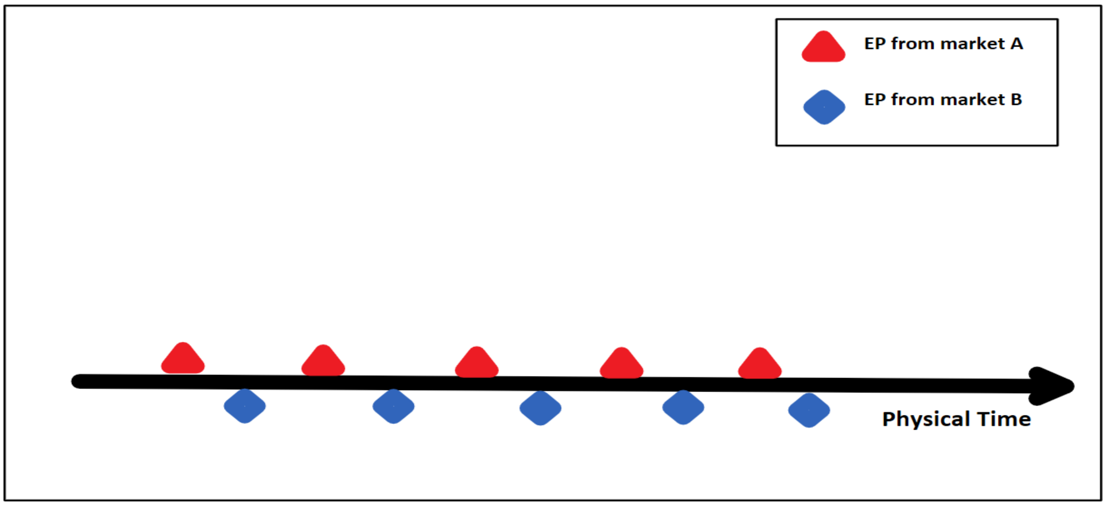

Figure 1. An example of one observed EP from market A is followed by another EP from market B.

If the price changes of market A is relatively more intense than market B, then we could observe more NDCs in market A than NDCs we observe in market B under the same threshold; Figure 2 shows one such example. In Figure 2, we observe two EPs in market A before we observe the first EP in market B. That was followed by three more EPs in market A, before the next two EPs in market B emerged. As we observe more EPs in market A than market B, we say that market A is relatively more intense than market B.

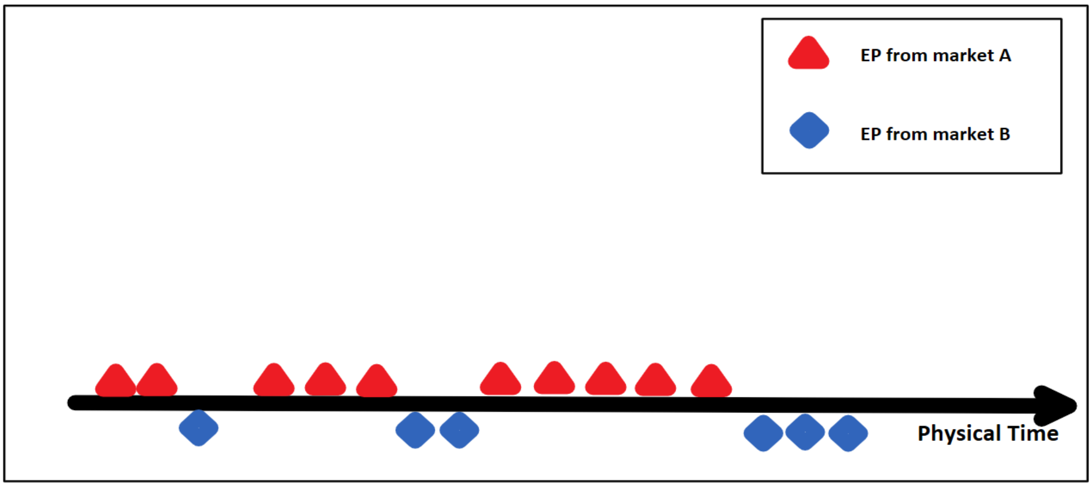

Figure 2. The *NDC* value of market A is greater than the *NDC* value of market B.

2.1 Formal definitions

Definition 1: A **DC extreme point**, **EP**, in a DC sequence is a double tuple which contains a time point EP_t with a price EP_p:

$$EP = (EP_t, EP_p). \qquad \blacksquare$$

A DC sequence S is a finite sequence of extreme points ordered by time EP_t. A DC sequence may be written as:

$$\mathrm{S} = (X_1, X_2, \dots, X_k, \dots, X_n).$$

where X_k is an extreme point, and $\mathrm{k} \in [1, \mathrm{n}]$.

As introduced above, $\forall X_i \in \mathrm{S}$, (we abuse the notation and treat S as a set here) we have a double tuple:

$$X_i = (x_{i.t}, x_{i.p}).$$

where:

- X_i is an extreme point EP.
- $x_{i.t}$ and $x_{i.p}$ denote the time and the price of X_i.
- All X_i's are ordered by time: $\forall\, x_{i.t}\colon (x_{i.t} - x_{i-1.t}) \geq 0$.

Let S_A^θ and S_B^θ be two DC sequences which were observed under the same threshold θ:

$$S_A^\theta = (A_1, A_2, \dots, A_k, \dots, A_{mA}).$$
$$S_B^\theta = (B_1, B_2, \dots, B_k, \dots, B_{nB}).$$

where:

- A_k and B_k are extreme points of two markets A and B.
- The subscripts m and n denote the total number of extreme points from market A and B respectively.

We measure the relative intensity between two markets under the same physical timeline. The examples of figure 1 and figure 2 illustrate the basic idea of this measure. Hence, we define a DC relative sequence to list the all EPs from both sequences S_A^θ and S_B^θ in chronological order. Based on this sequence we will compare the number of the occurrences between the adjacent EPs but from the different markets to observe the specific pattern.

Definition 2: A DC relative sequence (DCRS) is a sequence whose elements contain all the EPs of two DC sequences in chronological order. ■

Let ⊕ be an operator named 'combine'. Given two DC sequences S_A^θ and S_B^θ, the operation $S_A^\theta \oplus S_B^\theta$ combines all the elements of S_A^θ and S_B^θ ordered by time. That is,

$$DCRS_{S_A^\theta, S_B^\theta} = S_A^\theta \oplus S_B^\theta = (X_1, X_2, \dots, X_i, \dots, X_{mA+nB}).$$

where:

- $X_i \in \{A_1, A_2, \dots, A_k, \dots, A_{mA}, B_1, B_2, \dots, B_k, \dots, B_{nB}\}$
- $X_{i.t} = X_{i+1.t}$ (if two X_i's observed at the same time, we put the EP in S_A^θ before the EP in S_B^θ).

For example, the DCRS in Figure 1 above is:

DCRS1 = (A1, B1, A2, B2, A3, B3, A4, B4, A5, B5)

The DCRS in Figure 2 above is:

DCRS2 = (A1, A2, B1, A3, A4, A5, B2, B3, A6, A7, A8, A9, A10, B4, B5, B6)

Let us introduce an "identify symbol" I; Given an element X_i in $S_A^\theta \oplus S_B^\theta$, $I(X_i) = A$ if X_i belongs to S_A^θ, or $I(X_i) = B$ if X_i belongs to S_B^θ.

Definition 3: **A DCRS Group** (**DCRSG**) is generated by a division processes Γ, which divides a DCRS into z sub-sequences according to the identity of the elements (EPs). For any sub-sequence in DCRSG, all the elements only either belong to S_A^θ or S_B^θ. Let Y^θ the DCRSG of $S_A^\theta \oplus S_B^\theta$ be defined by

$$Y^\theta = DCRSG^{\ \theta} = \Gamma(S_A^\theta \oplus S_B^\theta) = (Y_1, Y_2, \dots, Y_j, \dots, Y_z) \qquad (\mathrm{z} \le mA + nB).$$

Note Y_j is a sub-sequence of DCRS $S_A^\theta \oplus S_B^\theta$. If all Y_j only contain one EP, $\mathrm{z} = mA + nB$ (see Figure 1). Otherwise, at least one Y_j contains the number of EPs, so $\mathrm{z} \le mA + nB$ (see Figure 2).

Every Y_j is a sub-sequence of the DCRS $S_A^\theta \oplus S_B^\theta$:

$$\forall\ \mathrm{j}{:}\ Y_j = (X_{j,i}, X_{j,i+1}, \dots, Y_{j,i+k}) \qquad (1 \le \mathrm{z} \le mA + nB, 0 \le \mathrm{k} \le mA + nB).$$

■

Based on the above definition, if all the elements of the sub-sequence Y_j belong to S_A^θ, then elements of the two adjacent sub-sequences Y_{j-1} and Y_{j+1} would both belong to S_B^θ:

$$\underbrace{\dots = I\left(X_{j-1,i+k}\right)}_{Y_{j-1}} \neq \underbrace{I\left(X_{j,i}\right) = I\left(X_{j,i+1}\right) = \dots = I\left(X_{j,i+k}\right)}_{Y_j} \neq \underbrace{I\left(X_{j+1,1}\right) = \dots}_{Y_{j+1}}$$

In a DCRSG, we count the value of *NDC*s for every Y_j, we have

$$N(DCRSG) = (N(Y_1^A), N(Y_2^B), \dots, N\left(Y_{j-1}^A\right), N\left(Y_j^B\right), \dots, N(Y_{z-1}^A), N(Y_z^B)).$$

where:

- N() is the counting function which counts the number of EPs for every sub-sequence Y_j.
- *A* and *B* are the identity of the two sequences S_A^θ and S_B^θ, respectively. The identity of the first element of DCRSG depends on which sub-sequence is first observed. The DCRSG above, we suppose the first observed sub-sequence is from S_A^θ, and vice versa. In the rest of this section, we keep this assumption.

For example, the DCRSG for DCRS1 above is:

DCRSG1 = ((A1), (B1), (A2), (B2), (A3), (B3), (A4), (B4), (A5), (B5))

N(DCRSG1) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)

The DCRSG for DCRS2 above is:

DCRSG2 = ((A1, A2), (B1), (A3, A4, A5), (B2, B3), (A6, A7, A8, A9, A10), (B4, B5, B6))

N(DCRSG2) = (2, 1, 3, 2, 5, 3)

Given $DCRSG$, we measure a difference of in the number of EPs between each adjacent sub-sequence and declare that DCRSGD denotes the result:

$$DCRSGD = ((N(Y_2^B) - N(Y_1^A)), (N(Y_4^B) - N(Y_3^A)), \dots, (N(Y_z^B) - N(Y_{z-1}^A))).$$

Let D denote any element of $DCRSGD$, so

$$D \ = N(Y_{2q}^B) - N(Y_{2q-1}^A) \qquad (1 \le q \le \tfrac{z}{2})$$

Definition 4: **A DCRSGD sequence** is defined by the difference of the adjacent sub-sequences of $DCRSG$, That is

$$DCRSGD = (D_1, D_2, \dots, D_{w-1}, D_w) \qquad (w = \tfrac{z}{2})$$

■

Here, for simplicity without loss of generality, we assume that (a) the DCRSG sequence starts with EPs in sequence A; and (b) the value of z is even. If DCRSG starts with EPs in DC sequence B, the leading group will be discarded. If the DCRSG ends with EPs in DC sequence A, the final group will be discarded.

For example, the DCRSGD for DCRSG1 above is:

DCRSGD1 = (0, 0, 0, 0, 0)

The DCRSGD for DCRSG2 above is:

DCRSGD2 = (-1, -1, -2)

$DCRSGD$ is a sequence which comprises the observed elements Ds chronologically under a given dataset. In a $DCRSGD$, the element D scale the level of the relative intensity, which is the difference of the NDC values between a pair of adjacent sub-sequences. According to the definition of $DCRSGD$ and the example in figure 3, we confirm the value of D can be extremely large when there are many EPs from S_A^θ versus a small number of EPs from S_B^θ in two adjacent sub-sequences, and vice versa. So, $D \in (-\infty, \infty)$, and there are three main conclusions for $DCRSGD$:

i. If $D_q \to 0$, the relative intensity is at the same level.
ii. If $D_q \gg 0$, the relative intensity of S_A^θ is lower than S_B^θ.
iii. If $D_q \ll 0$, the relative intensity of S_A^θ is higher than S_B^θ.

2.3 A limited range for the value of D

Based on the definition of $DCRSGD$, the interval for any D ranges from negative infinity to positive infinity, that is $D \in (-\infty, \infty)$. In practice, it is useless to determine an element of the absolute extremum from a DCRSGD because the value for any D can reach up to infinity. Hence, we normalize the value of each D, and the normalized D^* to limit the value range:

$$D^* = \frac{N(Y_{2q}^B) - N(Y_{2q-1}^A)}{N(Y_{2q}^B) + N(Y_{2q-1}^A)} \qquad (1 \le e \le \frac{z}{2})$$

A DCRSGD* is a measure of Relative Intensity between two DC sequences.

Definition 5: **A DCRSGD* sequence** is defined by the normalized difference of the adjacent sub-sequences of $DCCSG$:

*DCRSGD** = (D^*_1, D^*_2, …, D^*_{w-1}, D^*_w) $(w = \frac{z}{2})$

■

All D* values range between 1 and -1; that is $D^* \in (-1, 1)$.

For example, the DCRSGD* for DCRSG1 above is:

DCRSGD*1 = (0, 0, 0, 0, 0)

The DCRSGD* for DCRSG2 above is:

DCRSGD*2 = (-1/3, -1/5, -2/8) = (-0.333, -0.2, -0.25)

D* is a measure of **Relative Intensity** between the two adjacent DC sequence in a DCRSG. The range of D* values scale the magnitudes of the relative intensity:

i. If $D^* \to 0$, the relative intensity is at the same level.
ii. If $D^* \to 1$, the relative intensity of S_A^θ is enormously lower than S_B^θ.
iii. If $D^* \to -1$, the relative intensity of S_A^θ is enormously higher than S_B^θ.

3. Conclusion

This report presents a new measure, DCRI, for relative intensity between two market-periods. The concept of DCRI is built under the framework of directional change (DC). DCRI focusses on the number of extreme points (EPs) observed in the two market-periods. It counts the number of consecutive EPs in one market before EPs in the other market appears. The differences in the number of EPs in markets A and B form a sequence called DCRSGD (which stands for Directional Change Relative Sequence Group Differences, see Definition 4) or DCRSGD* (where differences are normalized to a range between -1 and 1, see Definition 5). We propose DCRSGD or DCRSGD* as a measure of relative intensity between two markets. In the comparison of two adjacent sub-sequences of DCRSG in market-periods A and B, a high D* value indicates higher volatility in market B compared to market A within those sub-sequences.

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